THEORIES OF AETHER AND ELECTRICITY.

DUBLIN UNIVERSITY PRESS SERIES.

A HISTORY

OF THE

EORIES OF AETHER AND ELECTRICITY

FROM THE AGE OF DESCARTES TO THE CLOSE OF THE NINETEENTH CENTURY.

BY

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MEMORANDUM ON NOTATION.

VECTORS are denoted by letters in clarendon type, as **E**.

The three components of a vector **E** are denoted by E_x, E_y, E_z; and the magnitude of the vector is denoted by E, so that

$$E^2 = E_x^2 + E_y^2 + E_z^2.$$

The *vector product* of two vectors **E** and **H**, which is denoted by [**E** . **H**], is the vector whose components are $(E_yH_z - E_zH_y, E_zH_x - E_xH_z, E_xH_y - E_yH_x)$. Its direction is at right angles to the direction of **E** and **H**, and its magnitude is represented by twice the area of the triangle formed by them.

The *scalar product* of **E** and **H** is $E_xH_x + E_yH_y + E_zH_z$. It is denoted by (**E** . **H**).

The quantity $\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$ is denoted by div **E**.

The vector whose components are

$$\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \quad \frac{\partial E_x}{\partial y} - \frac{\partial E_z}{\partial x}, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial z}\right)$$

is denoted by curl **E**.

If V denote a scalar quantity, the vector whose components are $\left(-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z}\right)$ is denoted by grad V.

The symbol ∇ is used to denote the vector operator whose components are $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$.

Differentiation with respect to the time is frequently indicated by a dot placed over the symbol of the variable which is differentiated.

THEORIES OF AETHER AND ELECTRICITY.

CHAPTER I.

THE THEORY OF THE AETHER IN THE SEVENTEENTH CENTURY.

THE observation of the heavens, which has been pursued continually from the earliest ages, revealed to the ancients the regularity of the planetary motions, and gave rise to the conception of a universal order. Modern research, building on this foundation, has shown how intimate is the connexion between the different celestial bodies. They are formed of the same kind of matter; they are similar in origin and history; and across the vast spaces which divide them they hold perpetual intercourse.

Until the seventeenth century the only influence which was known to be capable of passing from star to star was that of light. Newton added to this the force of gravity; and it is now recognized that the power of communicating across vacuous regions is possessed also by the electric and magnetic attractions.

It is thus erroneous to regard the heavenly bodies as isolated in vacant space; around and between them is an incessant conveyance and transformation of energy. To the vehicle of this activity the name *aether* has been given.

The aether is the solitary tenant of the universe, save for that infinitesimal fraction of space which is occupied by ordinary matter. Hence arises a problem which has long engaged attention, and is not yet completely solved: What relation subsists between the medium which fills the interstellar void and the condensations of matter that are scattered throughout it?

The history of this problem may be traced back continuously to the earlier half of the seventeenth century. It first emerged clearly in that reconstruction of ideas regarding the physical universe which was effected by René Descartes.

Descartes was born in 1596, the son of Joachim Descartes, Counsellor to the Parliament of Brittany. As a young man he followed the profession of arms, and served in the campaigns of Maurice of Nassau, and the Emperor; but his twenty-fourth year brought a profound mental crisis, apparently not unlike those which have been recorded of many religious leaders; and he resolved to devote himself thenceforward to the study of philosophy.

The age which preceded the birth of Descartes, and that in which he lived, were marked by events which greatly altered the prevalent conceptions of the world. The discovery of America, the circumnavigation of the globe by Drake, the overthrow of the Ptolemaic system of astronomy, and the invention of the telescope, all helped to loosen the old foundations and to make plain the need for a new structure. It was this that Descartes set himself to erect. His aim was the most ambitious that can be conceived; it was nothing less than to create from the beginning a complete system of human knowledge.

Of such a system the basis must necessarily be metaphysical; and this part of Descartes' work is that by which he is most widely known. But his efforts were also largely devoted to the mechanical explanation of nature, which indeed he regarded as one of the chief ends of Philosophy.*

The general character of his writings may be illustrated by a comparison with those of his most celebrated contemporary.† Bacon clearly defined the end to be sought for, and laid down the method by which it was to be attained; then, recognizing that to discover all the laws of nature is a task beyond the powers of one man or one generation, he left to posterity the work of filling in the framework which he had designed. Descartes, on the other hand, desired to leave as little as possible for his successors to do; his was a theory of the universe, worked out as far as possible in every detail. It is, however, impossible to derive such a theory inductively unless there are at hand sufficient observational data on which to base the induction; and as such data were not available in the age of Descartes, he was compelled to deduce phenomena from preconceived principles and causes, after the fashion of the older philosophers. To the inherent weakness of this method may be traced the errors that at last brought his scheme to ruin.

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* Of the works which bear on our present subject, the *Dioptrique* and the *Météores* were published at Leyden in 1638, and the *Principia Philosophiae* at Amsterdam in 1644, six years before the death of its author.

† The principal philosophical works of Bacon were written about eighteen years before those of Descartes.

The contrast between the systems of Bacon and Descartes is not unlike that between the Roman republic and the empire of Alexander. In the one case we have a career of aggrandizement pursued with patience for centuries; in the other a growth of fungus-like rapidity, a speedy dissolution, and an immense influence long exerted by the disunited fragments. The grandeur of Descartes' plan, and the boldness of its execution, stimulated scientific thought to a degree before unparalleled; and it was largely from its ruins that later philosophers constructed those more valid theories which have endured to our own time.

Descartes regarded the world as an immense machine, operating by the motion and pressure of matter. "Give me matter and motion," he cried, "and I will construct the universe." A peculiarity which distinguished his system from that which afterwards sprang from its decay was the rejection of all forms of action at a distance; he assumed that force cannot be communicated except by actual pressure or impact. By this assumption he was compelled to provide an explicit mechanism in order to account for each of the known forces of nature—a task evidently much more difficult than that which lies before those who are willing to admit action at a distance as an ultimate property of matter.

Since the sun interacts with the planets, in sending them light and heat and influencing their motions, it followed from Descartes' principle that interplanetary space must be a plenum, occupied by matter imperceptible to the touch but capable of serving as the vehicle of force and light. This conclusion in turn determined the view which he adopted on the all-important question of the nature of matter.

Matter, in the Cartesian philosophy, is characterized not by impenetrability, or by any quality recognizable by the senses, but simply by extension; extension constitutes matter, and matter constitutes space. The basis of all things is a primitive, elementary, unique type of matter, boundless in extent and infinitely divisible. In the process of evolution of the universe three distinct forms of this matter have originated, corresponding respectively to the luminous matter of the sun, the transparent matter of interplanetary space, and the dense, opaque matter of the earth. "The first is constituted by what has been scraped off the other particles of matter when they were rounded; it moves with so much velocity that when it meets other bodies the force of its agitation causes it to be broken and divided by them into a heap of small particles that are of such a figure as to fill exactly all the holes and small interstices which they find around these bodies. The next type includes most of the rest of matter; its particles are spherical, and are very small compared with the bodies we see on the earth; but nevertheless they have a finite magnitude, so that they can be divided into others yet smaller. There exists in addition a third type exemplified by some kinds of matter—namely, those which, on account of their size and figure, cannot be so easily moved as the preceding. I will endeavour to show that all the bodies of the visible world are composed of these three forms of matter, as of three distinct elements; in fact, that the sun and the fixed stars are formed of the first of these elements, the interplanetary spaces of the second, and the earth, with the planets and comets, of the third. For, seeing that the sun and the fixed stars emit light, the heavens transmit it, and the earth, the planets, and the comets reflect it, it appears to me that there is ground for using these three qualities of luminosity, transparence, and opacity, in order to distinguish the three elements of the visible world.*

According to Descartes' theory, the sun is the centre of an immense vortex formed of the first or subtlest kind of matter.† The vehicle of light in interplanetary space is matter of the second kind or element, composed of a closely packed assemblage of globules whose size is intermediate between that of the vortex-matter and that of ponderable matter. The globules of the second element, and all the matter of the first element, are constantly straining away from the centres around which they turn, owing to the centrifugal force of the vortices;‡ so that the globules are pressed in contact with each other, and tend to move outwards, although they do not actually so move.§ It is the transmission of this pressure which constitutes light; the action of light therefore extends on all sides round the sun and fixed stars, and travels instantaneously to any distance.‖ In the *Dioptrique*,¶ vision is compared to the perception of the presence of objects which a blind man obtains by the use of his stick; the transmission of pressure along the stick from the object to the hand being analogous to the transmission of pressure from a luminous object to the eye by the second kind of matter.

Descartes supposed the "diversities of colour and light" to be due to the different ways in which the matter moves.** In the *Météores*,†† the various colours are connected with different rotatory velocities of the globules, the particles which rotate most rapidly giving the sensation of red, the slower ones of yellow, and the slowest of green and blue—the order of colours being taken from the rainbow. The assertion of the dependence of colour on periodic time is a curious foreshadowing of one of the great discoveries of Newton.

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* *Principia*, Part iii, § 52.

† It is curious to speculate on the impression which would have been produced had the spirality of nebulæ been discovered before the overthrow of the Cartesian theory of vortices.

‡ *Ibid.*, §§ 55–59. § *Ibid.*, § 63. ‖ *Ibid.*, § 64. ¶ *Discours premier.*

** *Principia*, Part iv, § 195. †† *Discours Huitième.*

The general explanation of light on these principles was amplified by a more particular discussion of reflexion and refraction. The law of reflexion—that the angles of incidence and refraction are equal—had been known to the Greeks; but the law of refraction—that the sines of the angles of incidence and refraction are to each other in a ratio depending on the media—was now published for the first time.* Descartes gave it as his own; but he seems to have been under considerable obligations to Willebrord Snell (*b.* 1591, *d.* 1626), Professor of Mathematics at Leyden, who had discovered it experimentally (though not in the form in which Descartes gave it) about 1621. Snell did not publish his result, but communicated it in manuscript to several persons, and Huygens affirms that this manuscript had been seen by Descartes.

Descartes presents the law as a deduction from theory. This, however, he is able to do only by the aid of analogy; when rays meet ponderable bodies, "they are liable to be deflected or stopped in the same way as the motion of a ball or a stone impinging on a body"; for "it is easy to believe that the action or inclination to move, which I have said must be taken for light, ought to follow in this the same laws as motion."† Thus he replaces light, whose velocity of propagation he believes to be always infinite, by a projectile whose velocity varies from one medium to another. The law of refraction is then proved as follows‡:—

Let a ball thrown from A meet at B a cloth CBE, so weak that the ball is able to break through it and pass beyond, but with its resultant velocity reduced in some definite proportion, say $1 : k$.

Then if BI be a length measured on the refracted ray equal to AB, the projectile will take k times as long to describe BI as it took to describe AB. But the component of velocity parallel to the cloth must be unaffected by the impact; and therefore the projection BE of the refracted ray must be k times as long as the projection BC of the incident ray. So if i and r denote the angles of incidence and refraction, we have

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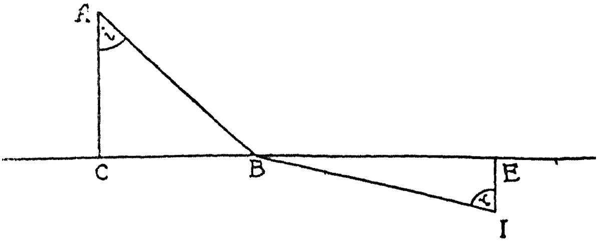
* *Dioptrique, Discours second.*

† *Ibid., Discours premier.*

‡ *Ibid., Discours second.*



$$\sin r = \frac{BE}{BI} = k \cdot \frac{BC}{BA} = k \sin i,$$

or the sines of the angles of incidence and refraction are in a constant ratio; this is the law of refraction.

Desiring to include all known phenomena in his system, Descartes devoted some attention to a class of effects which were at that time little thought of, but which were destined to play a great part in the subsequent development of Physics.

The ancients were acquainted with the curious properties possessed by two minerals, amber (ἤλεκτρον) and magnetic iron ore (ἡ λίθος Μαγνῆτις). The former, when rubbed, attracts light bodies: the latter has the power of attracting iron.

The use of the magnet for the purpose of indicating direction at sea does not seem to have been derived from classical antiquity; but it was certainly known in the time of the Crusades. Indeed, magnetism was one of the few sciences which progressed during the Middle Ages; for in the thirteenth century Petrus Peregrinus,* a native of Maricourt in Picardy, made a discovery of fundamental importance.

Taking a natural magnet or lodestone, which had been rounded into a globular form, he laid it on a needle, and marked the line along which the needle set itself. Then laying the needle on other parts of the stone, he obtained more lines in the same way. When the entire surface of the stone had been covered with such lines, their general disposition became evident; they formed circles, which girdled the stone in exactly the same way as meridians of longitude girdle the earth; and there were two points at opposite ends of the stone through which all the circles passed, just as all the meridians pass through the Arctic and Antarctic poles of the earth.* Struck by the analogy, Peregrinus proposed to call these two points the *poles* of the magnet: and he observed that the way in which magnets set themselves and attract each other depends solely on the position of their poles, as if these were the seat of the magnetic power. Such was the origin of those theories of poles and polarization which in later ages have played so great a part in Natural Philosophy.

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* His *Epistola* was written in 1269.

The observations of Peregrinus were greatly extended not long before the time of Descartes by William Gilberd or Gilbert† (*b.* 1540, *d.* 1603). Gilbert was born at Colchester: after studying at Cambridge, he took up medical practice in London, and had the honour of being appointed physician to Queen Elizabeth. In 1600 he published a work‡ on Magnetism and Electricity, with which the modern history of both subjects begins.

Of Gilbert's electrical researches we shall speak later: in magnetism he made the capital discovery of the reason why magnets set in definite orientations with respect to the earth; which is, that the earth is itself a great magnet, having one of its poles in high northern and the other in high southern latitudes. Thus the property of the compass was seen to be included in the general principle, that the north-seeking pole of every magnet attracts the south-seeking pole of every other magnet, and repels its north-seeking pole.

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* "Procul dubio omnes lineae hujusmodi in duo puncta concurrent sicut omnes orbes meridiani in duo concurrunt polos mundi oppositos."

† The form in the Colchester records is Gilberd.

‡ Gulielmi Gilberti de Magnete, Magneticisque corporibus, et de magno magnete tellure: London, 1600. An English translation by P. F. Mottelay was published in 1893.

Descartes attempted* to account for magnetic phenomena by his theory of vortices. A vortex of fluid matter was postulated round each magnet, the matter of the vortex entering by one pole and leaving by the other: this matter was supposed to act on iron and steel by virtue of a special resistance to its motion afforded by the molecules of those substances.

Crude though the Cartesian system was in this and many other features, there is no doubt that by presenting definite conceptions of molecular activity, and applying them to so wide a range of phenomena, it stimulated the spirit of inquiry, and prepared the way for the more accurate theories that came after. In its own day it met with great acceptance: the confusion which had resulted from the destruction of the old order was now, as it seemed, ended by a reconstruction of knowledge in a system at once credible and complete. Nor did its influence quickly wane; for even at Cambridge it was studied long after Newton had published his theory of gravitation;† and in the middle of the eighteenth century Euler and two of the Bernoullis based the explanation of magnetism on the hypothesis of vortices.‡

Descartes' theory of light rapidly displaced the conceptions which had held sway in the Middle Ages. The validity of his explanation of refraction was, however, called in question by his fellow-countryman Pierre de Fermat (*b.* 1601, *d.* 1665),§ and a controversy ensued, which was kept up by the Cartesians long after the death of their master. Fermat eventually introduced a new fundamental law, from which he proposed to deduce the paths of rays of light. This was the celebrated *Principle of Least Time,* enunciated[*] in the form, "Nature always acts by the shortest course." From it the law of reflexion can readily be derived, since the path described by light between a point on the incident ray and a point on the reflected ray is the shortest possible consistent with the condition of meeting the reflecting surfaces.[†] In order to obtain the law of refraction, Fermat assumed that "the resistance of the media is different," and applied his "method of maxima and minima" to find the path which would be described in the least time from a point of one medium to a point of the other. In 1661 he arrived at the solution.[‡] "The result of my work," he writes, "has been the most extraordinary, the most unforeseen, and the happiest, that ever was; for, after having performed all the equations, multiplications, antitheses, and other operations of my method, and having finally finished the problem, I have found that my principle gives exactly and precisely the same proportion for the refractions which Monsieur Descartes has established." His surprise was all the greater, as he had supposed light to move more slowly in dense than in rare media, whereas Descartes had (as will be evident from the demonstration given above) been obliged to make the contrary supposition.

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* *Principia*, Part iv, § 133 sqq.

† Whiston has recorded that, having returned to Cambridge after his ordination in 1693, he resumed his studies there, "particularly the Mathematicks, and the Cartesian Philosophy: which was alone in Vogue with us at that Time. But it was not long before I, with immense Pains, but no Assistance, set myself with the utmost Zeal to the study of Sir Isaac Newton's wonderful Discoveries."—Whiston's *Memoirs* (1749), i, p. 36.

‡ Their memoirs shared a prize of the French Academy in 1743, and were printed in 1752 in the *Recueil des pièces qui ont remporté les prix de l'Acad.*, tome v.

§ Renati Descartes Epistolae, Pars tertia; Amstelodami, 1683. The Fermat correspondence is comprised in letters XXIX to XLVI.

Although Fermat's result was correct, and, indeed, of high permanent interest, the principles from which it was derived were metaphysical rather than physical in character, and consequently were of little use for the purpose of framing a mechanical explanation of light. Descartes' theory therefore held the field until the publication in 1667[§] of the *Micrographia*

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[*] Epist. XLII, written at Toulouse in August, 1657, to Monsieur de la Chambre; reprinted in *Œuvres de Fermat* (ed. 1891), ii, p. 354.

[†] That reflected light follows the shortest path was no new result, for it had been affirmed (and attributed to Hero of Alexandria) in the *Κεφάλαια τῶν ὀπτικῶν* of Heliodorus of Larissa, a work of which several editions were published in the seventeenth century.

[‡] Epist. XLIII, written at Toulouse on Jan. 1, 1662; reprinted in *Œuvres de Fermat*, ii, p. 457; i, pp. 170, 173.

[§] The *imprimatur* of Viscount Brouncker, P.R.S., is dated Nov. 23, 1664.

Robert Hooke (*b.* 1635, *d.* 1703), one of the founders of the oyal Society, and at one time its Secretary.

Hooke, who was both an observer and a theorist, made two xperimental discoveries which concern our present subject; but both of these, as it appeared, he had been anticipated. The rst[*] was the observation of the iridescent colours which are een when light falls on a thin layer of air between two glass lates or lenses, or on a thin film of any transparent substance. hese are generally known as the "colours of thin plates," or Newton's rings"; they had been previously observed by Boyle.[†] ooke's second experimental discovery,[‡] made after the date of e *Micrographia*, was that light in air is not propagated exactly straight lines, but that there is some illumination within the eometrical shadow of an opaque body. This observation had een published in 1665 in a posthumous work[§] of Francesco aria Grimaldi (*b.* 1618, *d.* 1663), who had given to the phe-omenon the name *diffraction.*

Hooke's theoretical investigations on light were of great mportance, representing as they do the transition from the artesian system to the fully developed theory of undulations. e begins by attacking Descartes' proposition, that light is a endency to motion rather than an actual motion. "There is," e observes,[‖] "no luminous Body but has the parts of it in otion more or less"; and this motion is "exceeding quick." oreover, since some bodies (e.g. the diamond when rubbed or eated in the dark) shine for a considerable time without being wasted away, it follows that whatever is in motion is not per-anently lost to the body, and therefore that the motion must e of a to-and-fro or vibratory character. The amplitude of the vibrations must be exceedingly small, since some luminous bodies (e.g. the diamond again) are very hard, and so cannot yield or bend to any sensible extent.

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[*] *Micrographia*, p. 47.

[†] Boyle's *Works* (ed. 1772), i, p. 742.

[‡] Hooke's *Posthumous Works*, p. 186.

[§] *Physico-Mathesis de lumine, coloribus, et iride.* Bologna, 1665; book i, prop. i.

[‖] *Micrographia*, p. 55.

Concluding, then, that the condition associated with the emission of light by a luminous body is a rapid vibratory motion of very small amplitude, Hooke next inquires how light travels through space. "The next thing we are to consider," he says, "is the way or manner of the *trajection* of this motion through the interpos'd pellucid body to the eye: And here it will be easily granted—

"First, that it must be a body *susceptible* and *impartible* of this motion that will deserve the name of a Transparent; and next, that the parts of such a body must be *homogeneous*, or of the same kind.

"Thirdly, that the constitution and motion of the parts must be such that the appulse of the luminous body may be communicated or propagated through it to the greatest imaginable distance in the least imaginable time, though I see no reason to affirm that it must be in an instant.

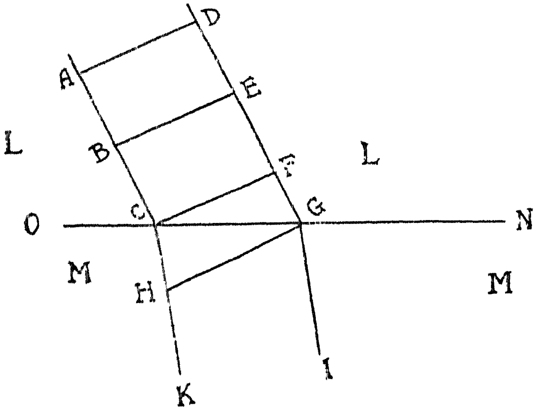
"Fourthly, that the motion is propagated every way through an *Homogeneous medium* by *direct* or *straight* lines extended every way like Rays from the centre of a Sphere.

"Fifthly, in an *Homogeneous medium* this motion is propagated every way with *equal velocity*, whence necessarily every *pulse* or *vibration* of the luminous body will generate a Sphere, which will continually increase, and grow bigger, just after the same manner (though indefinitely swifter) as the waves or rings on the surface of the water do swell into bigger and bigger circles about a point of it, where by the sinking of a Stone the motion was begun, whence it necessarily follows, that all the parts of these Spheres undulated through an *Homogeneous medium* cut the Rays at right angles."

Here we have a fairly definite mechanical conception. It resembles that of Descartes in postulating a medium as the vehicle of light; but according to the Cartesian hypothesis the disturbance is a statical pressure in this medium, while in Hooke's theory it is a rapid vibratory motion of small amplitude. In the above extract Hooke introduces, moreover, the idea of the *wave-surface*, or locus at any instant of a disturbance generated originally at a point, and affirms that it is a sphere, whose centre is the point in question, and whose radii are the rays of light issuing from the point.

Hooke's next effort was to produce a mechanical theory of refraction, to replace that given by Descartes. "Because," he says, "all transparent *mediums* are not *Homogeneous* to one another, therefore we will next examine how this pulse or motion will be propagated through differingly transparent *mediums*. And here, according to the most acute and excellent Philosopher *Des Cartes*, I suppose the sine of the angle of inclination in the first *medium* to be to the sine of refraction in the second, as the density of the first to the density of the second. By density, I mean not the density in respect of gravity (with which the refractions or transparency of *mediums* hold no proportion), but in respect only to the *trajection* of the Rays of light, in which respect they only differ in this, that the one propagates the pulse more easily and weakly, the other more slowly, but more strongly. But as for the pulses themselves, they will by the refraction acquire another property, which we shall now endeavour to explicate.

"We will suppose, therefore, in the first Figure, *ACFD* to be a physical Ray, or *ABC* and *DEF* to be two mathematical Rays, *trajected* from a very remote point of a luminous body through



an *Homogeneous* transparent medium *LL*, and *DA*, *EB*, *FC*, to be small portions of the orbicular impulses which must therefore cut the Rays at right angles: these Rays meeting with the plain surface *NO* of a *medium* that yields an easier *transitus* to the propagation of light, and falling *obliquely* on it, they will in the medium *MM* be refracted towards the perpendicular of the surface. And because this medium is more easily *trajected* than the former by a third, therefore the point *C* of the orbicular pulse *FC* will be moved to *H* four spaces in the same time that *F*, the other end of it, is moved to three spaces, therefore the whole refracted pulse to *H* shall be *oblique* to the refracted Rays *CHK* and *GI*."

Although this is not in all respects successful, it represents a decided advance on the treatment of the same problem by Descartes, which rested on a mere analogy. Hooke tries to determine what happens to the wave-front when it meets the interface between two media; and for this end he introduces the correct principle that the side of the wave-front which first meets the interface will go forward in the second medium with the velocity proper to that medium, while the other side of the wave-front which is still in the first medium is still moving with the old velocity: so that the wave-front will be deflected in the transition from one medium to the other.

This deflection of the wave-front was supposed by Hooke to be the origin of the prismatic colours. He regarded natural or white light as the simplest type of disturbance, being constituted by a simple and uniform pulse at right angles to the direction of propagation, and inferred that colour is generated by the distortion to which this disturbance is subjected in the process of refraction. "The Ray,"* he says, "is dispersed, split, and opened by its Refraction at the Superficies of a second medium, and from a line is opened into a diverging Superficies, and so obliquated, whereby the appearances of Colours are produced."

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* Hooke, *Posthumous Works*, p. 82.

"Colour," he says in another place,* "is nothing but the disturbance of light by the communication of the pulse to other transparent mediums, that is by the refraction thereof." His precise hypothesis regarding the different colours was† "that Blue is an impression on the Retina of an oblique and confus'd pulse of light, whose weakest part precedes, and whose strongest follows. And, that red is an impression on the Retina of an oblique and confus'd pulse of light, whose strongest part precedes, and whose weakest follows."

Hooke's theory of colour was completely overthrown, within a few years of its publication, by one of the earliest discoveries of Isaac Newton (*b.* 1642, *d.* 1727). Newton, who was elected a Fellow of Trinity College, Cambridge, in 1667, had in the beginning of 1666 obtained a triangular prism, "to try therewith the celebrated Phaenomena of Colours." For this purpose, "having darkened my chamber, and made a small hole in my window-shuts, to let in a convenient quantity of the Sun's light, I placed my Prisme at his entrance, that it might be thereby refracted to the opposite wall. It was at first a very pleasing divertisement, to view the vivid and intense colours produced thereby; but after a while applying myself to consider them more circumspectly, I became surprised to see them in an *oblong* form, which, according to the received laws of Refraction, I expected should have been *circular*." The length of the coloured spectrum was in fact about five times as great as its breadth.

This puzzling fact he set himself to study; and after more experiments the true explanation was discovered—namely, that ordinary white light is really a mixture of rays of every variety of colour, and that the elongation of the spectrum is due to the differences in the refractive power of the glass for these different rays.

"Amidst these thoughts," he tells us,‡ "I was forced from

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* To the Royal Society, February 15, 1671–2.

† *Micrographia*, p. 64.

‡ Phil. Trans., No. 80, February 19, 1671–2.

Cambridge by the intervening Plague"; this was in 1666, and his memoir on the subject was not presented to the Royal Society until five years later. In it he propounds a theory of colour directly opposed to that of Hooke. "Colours," he says, "are not *Qualifications of light* derived from Refractions, or Reflections of natural Bodies (as 'tis generally believed), but *Original and connate properties*, which in divers Rays are divers. Some Rays are disposed to exhibit a red colour and no other: some a yellow and no other, some a green and no other, and so of the rest. Nor are there only Rays proper and particular to the more eminent colours, but even to all their intermediate gradations.

"To the same degree of Refrangibility ever belongs the same colour, and to the same colour ever belongs the same degree of Refrangibility."

"The species of colour, and degree of Refrangibility proper to any particular sort of Rays, is not mutable by Refraction, nor by Reflection from natural bodies, nor by any other cause, that I could yet observe. When any one sort of Rays hath been well parted from those of other kinds, it hath afterwards obstinately retained its colour, notwithstanding my utmost endeavours to change it."

The publication of the new theory gave rise to an acute controversy. As might have been expected, Hooke was foremost among the opponents, and led the attack with some degree of asperity. When it is remembered that at this time Newton was at the outset of his career, while Hooke was an older man, with an established reputation, such harshness appears particularly ungenerous; and it is likely that the unpleasant consequences which followed the announcement of his first great discovery had much to do with the reluctance which Newton ever afterwards showed to publish his results to the world.

In the course of the discussion Newton found occasion to explain more fully the views which he entertained regarding the nature of light. Hooke charged him with holding the doctrine that light is a material substance. Now Newton had, as a matter of fact, a great dislike of the more imaginative kind of hypotheses; he altogether renounced the attempt to construct the universe from its foundations after the fashion of Descartes, and aspired to nothing more than a formulation of the laws which directly govern the actual phenomena. His theory of gravitation, for example, is strictly an expression of the results of observation, and involves no hypothesis as to the cause of the attraction which subsists between ponderable bodies; and his own desire in regard to optics was to present a theory free from speculation as to the hidden mechanism of light. Accordingly, in reply to Hooke's criticism, he protested* that his views on colour were in no way bound up with any particular conception of the ultimate nature of optical processes.

Newton was, however, unable to carry out his plan of connecting together the phenomena of light into a coherent and reasoned whole without having recourse to hypotheses. The hypothesis of Hooke, that light consists in vibrations of an aether, he rejected for reasons which at that time were perfectly cogent, and which indeed were not successfully refuted for over a century. One of these was the incompetence of the wave-theory to account for the rectilinear propagation of light, and another was its inability to embrace the facts—discovered, as we shall presently see, by Huygens, and first interpreted correctly by Newton himself—of polarization. On the whole, he seems to have favoured a scheme of which the following may be taken as a summary†:—

All space is permeated by an elastic medium or *aether*, which is capable of propagating vibrations in the same way as the air propagates the vibrations of sound, but with far greater velocity.

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* Phil. Trans. vii, 1672, p. 5086.

† Cf. Newton's memoir in Phil. Trans. vii, 1672; his memoir presented to the Royal Society in December, 1675, which is printed in Birch, iii, p. 247; his *Opticks*, especially Queries 18, 19, 20, 21, 23, 29; the Scholium at the end of the *Principia*; and a letter to Boyle, written in February, 1678–9, which is printed in Horsley's *Newtoni Opera*, p. 385.

In the *Principia*, Book I., section xiv, the analogy between rays of light and streams of corpuscles is indicated; but Newton does not commit himself to any theory of light based on this.

This aether pervades the pores of all material bodies, and is the cause of their cohesion; its density varies from one body to another, being greatest in the free interplanetary spaces. It is not necessarily a single uniform substance: but just as air contains aqueous vapour, so the aether may contain various "aethereal spirits," adapted to produce the phenomena of electricity, magnetism, and gravitation.

The vibrations of the aether cannot, for the reasons already mentioned, be supposed in themselves to constitute light. Light is therefore taken to be "something of a different kind, propagated from lucid bodies. They, that will, may suppose it an aggregate of various peripatetic qualities. Others may suppose it multitudes of unimaginable small and swift corpuscles of various sizes, springing from shining bodies at great distances one after another; but yet without any sensible interval of time, and continually urged forward by a principle of motion, which in the beginning accelerates them, till the resistance of the aethereal medium equals the force of that principle, much after the manner that bodies let fall in water are accelerated till the resistance of the water equals the force of gravity. But they, that like not this, may suppose light any other corporeal emanation, or any impulse or motion of any other medium or aethereal spirit diffused through the main body of aether, or what else they can imagine proper for this purpose. To avoid dispute, and make this hypothesis general, let every man here take his fancy; only whatever light be, I suppose it consists of rays differing from one another in contingent circumstances, as bigness, form, or vigour."*

In any case, light and aether are capable of mutual interaction; aether is in fact the intermediary between light and ponderable matter. When a ray of light meets a stratum of aether denser or rarer than that through which it has lately been passing, it is, in general, deflected from its rectilinear course; and differences of density of the aether between one material medium and another account on these principles for the reflexion and refraction of light. The condensation or rarefaction of the aether due to a material body extends to some little distance from the surface of the body, so that the inflexion due to it is really continuous, and not abrupt; and this further explains diffraction, which Newton took to be "only a new kind of refraction, caused, perhaps, by the external aether's beginning to grow rarer a little before it came at the opake body, than it was in free spaces."

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* Royal Society, Dec. 9, 1675.

Although the regular vibrations of Newton's aether were not supposed to constitute light, its irregular turbulence seems to have represented fairly closely his conception of heat. He supposed that when light is absorbed by a material body, vibrations are set up in the aether, and are recognizable as the heat which is always generated in such cases. The conduction of heat from hot bodies to contiguous cold ones he conceived to be effected by vibrations of the aether propagated between them; and he supposed that it is the violent agitation of aethereal motions which excites incandescent substances to emit light.

Assuming with Newton that light is not actually constituted by the vibrations of an aether, even though such vibrations may exist in close connexion with it, the most definite and easily conceived supposition is that rays of light are streams of corpuscles emitted by luminous bodies. Although this was not the hypothesis of Descartes himself, it was so thoroughly akin to his general scheme that the scientific men of Newton's generation, who were for the most part deeply imbued with the Cartesian philosophy, instinctively selected it from the wide choice of hypotheses which Newton had offered them; and by later writers it was generally associated with Newton's name. A curious argument in its favour was drawn from a phenomenon which had then been known for nearly half a century: Vincenzo Cascariolo, a shoemaker of Bologna, had discovered, about 1630, that a substance, which afterwards received the name of *Bologna stone* or *Bologna phosphorus*, has the property of shining in the dark after it has been exposed for some time to sunlight; and the storage of light which seemed to be here involved was more easily explicable on the corpuscular theory than on any other. The evidence in this quarter, however, pointed the other way when it was found that phosphorescent substances do not necessarily emit the same kind of light as that which was used to stimulate them.

In accordance with his earliest discovery, Newton considered colour to be an inherent characteristic of light, and inferred that it must be associated with some definite quality of the corpuscles or aether-vibrations. The corpuscles corresponding to different colours would, he remarked, like sonorous bodies of different pitch, excite vibrations of different types in the aether; and "if by any means those [aether-vibrations] of unequal bignesses be separated from one another, the largest beget a Sensation of a *Red* colour, the least or shortest of a deep *Violet*, and the intermediate ones, of intermediate colours; much after the manner that bodies, according to their several sizes, shapes, and motions, excite vibrations in the Air of various bignesses, which, according to those bignesses, make several Tones in Sound."*

This sentence is the first enunciation of the great principle that homogeneous light is essentially *periodic* in its nature, and that differences of period correspond to differences of colour. The analogy with Sound is obvious; and it may be remarked in passing that Newton's theory of periodic vibrations in an elastic medium, which he developed† in connexion with the explanation of Sound, would alone entitle him to a place among those who have exercised the greatest influence on the theory of light, even if he had made no direct contribution to the latter subject.

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* Phil. Trans. vii (1672), p. 5088.

† Newton's *Principia*, Book ii., Props. xliii.-l.

Newton devoted considerable attention to the colours of thin plates, and determined the empirical laws of the phenomena with great accuracy. In order to explain them, he supposed that "every ray of light, in its passage through any refracting surface, is put into a certain transient constitution or state, which, in the progress of the ray, returns at equal intervals, and disposes the ray, at every return, to be easily transmitted through the next refracting surface, and, between the returns, to be easily reflected by it."* The interval between two consecutive dispositions to easy transmission, or "length of fit," he supposed to depend on the colour, being greatest for red light and least for violet. If then a ray of homogeneous light falls on a thin plate, its fortunes as regards transmission and reflexion at the two surfaces will depend on the relation which the length of fit bears to the thickness of the plate; and on this basis he built up a theory of the colours of thin plates. It is evident that Newton's "length of fit" corresponds in some measure to the quantity which in the undulatory theory is called the wave-length of the light; but the suppositions of easy transmission and reflexion were soon found inadequate to explain all Newton's experimental results—at least without making other and more complicated additional assumptions.

At the time of the publication of Hooke's *Micrographia*, and Newton's theory of colours, it was not known whether light is propagated instantaneously or not. An attempt to settle the question experimentally had been made many years previously by Galileo,† who had stationed two men with lanterns at a considerable distance from each other; one of them was directed to observe when the other uncovered his light, and exhibit his own the moment he perceived it. But the interval of time required by the light for its journey was too small to be perceived in this way; and the discovery was ultimately made by an astronomer. It was observed in 1675 by Olof Roemer* (*b.* 1644, *d.* 1710) that the eclipses of the first satellites of Jupiter were apparently affected by an unknown disturbing cause; the time of the occurrence of the phenomenon was retarded when the earth and Jupiter, in the course of their orbital motions, happened to be most remote from each other, and accelerated in the contrary case. Roemer explained this by supposing that light requires a finite time for its propagation from the satellite to the earth; and by observations of eclipses, he calculated the interval required for its passage from the sun to the earth (the *light-equation*, as it is called) to be 11 minutes.†

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* *Opticks*, Book ii., Prop. 12.

† *Discorsi e dimostrazioni matematiche*, p. 43 of the Elzevir edition of 1638.

Shortly after Roemer's discovery, the wave-theory of light was greatly improved and extended by Christiaan Huygens (*b.* 1629, *d.* 1695). Huygens, who at the time was living in Paris, communicated his results in 1678 to Cassini, Roemer, De la Hire, and the other physicists of the French Academy, and prepared a manuscript of considerable length on the subject. This he proposed to translate into Latin, and to publish in that language together with a treatise on the Optics of Telescopes; but the work of translation making little progress, after a delay of twelve years, he decided to print the work on wave-theory in its original form. In 1690 it appeared at Leyden,‡ under the title *Traité de la lumière où sont expliquées les causes de ce qui luy arrive dans la réflexion et dans la réfraction. Et parti-*

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* Mém. de l'Acad. x. (1666–1699), p. 575.

† It was soon recognized that Roemer's value was too large: and the astronomers of the succeeding half-century reduced it to 7 minutes. Delambre, by an investigation whose details appear to have been completely destroyed, published in 1817 the value $493 \cdot 2^s$, from a discussion of eclipses of Jupiter's satellites during the previous 150 years. Glasenapp, in an inaugural dissertation published in 1875, discussed the eclipses of the first satellite between 1848 and 1870, and derived, by different assumptions, values between 496^s and 501^s, the most probable value being $500 \cdot 8^s$. Sampson, in 1909, derived $498 \cdot 64^s$ from his own readings of the Harvard Observations, and $498 \cdot 79^s$ from the Harvard readings, with probable errors of about $\pm 0 \cdot 02''$. The inequalities of Jupiter's surface give rise to some difficulty in exact determinations.

‡ Huygens had by this time returned to Holland.

culièrement dans l'étrange réfraction du cristal d'Islande. Par C.H.D.Z.[*]

The truth of Hooke's hypothesis, that light is essentially a form of motion, seemed to Huygens to be proved by the effects observed with burning-glasses; for in the combustion induced at the focus of the glass, the molecules of bodies are dissociated; which, as he remarked, must be taken as a certain sign of motion, if, in conformity to the Cartesian philosophy, we seek the cause of all natural phenomena in purely mechanical actions.

The question then arises as to whether the motion is that of a medium, as is supposed in Hooke's theory, or whether it may be compared rather to that of a flight of arrows, as in the corpuscular theory. Huygens decided that the former alternative is the only tenable one, since beams of light proceeding in directions inclined to each other do not interfere with each other in any way.

Moreover, it had previously been shown by Torricelli that light is transmitted as readily through a vacuum as through air; and from this Huygens inferred that the medium or aether in which the propagation takes place must penetrate all matter, and be present even in all so-called vacua.

The process of wave-propagation he discussed by aid of a principle which was now[†] introduced for the first time, and has since been generally known by his name. It may be stated thus: Consider a wave-front,[‡] or locus of disturbance, as it exists at a definite instant t_0: then each surface-element of the wave-front may be regarded as the source of a secondary wave, which in a homogeneous isotropic medium will be propagated outwards from the surface-element in the form of a sphere whose radius at any subsequent instant t is proportional to $(t-t_0)$; and the wave-front which represents the whole disturbance at the instant t is simply the envelope of the secondary waves which arise from the various surface elements of the original wave-front.* The introduction of this principle enabled Huygens to succeed where Hooke and other contemporary wave-theorists† had failed, in achieving the explanation of refraction and reflexion. His method was to combine his own principle with Hooke's device of following separately the fortunes of the right-hand and left-hand sides of a wave-front when it reaches the interface between two media. The actual explanation for the case of reflexion is as follows:—

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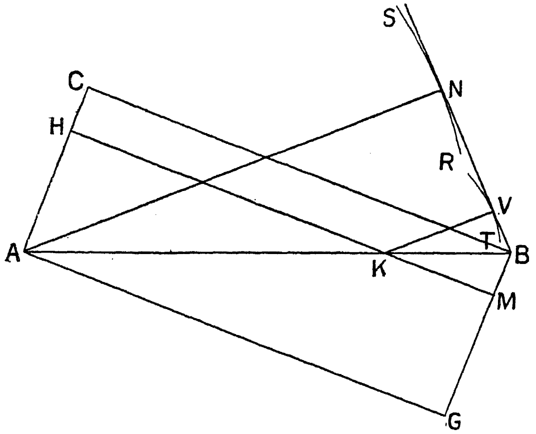
[*] i.e. Christiaan Huygens de Zuylichem. The custom of indicating names by initials was not unusual in that age.

[†] *Traité de la lum.*, p. 17.

[‡] It may be remarked that Huygens' "waves" are really what modern writers, following Hooke, call "pulses"; Huygens never considered true wave-trains having the property of periodicity.

Let AB represent the interface at which reflexion takes place, AHC the incident wave-front at an instant t_0, GMB the position which the wave-front would occupy at a later instant t if the propagation were not interrupted by reflexion. Then by



Huygens' principle the secondary wave from A is at the instant t a sphere RNS of radius equal to AG: the disturbance from H, after meeting the interface at K, will generate a secondary wave TV of radius equal to KM, and similarly the secondary wave corresponding to any other element of the original wave-front can be found. It is obvious that the envelope of these secondary waves, which constitutes the final wave-front, will be a plane BN, which will be inclined to AB at the same angle as AC. This gives the law of reflexion.

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* The justification for this was given long afterwards by Fresnel, *Annales de chimie*, xxi.

† e.g. Ignace Gaston Pardies and Pierre Ango, the latter of whom published a work on Optics at Paris in 1682.

The law of refraction is established by similar reasoning, on the supposition that the velocity of light depends on the medium in which it is propagated. Since a ray which passes from air to glass is bent inwards towards the normal, it may be inferred that light travels more slowly in glass than in air.

Huygens offered a physical explanation of the variation in velocity of light from one medium to another, by supposing that transparent bodies consist of hard particles which interact with the aethereal matter, modifying its elasticity. The opacity of metals he explained by an extension of the same idea, supposing that some of the particles of metals are hard (these account for reflexion) and the rest soft: the latter destroy the luminous motion by damping it.

The second half of the *Théorie de la lumière* is concerned with a phenomenon which had been discovered a few years previously by a Danish philosopher, Erasmus Bartholin (*b.* 1625, *d.* 1698). A sailor had brought from Iceland to Copenhagen a number of beautiful crystals which he had collected in the Bay of Röerford. Bartholin, into whose hands they passed, noticed* that any small object viewed through one of these crystals appeared double, and found the immediate cause of this in the fact that a ray of light entering the crystal gave rise in general to *two* refracted rays. One of these rays was subject to the ordinary law of refraction, while the other, which was called the *extraordinary* ray, obeyed a different law, which Bartholin did not succeed in determining.

The matter had arrived at this stage when it was taken up by Huygens. Since in his conception each ray of light corresponds to the propagation of a wave-front, the two rays in Iceland spar must correspond to two different wave-fronts propagated simultaneously. In this idea he found no difficulty; as he says: "It is certain that a space occupied by more than one kind of matter may permit the propagation of several kinds of waves, different in velocity; for this actually happens in air mixed with aethereal matter, where sound-waves and light-waves are propagated together."

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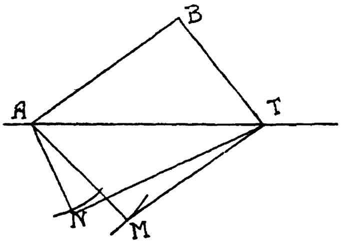
* *Experimenta cristalli Islandici disdiaclastici:* 1669.

Accordingly he supposed that a light-disturbance generated at any spot within a crystal of Iceland spar spreads out in the form of a wave-surface, composed of a sphere and a spheroid having the origin of disturbance as centre. The spherical wave-front corresponds to the ordinary ray, and the spheroid to the extraordinary ray; and the direction in which the extraordinary ray is refracted may be determined by a geometrical construction, in which the spheroid takes the place which in the ordinary construction is taken by the sphere.

Thus, let the plane of the figure be at right angles to the intersection of the wave-front with the surface of the crystal; let *AB* represent the trace of the incident wave-front; and suppose that in unit time the disturbance from *B* reaches the interface at *T*. In this unit-interval of time the disturbance from *A* will have spread out within the crystal into a sphere and spheroid: so the wave-front corresponding to the ordinary ray will be the tangent-plane to the sphere through the line whose trace is *T*, while the wave-front corresponding to the extraordinary ray will be the tangent-plane to the spheroid through the same line. The points of contact *N*



and M will determine the directions AN and AM of the two refracted rays* within the crystal.

Huygens did not in the *Théorie de la lumière* attempt a detailed physical explanation of the spheroidal wave, but communicated one later in a letter to Papin,† written in December, 1690. "As to the kinds of matter contained in Iceland crystal," he says, "I suppose one composed of small spheroids, and another which occupies the interspaces around these spheroids, and which serves to bind them together. Besides these, there is the matter of aether permeating all the crystal, both between and within the parcels of the two kinds of matter just mentioned; for I suppose both the little spheroids, and the matter which occupies the intervals around them, to be composed of small fixed particles, amongst which are diffused in perpetual motion the still finer particles of the aether. There is now no reason why the ordinary ray in the crystal should not be due to waves propagated in this aethereal matter. To account for the extraordinary refraction, I conceive another kind of waves, which have for vehicle both the aethereal matter and the two other kinds of matter constituting the crystal. Of these latter, I suppose that the matter of the small spheroids transmits the waves a little more quickly than the aethereal matter, while that around the spheroids transmits these waves a little more slowly than the same aethereal matter. . . . These same waves, when they travel in the direction of the breadth of the spheroids, meet with more of the matter of the spheroids, or at least pass with less obstruction, and so are propagated a little more quickly in this sense than in the other; thus the light-disturbance is propagated as a spheroidal sheet."

Huygens made another discovery‡ of capital importance when experimenting with the Iceland crystal. He observed that the two rays which are obtained by the double refraction of a single ray afterwards behave in a way different from ordinary light which has not experienced double refraction; and in particular, if one of these rays is incident on a second crystal of Iceland spar, it gives rise in some circumstances to two, and in others to only one, refracted ray. The behaviour of the ray at this second refraction can be altered by simply rotating the second crystal about the direction of the ray as axis; the ray undergoing the ordinary or extraordinary refraction according as the principal section of the crystal is in a certain direction or in the direction at right angles to this.

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* The word *ray* in the wave-theory is always applied to the line which goes from the centre of a wave (i.e. the origin of the disturbance) to a point on its surface, whatever may be the inclination of this line to the surface-element on which it abuts; for this line has the optical properties of the "rays" of the emission theory.

† Huygens' *Œuvres*, ed. 1905, x., p. 177.

‡ *Théorie de la lumière*, p. 89.

The first stage in the explanation of Huygens' observation was reached by Newton, who in 1717 showed[*] that a ray obtained by double refraction differs from a ray of ordinary light in the same way that a long rod whose cross-section is a rectangle differs from a long rod whose cross-section is a circle: in other words, the properties of a ray of ordinary light are the same with respect to all directions at right angles to its direction of propagation, whereas a ray obtained by double refraction must be supposed to have *sides*, or properties related to special directions at right angles to its own direction. The refraction of such a ray at the surface of a crystal depends on the relation of its sides to the principal plane of the crystal.

That a ray of light should possess such properties seemed to Newton[†] an insuperable objection to the hypothesis which regarded waves of light as analogous to waves of sound. On this point he was in the right: his objections are perfectly valid against the wave-theory as it was understood by his contemporaries[‡], although not against the theory[§] which was put forward a century later by Young and Fresnel.

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[*] The second edition of Newton's *Opticks*, Query 26.

[†] *Opticks*, Query 28.

[‡] In which the oscillations are performed in the direction in which the wave advances.

[§] In which the oscillations are performed in a direction at right angles to that in which the wave advances.

CHAPTER II.

ELECTRIC AND MAGNETIC SCIENCE PRIOR TO THE INTRODUCTION OF THE POTENTIALS.

THE magnetic discoveries of Peregrinus and Gilbert, and the vortex-hypothesis by which Descartes had attempted to explain them,* had raised magnetism to the rank of a separate science by the middle of the seventeenth century. The kindred science of electricity was at that time in a less developed state; but it had been considerably advanced by Gilbert, whose researches in this direction will now be noticed.

For two thousand years the attractive power of amber had been regarded as a virtue peculiar to that substance, or possessed by at most one or two others. Gilbert proved† this view to be mistaken, showing that the same effects are induced by friction in quite a large class of bodies; among which he mentioned glass, sulphur, sealing-wax, and various precious stones.

A force which was manifested by so many different kinds of matter seemed to need a name of its own; and accordingly Gilbert gave to it the name *electric*, which it has ever since retained.

Between the magnetic and electric forces Gilbert remarked many distinctions. The lodestone requires no stimulus of friction such as is needed to stir glass and sulphur into activity. The lodestone attracts only magnetizable substances, whereas electrified bodies attract everything. The magnetic attraction between two bodies is not affected by interposing a sheet of paper, or a linen cloth, or by immersing the bodies in water; whereas the electric attraction is readily destroyed by screens. Lastly, the magnetic force tends to arrange bodies in definite orientations; while the electric force merely tends to heap them together in shapeless clusters.

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* Cf. pp. 7–9.

† *De Magnete*, lib. ii., cap. 2.

These facts appeared to Gilbert to indicate that electric phenomena are due to something of a material nature, which under the influence of friction is liberated from the glass or amber in which under ordinary circumstances it is imprisoned. In support of this view he adduced evidence from other quarters. Being a physician, he was well acquainted with the doctrine that the human body contains various humours or kinds of moisture—phlegm, blood, choler, and melancholy,—which, as they predominated, were supposed to determine the temper of mind; and when he observed that electrifiable bodies were almost all hard and transparent, and therefore (according to the ideas of that time) formed by the consolidation of watery liquids, he concluded that the common menstruum of these liquids must be a particular kind of humour, to the possession of which the electrical properties of bodies were to be referred. Friction might be supposed to warm or otherwise excite or liberate the humour, which would then issue from the body as an effluvium and form an atmosphere around it. The effluvium must, he remarked, be very attenuated, for its emission cannot be detected by the senses.

The existence of an atmosphere of effluvia round every electrified body might indeed have been inferred, according to Gilbert's ideas, from the single fact of electric attraction. For he believed that matter cannot act where it is not; and hence if a body acts on all surrounding objects without appearing to touch them, something must have proceeded out of it unseen.

The whole phenomenon appeared to him to be analogous to the attraction which is exercised by the earth on falling bodies. For in the latter case he conceived of the atmospheric air as the effluvium by which the earth draws all things downwards to itself.

Gilbert's theory of electrical emanations commended itself generally to such of the natural philosophers of the seventeenth century as were interested in the subject; among whom were numbered Niccolo Cabeo (*b.* 1585, *d.* 1650), an Italian Jesuit who was perhaps the first to observe that electrified bodies repel as well as attract; the English royalist exile, Sir Kenelm Digby (*b.* 1603, *d.* 1665); and the celebrated Robert Boyle (*b.* 1627, *d.* 1691). There were, however, some differences of opinion as to the manner in which the effluvia acted on the small bodies and set them in motion towards the excited electric; Gilbert himself had supposed the emanations to have an inherent tendency to reunion with the parent body; Digby likened their return to the condensation of a vapour by cooling; and other writers pictured the effluvia as forming vortices round the attracted bodies in the Cartesian fashion.

There is a well-known allusion to Gilbert's hypothesis in Newton's *Opticks.*[*]

"Let him also tell me, how an electrick body can by friction emit an exhalation so rare and subtle,[†] and yet so potent, as by its emission to cause no sensible diminution of the weight of the electrick body, and to be expanded through a sphere, whose diameter is above two feet, and yet to be able to agitate and carry up leaf copper, or leaf gold, at a distance of above a foot from the electrick body?"

It is, perhaps, somewhat surprising that the Newtonian doctrine of gravitation should not have proved a severe blow to the emanation theory of electricity; but Gilbert's doctrine was now so firmly established as to be unshaken by the overthrow of the analogy by which it had been originally justified. It was, however, modified in one particular about the beginning of the eighteenth century. In order to account for the fact that electrics are not perceptibly wasted away by excitement, the earlier writers had supposed all the emanations to return ultimately to the body which had emitted them; but the corpuscular theory of light accustomed philosophers to the idea of emissions so subtle as to cause no perceptible loss; and after the time of Newton the doctrine of the return of the electric effluvia gradually lost credit.

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[*] Query 22.

[†] "Subtlety," says Johnson, "which in its original import means exility of particles, is taken in its metaphorical meaning for nicety of distinction."

Newton died in 1727. Of the expositions of his philosophy which were published in his lifetime by his followers, one at least deserves to be noticed for the sake of the insight which it affords into the state of opinion regarding light, heat, and electricity in the first half of the eighteenth century. This was the *Physices elementa mathematica experimentis confirmata* of Wilhelm Jacob s'Gravesande (*b.* 1688, *d.* 1742), published at Leyden in 1720. The Latin edition was afterwards reprinted several times, and was, moreover, translated into French and English: it seems to have exercised a considerable and, on the whole, well-deserved influence on contemporary thought.

s'Gravesande supposed light to consist in the projection of corpuscles from luminous bodies to the eye; the motion being very swift, as is shown by astronomical observations. Since many bodies, e.g. the metals, become luminous when they are heated, he inferred that every substance possesses a natural store of corpuscles, which are expelled when it is heated to incandescence; conversely, corpuscles may become united to a material body; as happens, for instance, when the body is exposed to the rays of a fire. Moreover, since the heat thus acquired is readily conducted throughout the substance of the body, he concluded that corpuscles can penetrate all substances, however hard and dense they be.

Let us here recall the ideas then current regarding the nature of material bodies. From the time of Boyle (1626–1691) it had been recognized generally that substances perceptible to the senses may be either *elements* or *compounds* or *mixtures*; the compounds being chemical individuals, distinct from mere mixtures of elements. But the substances at that time accepted as elements were very different from those which are now known by the name. Air and the calces* of the metals figured in the list, while almost all the chemical elements now recognized were omitted from it; some of them, such as oxygen and hydrogen, because they were as yet undiscovered, and others, such as the metals, because they were believed to be compounds.

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* i.e. oxides.

Among the chemical elements, it became customary after the time of Newton to include light-corpuscles.* That something which is confessedly imponderable should ever have been admitted into this class may at first sight seem surprising. But it must be remembered that questions of ponderability counted for very little with the philosophers of the period. Three-quarters of the eighteenth century had passed before Lavoisier enunciated the fundamental doctrine that the total weight of the substances concerned in a chemical reaction is the same after the reaction as before it. As soon as this principle came to be universally applied, light parted company from the true elements in the scheme of chemistry.

We must now consider the views which were held at this time regarding the nature of heat. These are of interest for our present purpose, on account of the analogies which were set up between heat and electricity.

The various conceptions which have been entertained concerning heat fall into one or other of two classes, according as heat is represented as a mere condition producible in bodies, or as a distinct species of matter. The former view, which is that universally held at the present day, was advocated by the great philosophers of the seventeenth century. Bacon maintained it in the *Novum Organum*: "Calor," he wrote, "est motus expansivus, cohibitus, et nitens per partes minores."† Boyle‡ affirmed that the "Nature of Heat" consists in "a various, vehement, and intestine commotion of the Parts among themselves." Hooke§ declared that "Heat is a property of a body arising from the motion or agitation of its parts." And Newton‖ asked: "Do not all fixed Bodies, when heated beyond a certain Degree, emit light and shine; and is not this Emission performed by the vibrating Motion of their Parts?" and, moreover, suggested the converse of this, namely, that when light is absorbed by a material body, vibrations are set up which are perceived by the senses as heat.

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* Newton himself (*Opticks*, p. 349) suspected that light-corpuscles and ponderable matter might be transmuted into each other: much later, Boscovich (*Theoria*, pp. 215, 217) regarded the matter of light as a principle or element in the constitution of natural bodies.

† *Nov. Org.*, Lib. II., Aphor. XX.

‡ *Mechanical Production of Heat and Cold.*

§ *Micrographia*, p. 37.

‖ *Opticks*.

The doctrine that heat is a material substance was maintained in Newton's lifetime by a certain school of chemists. The most conspicuous member of the school was Wilhelm Homberg (*b.* 1652, *d.* 1715) of Paris, who* identified heat and light with the *sulphureous principle*, which he supposed to be one of the primary ingredients of all bodies, and to be present even in the interplanetary spaces. Between this view and that of Newton it might at first seem as if nothing but sharp opposition was to be expected.† But a few years later the professed exponents of the *Principia* and the *Opticks* began to develop their system under the evident influence of Homberg's writings. This evolution may easily be traced in s'Gravesande, whose starting-point is the admittedly Newtonian idea that heat bears to light a relation similar to that which a state of turmoil bears to regular rectilinear motion; whence, conceiving light as a projection of corpuscles, he infers that in a hot body the material particles and the light-corpuscles‡ are in a state of agitation, which becomes more violent as the body is more intensely heated.

s'Gravesande thus holds a position between the two opposite camps. On the one hand he interprets heat as a mode of motion; but on the other he associates it with the presence of a particular kind of matter, which he further identifies with the matter of light. After this the materialistic hypothesis made rapid progress. It was frankly advocated by another member of the Dutch school, Hermann Boerhaave* (*b.* 1668, *d.* 1738), Professor in the University of Leyden, whose treatise on chemistry was translated into English in 1727.

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* Mém. de l'Acad., 1705, p. 88.

† Though it reminds us of a curious conjecture of Newton's: "Is not the strength and vigour of the action between light and sulphureous bodies one reason why sulphureous bodies take fire more readily and burn more vehemently than other bodies do?"

‡ I have thought it best to translate s'Gravesande's *ignis* by "light-corpuscles." This is, I think, fully justified by such of his statements as *Quando ignis per lineas rectas oculos nostros intrat, ex motu quem fibris in fundo oculi communicat ideam luminis excitat.*

Somewhat later it was found that the heating effects of the rays from incandescent bodies may be separated from their luminous effects by passing the rays through a plate of glass, which transmits the light, but absorbs the heat. After this discovery it was no longer possible to identify the matter of heat with the corpuscles of light; and the former was consequently accepted as a distinct element, under the name of *caloric.*† In the latter part of the eighteenth and early part of the nineteenth centuries‡ caloric was generally conceived as occupying the interstices between the particles of ponderable matter—an idea which fitted in well with the observation that bodies commonly expand when they are absorbing heat, but which was less competent to explain the fact§ that water expands when freezing. The latter difficulty was overcome by supposing the union between a body and the caloric absorbed in the process of melting to be of a chemical nature; so that the consequent changes in volume would be beyond the possibility of prediction.

As we have already remarked, the imponderability of heat did not appear to the philosophers of the eighteenth century to be a sufficient reason for excluding it from the list of chemical elements; and in any case there was considerable doubt as to whether caloric was ponderable or not. Some experimenters believed that bodies were heavier when cold than when hot; others that they were heavier when hot than when cold. The century was far advanced before Lavoisier and Rumford finally proved that the temperature of a body is without sensible influence on its weight.

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* Boerhaave followed Homberg in supposing the matter of heat to be present in all so-called vacuous spaces.

† Scheele in 1777 supposed caloric to be a compound of oxygen and phlogiston, and light to be oxygen combined with a greater proportion of phlogiston.

‡ In spite of the experiments of Benjamin Thompson, Count Rumford (*b.* 1753, *d.* 1814), in the closing years of the eighteenth century. These should have sufficed to re-establish the older conception of heat.

§ This had been known since the time of Boyle.

Perhaps nothing in the history of natural philosophy is more amazing than the vicissitudes of the theory of heat. The true hypothesis, after having met with general acceptance throughout a century, and having been approved by a succession of illustrious men, was deliberately abandoned by their successors in favour of a conception utterly false, and, in some of its developments, grotesque and absurd.

We must now return to s'Gravesande's book. The phenomena of combustion he explained by assuming that when a body is sufficiently heated the light-corpuscles interact with the material particles, some constituents being in consequence separated and carried away with the corpuscles as flame and smoke. This view harmonizes with the theory of calcination which had been developed by Becher and his pupil Stahl at the end of the seventeenth century, according to which the metals were supposed to be composed of their calces and an element *phlogiston*. The process of combustion, by which a metal is changed into its calx, was interpreted as a decomposition, in which the phlogiston separated from the metal and escaped into the atmosphere; while the conversion of the calx into the metal was regarded as a union with phlogiston.*

s'Gravesande attributed electric effects to vibrations induced in effluvia, which he supposed to be permanently attached to such bodies as amber. "Glass," he asserted, "contains in it, and has about its surface, a certain atmosphere, which is excited by Friction and put into a vibratory motion; for it attracts and repels light Bodies. The smallest parts of the glass are agitated by the Attrition, and by reason of their elasticity, their motion is vibratory, which is communicated to the Atmosphere abovementioned: and therefore that Atmosphere exerts its action the further, the greater agitation the Parts of the Glass receive when a greater attrition is given to the glass."

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* The correct idea of combustion had been advanced by Hooke. "The dissolution of inflammable bodies," he asserts in the *Micrographia*, "is performed by a substance inherent in and mixed with the air, that is like, if not the very same with, that which is fixed in saltpetre." But this statement met with little favour at the time, and the doctrine of the compound nature of metals survived in full vigour until the discovery of oxygen by Priestley and Scheele in 1771-5. In 1775 Lavoisier reaffirmed Hooke's principle that a metallic calx is not the metal minus phlogiston, but the metal plus oxygen; and this idea, which carried with it the recognition of the elementary nature of metals, was generally accepted by the end of the eighteenth century.

The English translator of s'Gravesande's work was himself destined to play a considerable part in the history of electrical science. Jean Théophile Desaguliers (*b.* 1683, *d.* 1744) was an Englishman only by adoption. His father had been a Huguenot pastor, who, escaping from France after the revocation of the Edict of Nantes, brought away the boy from La Rochelle, concealed, it is said, in a tub. The young Desaguliers was afterwards ordained, and became chaplain to that Duke of Chandos who was so ungratefully ridiculed by Pope. In this situation he formed friendships with some of the natural philosophers of the capital, and amongst others with Stephen Gray, an experimenter of whom little is known* beyond the fact that he was a pensioner of the Charterhouse.

In 1729 Gray communicated, as he says,† "to Dr. Desaguliers and some other Gentlemen" a discovery he had lately made, "showing that the Electrick Vertue of a Glass Tube may be conveyed to any other Bodies so as to give them the same Property of attracting and repelling light Bodies as the Tube does, when excited by rubbing: and that this attractive Vertue might be carried to Bodies that were many Feet distant from the Tube."

This was a result of the greatest importance, for previous workers had known of no other way of producing the attractive emanations than by rubbing the body concerned.‡ It was found that only a limited class of substances, among which the metals were conspicuous, had the capacity of acting as channels for the transport of the electric power; to these Desaguliers, who continued the experiments after Gray's death in 1736, gave* the name *non-electrics* or *conductors.*

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* Those who are interested in the literary history of the eighteenth century will recall the controversy as to whether the verses on the death of Stephen Gray were written by Anna Williams, whose name they bore, or by her patron Johnson.

† Phil. Trans. xxxvii (1731), pp. 18, 227, 285, 397.

‡ Otto von Guericke (*b.* 1602, *d.* 1686) had, as a matter of fact, observed the conduction of electricity along a linen thread; but this experiment does not seem to have been followed up. Cf. *Experimenta nova magdeburgica*, 1672.

After Gray's discovery it was no longer possible to believe that the electric effluvia are inseparably connected with the bodies from which they are evoked by rubbing; and it became necessary to admit that these emanations have an independent existence, and can be transferred from one body to another. Accordingly we find them recognized, under the name of the *electric fluid*,† as one of the substances of which the world is constituted. The imponderability of this fluid did not, for the reasons already mentioned, prevent its admission by the side of light and caloric into the list of chemical elements.

The question was actively debated as to whether the electric fluid was an element *sui generis,* or, as some suspected, was another manifestation of that principle whose operation is seen in the phenomena of heat. Those who held the latter view urged that the electric fluid and heat can both be induced by friction, can both induce combustion, and can both be transferred from one body to another by mere contact; and, moreover, that the best conductors of heat are also in general the best conductors of electricity. On the other hand it was contended that the electrification of a body does not cause any appreciable rise in its temperature; and an experiment of Stephen Gray's brought to light a yet more striking difference. Gray,‡ in 1729, made two oaken cubes, one solid and the other hollow, and showed that when electrified in the same way they produced exactly similar effects; whence he concluded that it was only the surfaces which had taken part in the phenomena. Thus while heat is disseminated throughout the substance of a body, the electric fluid resides at or near its surface. In the middle of the eighteenth century it was generally compared to an enveloping atmosphere. "The electricity which a non-electric of great length (for example, a hempen string 800 or 900 feet long) receives, runs from one end to the other in a sphere of electrical *Effluvia*," says Desaguliers in 1740;[*] and a report of the French Academy in 1733 says:[†] "Around an electrified body there is formed a vortex of exceedingly fine matter in a state of agitation, which urges towards the body such light substances as lie within its sphere of activity. The existence of this vortex is more than a mere conjecture; for when an electrified body is brought close to the face it causes a sensation like that of encountering a cobweb."[‡]

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* Phil. Trans. xli. (1739), pp. 186, 193, 200, 209: *Dissertation concerning Electricity*, 1742.

† The Cartesians defined a fluid to be a body whose minute parts are in a continual agitation.

‡ Phil. Trans. xxxvii., p. 35.

The report from which this is quoted was prepared in connexion with the discoveries of Charles-François du Fay (*b.* 1698, *d.* 1739), superintendent of gardens to the King of France. Du Fay[§] accounted for the behaviour of gold leaf when brought near to an electrified glass tube by supposing that at first the vortex of the tube envelopes the gold-leaf, and so attracts it towards the tube. But when contact occurs, the gold-leaf acquires the electric virtue, and so becomes surrounded by a vortex of its own. The two vortices, striving to extend in contrary senses, repel each other, and the vortex of the tube, being the stronger, drives away that of the gold-leaf. "It is then certain," says du Fay,[‖] "that bodies which have become electric by contact are repelled by those which have rendered them electric; but are they repelled likewise by other electrified bodies of all kinds? And do electrified bodies differ from each other in no respect save their intensity of electrification? An examination of this matter has led me to a discovery which I should never have foreseen, and of which I believe no one hitherto has had the least idea."

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[*] Phil. Trans. xli., p. 636.

[†] Hist. de l'Acad., 1733, p. 6.

[‡] This observation had been made first by Hawksbee at the beginning of the century.

[§] Mém. de l'Acad. des Sciences, 1733, pp. 23, 73, 233, 457; 1734, pp. 341, 503; 1737, p. 86; Phil. Trans. xxxviii. (1734), p. 258.

[‖] Mém. de l'Acad., 1733, p. 464.

He found, in fact, that when gold-leaf which had been electrified by contact with excited glass was brought near to an excited piece of copal,* an *attraction* was manifested between them. "I had expected," he writes, "quite the opposite effect, since, according to my reasoning, the copal and gold-leaf, which were both electrified, should have repelled each other." Proceeding with his experiments he found that the gold-leaf, when electrified and repelled by glass, was attracted by all electrified resinous substances, and that when repelled by the latter it was attracted by the glass. "We see, then," he continues, "that there are two electricities of a totally different nature—namely, that of transparent solids, such as glass, crystal, &c., and that of bituminous or resinous bodies, such as amber, copal, sealing-wax, &c. Each of them repels bodies which have contracted an electricity of the same nature as its own, and attracts those whose electricity is of the contrary nature. We see even that bodies which are not themselves electrics can acquire either of these electricities, and that then their effects are similar to those of the bodies which have communicated it to them."

To the two kinds of electricity whose existence was thus demonstrated, du Fay gave the names *vitreous* and *resinous*, by which they have ever since been known.

An interest in electrical experiments seems to have spread from du Fay to other members of the Court circle of Louis XV; and from 1745 onwards the Memoirs of the Academy contain a series of papers on the subject by the Abbé Jean-Antoine Nollet (*b.* 1700, *d.* 1770), afterwards preceptor in natural philosophy to the Royal Family. Nollet attributed electric phenomena to the movement in opposite directions of two currents of a fluid, "very subtle and inflammable," which he supposed to be present in all bodies under all circumstances.† When an electric is excited by friction, part of this fluid escapes from its pores, forming an *effluent stream*; and this loss is repaired by an

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* A hard transparent resin, used in the preparation of varnish.

† Cf. Nollet's *Recherches*, 1749, p. 245.

affluent stream of the same fluid entering the body from outside. Light bodies in the vicinity, being caught in one or other of these streams, are attracted or repelled from the excited electric.

Nollet's theory was in great vogue for some time; but six or seven years after its first publication, its author came across a work purporting to be a French translation of a book printed originally in England, describing experiments said to have been made at Philadelphia, in America, by one Benjamin Franklin. "He could not at first believe," as Franklin tells us in his *Autobiography*, "that such a work came from America, and said it must have been fabricated by his enemies at Paris to decry his system. Afterwards, having been assured that there really existed such a person as Franklin at Philadelphia, which he had doubted, he wrote and published a volume of letters, chiefly addressed to me, defending his theory, and denying the verity of my experiments, and of the positions deduced from them."

We must now trace the events which led up to the discovery which so perturbed Nollet.

In 1745 Pieter van Musschenbroek (*b.* 1692, *d.* 1761), Professor at Leyden, attempted to find a method of preserving electric charges from the decay which was observed when the charged bodies were surrounded by air. With this purpose he tried the effect of surrounding a charged mass of water by an envelope of some non-conductor, e.g., glass. In one of his experiments, a phial of water was suspended from a gun-barrel by a wire let down a few inches into the water through the cork; and the gun-barrel, suspended on silk lines, was applied so near an excited glass globe that some metallic fringes inserted into the gun-barrel touched the globe in motion. Under these circumstances a friend named Cunaeus, who happened to grasp the phial with one hand, and touch the gun-barrel with the other, received a violent shock; and it became evident that a method of accumulating or intensifying the electric power had been discovered.*

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* The discovery was made independently in the same year by Ewald Georg von Kleist, Dean of Kummin.

Shortly after the discovery of the *Leyden phial*, as it was named by Nollet, had become known in England, a London apothecary named William Watson (*b.* 1715, *d.* 1787)[*] noticed that when the experiment is performed in this fashion the observer feels the shock "in no other parts of his body but his arms and breast"; whence he inferred that in the act of discharge there is a transference of something which takes the shortest or best-conducting path between the gun-barrel and the phial. This idea of transference seemed to him to bear some similarity to Nollet's doctrine of afflux and efflux; and there can indeed be little doubt that the Abbé's hypothesis, though totally false in itself, furnished some of the ideas from which Watson, with the guidance of experiment, constructed a correct theory. In a memoir[†] read to the Royal Society in October, 1746, he propounded the doctrine that electrical actions are due to the presence of an "electrical aether," which in the charging or discharging of a Leyden jar is *transferred*, but is not created or destroyed. The excitation of an electric, according to this view, consists not in the evoking of anything from within the electric itself without compensation, but in the accumulation of a surplus of electrical aether by the electric *at the expense of some other body*, whose stock is accordingly depleted. All bodies were supposed to possess a certain natural store, which could be drawn upon for this purpose.

"I have shewn," wrote Watson, "that electricity is the effect of a very subtil and elastic fluid, occupying all bodies in contact with the terraqueous globe; and that every-where, in its natural state, it is of the same degree of density; and that glass and other bodies, which we denominate electrics *per se*, have the power, by certain known operations, of taking this fluid from one body, and conveying it to another, in a quantity sufficient to be obvious to all our senses; and that, under certain circumstances, it was possible to render the electricity in some bodies more rare than it naturally is, and, by communicating this to other bodies, to give them an additional quantity, and make their electricity more dense."

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[*] Watson afterwards rose to eminence in the medical profession, and was knighted.

[†] Phil. Trans. xliv., p. 718. It may here be noted that it was Watson who improved the phial by coating it nearly to the top, both inside and outside, with tinfoil.

In the same year in which Watson's theory was proposed, a certain Dr. Spence, who had lately arrived in America from Scotland, was showing in Boston some electrical experiments. Among his audience was a man who already at forty years of age was recognized as one of the leading citizens of the English colonies in America, Benjamin Franklin of Philadelphia (*b.* 1706, *d.* 1790). Spence's experiments "were," writes Franklin,* "imperfectly performed, as he was not very expert; but, being on a subject quite new to me, they equally surprised and pleased me." Soon after this, the "Library Company" of Philadelphia (an institution founded by Franklin himself) received from Mr. Peter Collinson of London a present of a glass tube, with some account of its use. In a letter written to Collinson on July 11th, 1747,† Franklin described experiments made with this tube, and certain deductions which he had drawn from them.

If one person A, standing on wax so that electricity cannot pass from him to the ground, rubs the tube, and if another person B, likewise standing on wax, passes his knuckle along near the glass so as to receive its electricity, then both A and B will be capable of giving a spark to a third person C standing on the floor; that is, they will be electrified. If, however, A and B touch each other, either during or after the rubbing, they will not be electrified.

This observation suggested to Franklin the same hypothesis that (unknown to him) had been propounded a few months previously by Watson: namely, that electricity is an element present in a certain proportion in all matter in its normal condition; so that, before the rubbing, each of the persons A, B, and C has an equal share. The effect of the rubbing is to transfer some of A's electricity to the glass, whence it is transferred to B. Thus A has a deficiency and B a superfluity of electricity; and if either of them approaches C, who has the normal amount, the distribution will be equalized by a spark. If, however, A and B are in contact, electricity flows between them so as to re-establish the original equality, and neither is then electrified with reference to C.

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* Franklin's *Autobiography.*

† Franklin's *New Experiments and Observations on Electricity*, letter ii.

Thus electricity is not created by rubbing the glass, but only transferred to the glass from the rubber, so that the rubber loses exactly as much as the glass gains; *the total quantity of electricity in any insulated system is invariable.* This assertion is usually known as the *principle of conservation of electric charge.*

The condition of A and B in the experiment can evidently be expressed by plus and minus signs: A having a deficiency $-e$ and B a superfluity $+e$ of electricity. Franklin, at the commencement of his own experiments, was not acquainted with du Fay's discoveries; but it is evident that the electric fluid of Franklin is identical with the vitreous electricity of du Fay, and that du Fay's resinous electricity is, in Franklin's theory, merely the deficiency of a stock of vitreous electricity supposed to be possessed naturally by all ponderable bodies. In Franklin's theory we are spared the necessity for admitting that two quasi-material bodies can by their union annihilate each other, as vitreous and resinous electricity were supposed to do.

Some curiosity will naturally be felt as to the considerations which induced Franklin to attribute the positive character to vitreous rather than to resinous electricity. They seem to have been founded on a comparison of the brush discharges from conductors charged with the two electricities; when the electricity was resinous, the discharge was observed to spread over the surface of the opposite conductor "as if it flowed from it." Again, if a Leyden jar whose inner coating is electrified vitreously is discharged silently by a conductor, of whose pointed ends one is near the knob and the other near the outer coating, the point which is near the knob is seen in the dark to be illuminated with a star or globule, while the point which is near the outer coating is illuminated with a pencil of rays; which suggested to Franklin that the electric fluid, going from the inside to the outside of the jar, enters at the former point and issues from the latter. And yet again, in some cases the flame of a wax taper is blown away from a brass ball which is discharging vitreous electricity, and towards one which is discharging resinous electricity. But Franklin remarks that the interpretation of these observations is somewhat conjectural, and that whether vitreous or resinous electricity is the actual electric fluid is not certainly known.

Regarding the physical nature of electricity, Franklin held much the same ideas as his contemporaries; he pictured it as an elastic* fluid, consisting of "particles extremely subtile, since it can permeate common matter, even the densest metals, with such ease and freedom as not to receive any perceptible resistance." He departed, however, to some extent from the conceptions of his predecessors, who were accustomed to ascribe all electrical repulsions to the diffusion of effluvia from the excited electric to the body acted on; so that the tickling sensation which is experienced when a charged body is brought near to the human face was attributed to a direct action of the effluvia on the skin. This doctrine, which, as we shall see, practically ended with Franklin, bears a suggestive resemblance to that which nearly a century later was introduced by Faraday; both explained electrical phenomena without introducing action at a distance, by supposing that something which forms an essential part of the electrified system is present at the spot where any electric action takes place; but in the older theory this something was identified with the electric fluid itself, while in the modern view it is identified with a state of stress in the aether. In the interval between the fall of one school and the rise of the other, the theory of action at a distance was dominant.

The germs of the last-mentioned theory may be found in

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* i.e., repulsive of its own particles.

Franklin's own writings. It originated in connexion with the explanation of the Leyden jar, a matter which is discussed in his third letter to Collinson, of date September 1st, 1747. In charging the jar, he says, a quantity of electricity is taken away from one side of the glass, by means of the coating in contact with it, and an equal quantity is communicated to the other side, by means of the other coating. The glass itself he supposes to be impermeable to the electric fluid, so that the deficiency on the one side can permanently coexist with the redundancy on the other, so long as the two sides are not connected with each other; but when a connexion is set up, the distribution of fluid is equalized through the body of the experimenter, who receives a shock.

Compelled by this theory of the jar to regard glass as impenetrable to electric effluvia, Franklin was nevertheless well aware* that the interposition of a glass plate between an electrified body and the objects of its attraction does not shield the latter from the attractive influence. He was thus driven to suppose† that the surface of the glass which is nearest the excited body is directly affected, and is able to exert an influence through the glass on the opposite surface; the latter surface, which thus receives a kind of secondary or derived excitement, is responsible for the electric effects beyond it.

This idea harmonized admirably with the phenomena of the jar; for it was now possible to hold that the excess of electricity on the inner face exercises a repellent action through the substance of the glass, and so causes a deficiency on the outer faces by driving away the electricity from it.‡

Franklin had thus arrived at what was really a theory of action at a distance between the particles of the electric fluid; and this he was able to support by other experiments. "Thus," he writes,§ "the stream of a fountain, naturally dense and continual, when electrified, will separate and spread in the form of a brush, every drop endeavouring to recede from every other drop.' In order to account for the attraction between oppositely charged bodies, in one of which there is an excess of electricity as compared with ordinary matter, and in the other an excess of ordinary matter as compared with electricity, he assumed that "though the particles of electrical matter do repel each other, they are strongly attracted by all other matter"; so that "common matter is as a kind of spunge to the electrical fluid."

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* *New Experiments*, 1750, § 28.

† *Ibid.*, 1750, § 34.

‡ *Ibid.*, 1750, § 32.

§ Letter v.

These repellent and attractive powers he assigned only to the actual (vitreous) electric fluid; and when later on the mutual repulsion of resinously electrified bodies became known to him,* it caused him considerable perplexity.† As we shall see, the difficulty was eventually removed by Aepinus.

In spite of his belief in the power of electricity to act at a distance, Franklin did not abandon the doctrine of effluvia. "The form of the electrical atmosphere," he says,‡ "is that of the body it surrounds. This shape may be rendered visible in a still air, by raising a smoke from dry rosin dropt into a hot tea-spoon under the electrified body, which will be attracted, and spread itself equally on all sides, covering and concealing the body, And this form it takes, because it is attracted by all parts of the surface of the body, though it cannot enter the substance already replete. Without this attraction, it would not remain round the body, but dissipate in the air." He observed, however, that electrical effluvia do not seem to affect, or be affected by, the air; since it is possible to breathe freely in the neighbourhood of electrified bodies; and moreover a current of dry air does not destroy electric attractions and repulsions.§

Regarding the suspected identity of electricity with the matter of heat, as to which Nollet had taken the affirmative position, Franklin expressed no opinion. "Common fire," he writes,[*] "is in all bodies, more or less, as well as electrical fire. Perhaps they may be different modifications of the same element; or they may be different elements. The latter is by some suspected. If they are different things, yet they may and do subsist together in the same body."

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* He refers to it in his Paper read to the Royal Society, December 18, 1755.

† Cf. letters xxxvii and xxxviii, dated 1761 and 1762.

‡ *New Experiments*, 1750, § 15.

§ Letter vii, 1751.

Franklin's work did not at first receive from European philosophers the attention which it deserved; although Watson generously endeavoured to make the colonial writer's merits known,[†] and inserted some of Franklin's letters in one of his own papers communicated to the Royal Society. But an account of Franklin's discoveries, which had been printed in England, happened to fall into the hands of the naturalist Buffon, who was so much impressed that he secured the issue of a French translation of the work; and it was this publication which, as we have seen, gave such offence to Nollet. The success of a plan proposed by Franklin for drawing lightning from the clouds soon engaged public attention everywhere; and in a short time the triumph of the *one-fluid theory* of electricity, as the hypothesis of Watson and Franklin is generally called, was complete. Nollet, who was obdurate, "lived to see himself the last of his sect, except Monsieur B— of Paris, his élève and immediate disciple."[‡]

The theory of effluvia was finally overthrown, and replaced by that of action at a distance, by the labours of one of Franklin's continental followers, Francis Ulrich Theodore Aepinus[§] (*b.* 1724, *d.* 1802). The doctrine that glass is impermeable to electricity, which had formed the basis of Franklin's theory of the Leyden phial, was generalized by Aepinus[‖] and his co-worker Johann Karl Wilcke (*b.* 1732, *d.* 1796) into the law that all non-conductors are impermeable to the electric fluid. That this applies even to air they proved by constructing a machine analogous to the Leyden jar, in which, however, air took the place of glass as the medium between two oppositely charged surfaces. The success of this experiment led Aepinus to deny altogether the existence of electric effluvia surrounding charged bodies:* a position which he regarded as strengthened by Franklin's observation, that the electric field in the neighbourhood of an excited body is not destroyed when the adjacent air is blown away. The electric fluid must therefore be supposed not to extend beyond the excited bodies themselves. The experiment of Gray, to which we have already referred, showed that it does not penetrate far into their substance; and thus it became necessary to suppose that the electric fluid, in its state of rest, is confined to thin layers on the surfaces of the excited bodies. This being granted, the attractions and repulsions observed between the bodies compel us to believe that electricity acts at a distance across the intervening air.

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[*] Letter v.

[†] Phil. Trans. xlvii, p. 202. Watson agreed with Nollet in rejecting Franklin's theory of the impermeability of glass.

[‡] Franklin's *Autobiography*.

[§] This philosopher's surname had been hellenized from its original form *Hoeck* to *αιπεινος* by one of his ancestors, a distinguished theologian.

[‖] F. V. T. Aepinus *Tentamen Theoriae Electricitatis et Magnetismi*: St. Petersburg, 1759.

Since two vitreously charged bodies repel each other, the force between two particles of the electric fluid must (on Franklin's one-fluid theory, which Aepinus adopted) be repulsive: and since there is an attraction between oppositely charged bodies, the force between electricity and ordinary matter must be attractive. These assumptions had been made, as we have seen, by Franklin; but in order to account for the repulsion between two resinously charged bodies, Aepinus introduced a new supposition—namely, that the particles of ordinary matter repel each other. This, at first, startled his contemporaries; but, as he pointed out, the "unelectrified" matter with which we are acquainted is really matter saturated with its natural quantity of the electric fluid, and the forces due to the matter and fluid balance each other; or perhaps, as he suggested, a slight want of equality between these forces might give, as a residual, the force of gravitation.

Assuming that the attractive and repellent forces increase as the distance between the acting charges decreases, Aepinus applied his theory to explain a phenomenon which had been more or less indefinitely observed by many previous writers, and specially studied a short time previously by John Canton* (*b.* 1718, *d.* 1772) and by Wilcke†—namely, that if a conductor is brought into the neighbourhood of an excited body without actually touching it, the remoter portion of the conductor acquires an electric charge of the same kind as that of the excited body, while the nearer portion acquires a charge of the opposite kind. This effect, which is known as the *induction* of electric charges, had been explained by Canton himself and by Franklin‡ in terms of the theory of electric effluvia. Aepinus showed that it followed naturally from the theory of action at a distance, by taking into account the mobility of the electric fluid in conductors; and by discussing different cases, so far as was possible with the means at his command, he laid the foundations of the mathematical theory of electrostatics.

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* This was also maintained about the same time by Giacomo Battista Beccaria of Turin (*b.* 1716, *d.* 1781).

Aepinus did not succeed in determining the law according to which the force between two electric charges varies with the distance between them; and the honour of having first accomplished this belongs to Joseph Priestley (*b.* 1733, *d.* 1804), the discoverer of oxygen. Priestley, who was a friend of Franklin's, had been informed by the latter that he had found cork balls to be wholly unaffected by the electricity of a metal cup within which they were held; and Franklin desired Priestley to repeat and ascertain the fact. Accordingly, on December 21st, 1766, Priestley instituted experiments, which showed that, when a hollow metallic vessel is electrified, there is no charge on the inner surface (except near the opening), and no electric force in the air inside. From this he at once drew the correct conclusion, which was published in 1767.§ "May we not infer," he says, "from this experiment that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances; since it is easily demonstrated that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another?"

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* Phil. Trans. xlviii (1753), p. 350.

† *Disputatio physica experimentalis de electricitatibus contrariis:* Rostock, 1757.

‡ In his paper read to the Royal Society on Dec. 18th, 1755.

§ J. Priestley, *The History and Present State of Electricity, with Original Experiments*; London, 1767; page 732. That electrical attraction follows the law of the inverse square had been suspected by Daniel Bernoulli in 1760: Cf. Socin's Experiments, *Acta Helvetica*, iv, p. 214.

This brilliant inference seems to have been insufficiently studied by the scientific men of the day; and, indeed, its author appears to have hesitated to claim for it the authority of a complete and rigorous proof. Accordingly we find that the question of the law of force was not regarded as finally settled for eighteen years afterwards.*

By Franklin's law of the conservation of electric charge, and Priestley's law of attraction between charged bodies, electricity was raised to the position of an exact science. It is impossible to mention the names of these two friends in such a connexion without reflecting on the curious parallelism of their lives. In both men there was the same combination of intellectual boldness and power with moral earnestness and public spirit. Both of them carried on a long and tenacious struggle with the reactionary influences which dominated the English Government in the reign of George III; and both at last, when overpowered in the conflict, reluctantly exchanged their native flag for that of the United States of America. The names of both have been held in honour by later generations, not more for their scientific discoveries than for their services to the cause of religious, intellectual, and political freedom.

The most celebrated electrician of Priestley's contemporaries in London was the Hon. Henry Cavendish (*b.* 1731, *d.* 1810), whose interest in the subject was indeed hereditary, for his father, Lord Charles Cavendish, had assisted in Watson's experiments of 1747.† In 1771 Cavendish‡ presented to the Royal Society an "Attempt to explain some of the principal phenomena of Electricity, by means of an elastic fluid." The hypothesis adopted is that of the one-fluid theory, in much the same form as that of Aepinus. It was, as he tells us, discovered independently, although he became acquainted with Aepinus' work before the publication of his own paper.

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* In 1769 Dr. John Robison (*b.* 1739, *d.* 1805), of Edinburgh, endeavoured to determine the law of force by direct experiment, and found it to be that of the inverse $2{\cdot}06^{\text{th}}$ power of the distance.

† Phil. Trans. xlv, p. 67 (1750).

‡ Phil. Trans. lxi, p. 584 (1771).

In this memoir Cavendish makes no assumption regarding the law of force between electric charges, except that it is "inversely as some less power of the distance than the cube"; but he evidently inclines to believe in the law of the inverse square. Indeed, he shows it to be "likely, that if the electric attraction or repulsion is inversely as the square of the distance, almost all the redundant fluid in the body will be lodged close to the surface, and there pressed close together, and the rest of the body will be saturated"; which approximates closely to the discovery made four years previously by Priestley. Cavendish did, as a matter of fact, rediscover the inverse square law shortly afterwards; but, indifferent to fame, he neglected to communicate to others this and much other work of importance. The value of his researches was not realized until the middle of the nineteenth century, when William Thomson (Lord Kelvin) found in Cavendish's manuscripts the correct value for the ratio of the electric charges carried by a circular disk and a sphere of the same radius which had been placed in metallic connexion. Thomson urged that the papers should be published; which came to pass* in 1879, a hundred years from the date of the great discoveries which they enshrined. It was then seen that Cavendish had anticipated his successors in several of the ideas which will presently be discussed—amongst others, those of electrostatic capacity and specific inductive capacity.

In the published memoir of 1771 Cavendish worked out the consequences of his fundamental hypothesis more completely than Aepinus; and, in fact, virtually introduced the notion of electric potential, though, in the absence of any definite assumption as to the law of force, it was impossible to develop this idea to any great extent.

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* *The Electrical Researches of the Hon. Henry Cavendish*, edited by J. Clerk Maxwell, 1879.

One of the investigations with which Cavendish occupied himself was a comparison between the conducting powers of different materials for electrostatic discharges. The question had been first raised by Beccaria, who had shown* in 1753 that when the circuit through which a discharge is passed contains tubes of water, the shock is more powerful when the cross-section of the tubes is increased. Cavendish went into the matter much more thoroughly, and was able, in a memoir presented to the Royal Society in 1775,† to say: "It appears from some experiments, of which I propose shortly to lay an account before this Society, that iron wire conducts about 400 million times better than rain or distilled water—that is, the electricity meets with no more resistance in passing through a piece of iron wire 400,000,000 inches long than through a column of water of the same diameter only one inch long. Sea-water, or a solution of one part of salt in 30 of water, conducts 100 times, or a saturated solution of sea-salt about 720 times, better than rain-water."

The promised account of the experiments was published in the volume edited in 1879. It appears from it that the method of testing by which Cavendish obtained these results was simply that of physiological sensation; but the figures given in the comparison of iron and sea-water are remarkably exact.

While the theory of electricity was being established on a sure foundation by the great investigators of the eighteenth century, a no less remarkable development was taking place in the kindred science of magnetism, to which our attention must now be directed.

The law of attraction between magnets was investigated at an earlier date than the corresponding law for electrically charged bodies. Newton, in the *Principia*,‡ says: "The power of gravity is of a different nature from the power of magnetism. For the magnetic attraction is not as the matter attracted. Some bodies are attracted more by the magnet, others less; most bodies not at all. The power of magnetism, in one and the same body, may be increased and diminished; and is sometimes far stronger, for the quantity of matter, than the power of gravity; and in receding from the magnet, decreases not in the duplicate, but almost in the triplicate proportion of the distance, as nearly as I could judge from some rude observations."

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* G. B. Beccaria, *Dell' elettricismo artificiale e naturale*, Turin, 1753, p. 113.

† Phil. Trans. lxvi (1776), p. 196.

‡ Book iii, Prop. vi, cor. 5.

The edition of the *Principia* which was published in 1742 by Thomas Le Seur and Francis Jacquier contains a note on this corollary, in which the correct result is obtained that the directive couple exercised on one magnet by another is proportional to the inverse cube of the distance.

The first discoverer of the law of force between magnetic poles was John Michell (*b.* 1724, *d.* 1793), at that time a young Fellow of Queen's College, Cambridge,* who in 1750 published *A Treatise of Artificial Magnets; in which is shown an easy and expeditious method of making them superior to the best natural ones.* In this he states the principles of magnetic theory as follows† :—

"Wherever any Magnetism is found, whether in the Magnet itself, or any piece of Iron, etc., excited by the Magnet, there are always found two Poles, which are generally called North and South; and the North Pole of one Magnet always attracts the South Pole, and repels the North Pole of another: and *vice versa.*" This is of course adopted from Gilbert.

"Each Pole attracts or repels exactly equally, at equal distances, in every direction." This, it may be observed, overthrows the theory of vortices, with which it is irreconcilable. "The Magnetical Attraction and Repulsion are exactly equal to each other." This, obvious though it may seem to us, was really a most important advance, for, as he remarks, "Most people, who have mention'd any thing relating to this property of the Magnet, have agreed, not only that the Attraction and Repulsion of Magnets are not equal to each other, but that also, they do not observe the same rule of increase and decrease."

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* Michell had taken his degree only two years previously. Later in life he was on terms of friendship with Priestley, Cavendish, and William Herschel; it was he who taught Herschel the art of grinding mirrors for telescopes. The plan of determining the density of the earth, which was carried out by Cavendish in 1798, and is generally known as the "Cavendish Experiment," was due to Michell. Michell was the first inventor of the torsion-balance; he also made many valuable contributions to Astronomy. In 1767 he became Rector of Thornhill, Yorks, and lived there until his death.

† Loc. cit., p. 17.

"The Attraction and Repulsion of Magnets decreases, as the Squares of the distances from the respective poles increase." This great discovery, which is the basis of the mathematical theory of Magnetism, was deduced partly from his own observations, and partly from those of previous investigators (e.g. Dr. Brook Taylor and P. Muschenbroek), who, as he observes, had made accurate experiments, but had failed to take into account all the considerations necessary for a sound theoretical discussion of them.

After Michell the law of the inverse square was maintained by Tobias Mayer* of Göttingen (*b.* 1723, *d.* 1762), better known as the author of Lunar Tables which were long in use; and by the celebrated mathematician, Johann Heinrich Lambert† (*b.* 1728, *d.* 1777).

The promulgation of the one-fluid theory of electricity, in the middle of the eighteenth century, naturally led to attempts to construct a similar theory of magnetism; this was effected in 1759 by Aepinus‡, who supposed the "poles" to be places at which a *magnetic fluid* was present in amount exceeding or falling short of the normal quantity. The permanence of magnets was accounted for by supposing the fluid to be entangled in their pores, so as to be with difficulty displaced. The particles of the fluid were assumed to repel each other, and to attract the particles of iron and steel; but, as Aepinus saw, in order to satisfactorily explain magnetic phenomena it was necessary to assume also a mutual repulsion among the material particles of the magnet.

Subsequently *two* imponderable magnetic fluids, to which the names *boreal* and *austral* were assigned, were postulated by the Hollander Anton Brugmans (*b.* 1732, *d.* 1789) and by Wilcke. These fluids were supposed to have properties of mutual attraction and repulsion similar to those possessed by vitreous and resinous electricity.

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* Noticed in *Göttinger Gelehrter Anzeiger*, 1760: cf. Aepinus, *Nov. Comm. Acad. Petrop.*, 1768, and Mayer's *Opera Inedita*, herausg. von G. C. Lichtenberg.

† *Histoire de l'Acad. de Berlin*, 1766, pp. 22, 49.

‡ In the *Tentamen*, to which reference has already been made.

The writer who next claims our attention for his services both to magnetism and to electricity is the French physicist, Charles Augustin Coulomb* (*b.* 1736, *d.* 1806). By aid of the torsion-balance, which was independently invented by Michell and himself, he verified in 1785 Priestley's fundamental law that the repulsive force between two small globes charged with the same kind of electricity is in the inverse ratio of the square of the distance of their centres. In the second memoir he extended this law to the attraction of opposite electricities.

Coulomb did not accept the one-fluid theory of Franklin, Aepinus, and Cavendish, but preferred a rival hypothesis which had been proposed in 1759 by Robert Symmer.† "My notion," said Symmer, "is that the operations of electricity do not depend upon one single positive power, according to the opinion generally received; but upon two distinct, positive, and active powers, which, by contrasting, and, as it were, counteracting each other, produce the various phenomena of electricity; and that, when a body is said to be positively electrified, it is not simply that it is possessed of a larger share of electric matter than in a natural state; nor, when it is said to be negatively electrified, of a less; but that, in the former case, it is possessed of a larger portion of one of those active powers, and in the latter, of a larger portion of the other; while a body in its natural state remains unelectrified, from an equal ballance of those two powers within it."

Coulomb developed this idea: "Whatever be the cause of electricity," he says,‡ "we can explain all the phenomena by supposing that there are two electric fluids, the parts of the same fluid repelling each other according to the inverse square of the distance, and attracting the parts of the other fluid according to the same inverse square law." "The supposition of two fluids," he adds, "is moreover in accord with all those discoveries of modern chemists and physicists, which have made known to us various pairs of gases whose elasticity is destroyed by their admixture in certain proportions—an effect which could not take place without something equivalent to a repulsion between the parts of the same gas, which is the cause of its elasticity, and an attraction between the parts of different gases, which accounts for the loss of elasticity on combination."

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* Coulomb's First, Second, and Third Memoirs appear in Mémoires de l'Acad., 1785; the Fourth in 1786, the Fifth in 1787, the Sixth in 1788, and the Seventh in 1789.

† Phil. Trans. li (1759), p. 371.

‡ Sixth Memoir, p. 561.

According, then, to the two-fluid theory, the "natural fluid" contained in all matter can be decomposed, under the influence of an electric field, into equal quantities of vitreous and resinous electricity, which, if the matter be conducting, can then fly to the surface of the body. The abeyance of the characteristic properties of the opposite electricities when in combination was sometimes further compared to the neutrality manifested by the compound of an acid and an alkali.

The publication of Coulomb's views led to some controversy between the partisans of the one-fluid and two-fluid theories; the latter was soon generally adopted in France, but was stoutly opposed in Holland by Van Marum and in Italy by Volta. The chief difference between the rival hypotheses is that, in the two-fluid theory, both the electric fluids are movable within the substance of a solid conductor; while in the one-fluid theory the actual electric fluid is mobile, but the particles of the conductor are fixed. The dispute could therefore be settled only by a determination of the actual motion of electricity in discharges; and this was beyond the reach of experiment.

In his Fourth Memoir Coulomb showed that electricity in equilibrium is confined to the surface of conductors, and does not penetrate to their interior substance; and in the Sixth Memoir* he virtually establishes the result that the electric force near a conductor is proportional to the surface-density of electrification.

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* Page 677.

Since the overthrow of the doctrine of electric effluvia by Aepinus, the aim of electricians had been to establish their science upon the foundation of a law of action at a distance, resembling that which had led to such triumphs in Celestial Mechanics. When the law first stated by Priestley was at length decisively established by Coulomb, its simplicity and beauty gave rise to a general feeling of complete trust in it as the best attainable conception of electrostatic phenomena. The result was that attention was almost exclusively focused on action-at-a-distance theories, until the time, long afterwards, when Faraday led natural philosophers back to the right path.

Coulomb rendered great services to magnetic theory. It was he who in 1777, by simple mechanical reasoning, completed the overthrow of the hypothesis of vortices.* He also, in the second of the Memoirs already quoted,† confirmed Michell's law, according to which the particles of the magnetic fluids attract or repel each other with forces proportional to the inverse square of the distance. Coulomb, however, went beyond this, and endeavoured to account for the fact that the two magnetic fluids, unlike the two electric fluids, cannot be obtained separately; for when a magnet is broken into two pieces, one containing its north and the other its south pole, it is found that each piece is an independent magnet possessing two poles of its own, so that it is impossible to obtain a north or south pole in a state of isolation. Coulomb explained this by supposing‡ that the magnetic fluids are permanently imprisoned within the molecules of magnetic bodies, so as to be incapable of crossing from one molecule to the next; each molecule therefore under all circumstances contains as much of the boreal as of the cal fluid, and magnetization consists simply in a separation he two fluids to opposite ends of each molecule. Such ypothesis evidently accounts for the impossibility of rating the two fluids to opposite ends of a body of finite The same idea, here introduced for the first time, has e been applied with success in other departments of rical philosophy.

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* *Mém. présentés par divers Savans*, ix (1780), p. 165.

† Mém de l'Acad., 1785, p. 593. Gauss finally established the law by a much more refined method.

‡ In his Seventh Memoir, Mém, de l'Acad., 1789, p. 488.

In spite of the advances which have been recounted, mathematical development of electric and magnetic theory scarcely begun at the close of the eighteenth century; and y erroneous notions were still widely entertained. In a ort* which was presented to the French Academy in 1800, as assumed that the mutual repulsion of the particles of tricity on the surface of a body is balanced by the stance of the surrounding air; and for long afterwards electric force outside a charged conductor was confused n a supposed additional pressure in the atmosphere.

Electrostatical theory was, however, suddenly advanced to te a mature state of development by Siméon Denis Poisson 1781, *d.* 1840), in a memoir which was read to the French demy in 1812.† As the opening sentences show, he accepted conceptions of the two-fluid theory.

" The theory of electricity which is most generally accepted," says, " is that which attributes the phenomena to two erent fluids, which are contained in all material bodies. is supposed that molecules of the same fluid repel each er and attract the molecules of the other fluid; these ces of attraction and repulsion obey the law of the inverse are of the distance; and at the same distance the attractive wer is equal to the repellent power; whence it follows it, when all the parts of a body contain equal quantities the two fluids, the latter do not exert any influence on e fluids contained in neighbouring bodies, and consequently electrical effects are discernible. This equal and uniform distribution of the two fluids is called the *natural state*; when this state is disturbed in any body, the body is said to be *electrified*, and the various phenomena of electricity begin to take place.

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* On Volta's discoveries.

† Mém. de l'Institut, 1811, Part i., p. 1, Part ii., p. 163.

"Material bodies do not all behave in the same way with respect to the electric fluid: some, such as the metals, do not appear to exert any influence on it, but permit it to move about freely in their substance; for this reason they are called *conductors*. Others, on the contrary—very dry air, for example—oppose the passage of the electric fluid in their interior, so that they can prevent the fluid accumulated in conductors from being dissipated throughout space."

When an excess of one of the electric fluids is communicated to a metallic body, this charge distributes itself over the surface of the body, forming a layer whose thickness at any point depends on the shape of the surface. The resultant force due to the repulsion of all the particles of this surface-layer must vanish at any point in the interior of the conductor, since otherwise the natural state existing there would be disturbed; and Poisson showed that by aid of this principle it is possible in certain cases to determine the distribution of electricity in the surface-layer. For example, a well-known proposition of the theory of Attractions asserts that a hollow shell whose bounding surfaces are two similar and similarly situated ellipsoids exercises no attractive force at any point within the interior hollow; and it may thence be inferred that, if an electrified metallic conductor has the form of an ellipsoid, the charge will be distributed on it proportionally to the normal distance from the surface to an adjacent similar and similarly situated ellipsoid.

Poisson went on to show that this result was by no means all that might with advantage be borrowed from the theory of Attractions. Lagrange, in a memoir on the motion of gravitating bodies, had shown* that the components of the attractive force at any point can be simply expressed as the derivates of the function which is obtained by adding together the masses of all the particles of an attracting system, each divided by its distance from the point; and Laplace had shown* that this function V satisfies the equation

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* *Mém. de Berlin*, 1777. The theorem was afterwards published, and ascribed to Laplace, in a memoir by Legendre on the Attractions of Spheroids, which will be found in the *Mém. par divers Savans*, published in 1785.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

in space free from attracting matter. Poisson himself showed later, in 1813,† that when the point (x, y, z) is within the substance of the attracting body, this equation of Laplace must be replaced by

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho,$$

where ρ denotes the density of the attracting matter at the point. In the present memoir Poisson called attention to the utility of this function V in electrical investigations, remarking that its value over the surface of any conductor must be constant.

The known formulae for the attractions of spheroids show that when a charged conductor is spheroidal, the repellent force acting on a small charged body immediately outside it will be directed at right angles to the surface of the spheroid, and will be proportional to the thickness of the surface-layer of electricity at this place. Poisson suspected that this theorem might be true for conductors not having the spheroidal form—a result which, as we have seen, had been already virtually given by Coulomb; and Laplace suggested to Poisson the following proof, applicable to the general case. The force at a point immediately outside the conductor can be divided into a part s due to the part of the charged surface immediately adjacent to the point, and a part S due to the rest of the surface. At a point close to this, but just inside the conductor, the force S will still act; but the force s will evidently be reversed in direction. Since the resultant force at the latter point vanishes, we must have $S = s$; so the resultant force at the exterior point is $2s$. But s is proportional to the charge per unit area of the surface, as is seen by considering the case of an infinite plate; which establishes the theorem.

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* Mém. de l'Acad., 1782 (published in 1785), p. 113.

† Bull. de la Soc. Philomathique. iii. (1813), p. 388.

When several conductors are in presence of each other, the distribution of electricity on their surfaces may be determined by the principle, which Poisson took as the basis of his work, that at any point in the interior of any one of the conductors, the resultant force due to all the surface-layers must be zero. He discussed, in particular, one of the classical problems of electrostatics—namely, that of determining the surface-density on two charged conducting spheres placed at any distance from each other. The solution depends on Double Gamma Functions in the general case; when the two spheres are in contact, it depends on ordinary Gamma Functions. Poisson gave a solution in terms of definite integrals, which is equivalent to that in terms of Gamma Functions; and after reducing his results to numbers, compared them with Coulomb's experiments.

The rapidity with which in a single memoir Poisson passed from the barest elements of the subject to such recondite problems as those just mentioned may well excite admiration. His success is, no doubt, partly explained by the high state of development to which analysis had been advanced by the great mathematicians of the eighteenth century; but even after allowance has been made for what is due to his predecessors, Poisson's investigation must be accounted a splendid memorial of his genius.

Some years later Poisson turned his attention to magnetism; and, in a masterly paper* presented to the French Academy in 1824, gave a remarkably complete theory of the subject.

His starting-point is Coulomb's doctrine of two imponderable magnetic fluids, arising from the decomposition of a neutral fluid, and confined in their movements to the individual elements of the magnetic body, so as to be incapable of passing from one element to the next.

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* Mem. de l'Acad., v, p. 247.

Suppose that an amount m of the positive magnetic fluid is located at a point (x, y, z); the components of the *magnetic intensity*, or force exerted on unit magnetic pole, at a point (ξ, η, ζ) will evidently be

$$-m\frac{\partial}{\partial\xi}\left(\frac{1}{r}\right), \qquad -m\frac{\partial}{\partial\eta}\left(\frac{1}{r}\right), \qquad -m\frac{\partial}{\partial\zeta}\left(\frac{1}{r}\right),$$

where r denotes $\{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\}^{\frac{1}{2}}$. Hence if we consider next a magnetic element in which equal quantities of the two magnetic fluids are displaced from each other parallel to the x axis, the components of the magnetic intensity at (ξ, η, ζ) will be the negative derivates, with respect to ξ, η, ζ respectively, of the function

$$A\frac{\partial}{\partial x}\left(\frac{1}{r}\right),$$

where the quantity A, which does not involve (ξ, η, ζ), may be called the *magnetic moment* of the element: it may be measured by the couple required to maintain the element in equilibrium at a definite angular distance from the magnetic meridian.

If the displacement of the two fluids from each other in the element is not parallel to the axis of x, it is easily seen that the expression corresponding to the last is

$$A\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + B\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + C\frac{\partial}{\partial z}\left(\frac{1}{r}\right),$$

where the vector (A, B, C) now denotes the magnetic moment of the element.

Thus the magnetic intensity at an external point (ξ, η, ζ) due to any magnetic body has the components

$$\left(-\frac{\partial V}{\partial\xi}, \quad -\frac{\partial V}{\partial\eta}, \quad -\frac{\partial V}{\partial\zeta}\right),$$

where

$$V = \iiint\left(A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y} + C\frac{\partial}{\partial z}\right)\left(\frac{1}{r}\right)dx\,dy\,dz$$

integrated throughout the substance of the magnetic body, and where the vector (A, B, C) or **I** represents the magnetic moment per unit-volume, or, as it is generally called, the *magnetization*. The function V was afterwards named by Green the *magnetic potential.*

Poisson, by integrating by parts the preceding expression for the magnetic potential, obtained it in the form

$$V = \iint (\mathbf{I}\,.\,d\mathbf{S}).\frac{1}{r} - \iiint \frac{1}{r} \operatorname{div} \mathbf{I}\, dx\, dy\, dz,^*$$

the first integral being taken over the surface S of the magnetic body, and the second integral being taken throughout its volume. This formula shows that the magnetic intensity produced by the body in external space is the same as would be produced by a fictitious distribution of magnetic fluid, consisting of a layer over its surface, of surface-charge $(\mathbf{I}\,.\,d\mathbf{S})$ per element dS, together with a volume-distribution of density $-\operatorname{div}\mathbf{I}$ throughout its substance. These fictitious magnetizations are generally known as *Poisson's equivalent surface- and volume-distributions of magnetism.*

Poisson, moreover, perceived that at a point in a very small cavity excavated within the magnetic body, the magnetic potential has a limiting value which is independent of the shape of the cavity as the dimensions of the cavity tend to zero; but that this is not true of the magnetic intensity, which in such a small cavity depends on the shape of the cavity. Taking the cavity to be spherical, he showed that the magnetic intensity within it is

$$\operatorname{grad} V + \tfrac{4}{3}\pi\mathbf{I},^\dagger$$

where **I** denotes the magnetization at the place.

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* If the components of a vector **a** are denoted by (a_x, a_y, a_z), the quantity $a_xb_x + a_yb_y + a_zb_z$ is called the *scalar product* of two vectors **a** and **b**, and is denoted by $(\mathbf{a}\,.\,\mathbf{b})$.

The quantity $\frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$ is called the *divergence* of the vector **a**, and is denoted by div **a**.

† The vector whose components are $-\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z}$ is denoted by grad V.

This memoir also contains a discussion of the magnetism temporarily induced in soft iron and other magnetizable metals by the approach of a permanent magnet. Poisson accounted for the properties of temporary magnets by assuming that they contain embedded in their substance a great number of small spheres, which are perfect conductors for the magnetic fluids; so that the resultant magnetic intensity in the interior of one of these small spheres must be zero. He showed that such a sphere, when placed in a field of magnetic intensity $\mathbf{F}$,[*] must acquire a magnetic moment of amount $\frac{3}{4\pi}\mathbf{F} \times$ the volume of the sphere, in order to counteract within the sphere the force $\mathbf{F}$. Thus if k_p denote the total volume of these spheres contained within a unit volume of the temporary magnet, the magnetization will be $\mathbf{I}$, where

$$\tfrac{4}{3}\pi\mathbf{I} = k_p\mathbf{F},$$

and $\mathbf{F}$ denotes the magnetic intensity within a spherical cavity excavated in the body. This is *Poisson's law of induced magnetism.*

It is known that some substances acquire a greater degree of temporary magnetization than others when placed in the same circumstances: Poisson accounted for this by supposing that the quantity k_p varies from one substance to another. But the experimental data show that for soft iron k_p must have a value very near unity, which would obviously be impossible if k_p is to mean the ratio of the volume of spheres contained within a region to the total volume of the region.[†] The physical interpretation assigned by Poisson to his formulae must therefore be rejected, although the formulae themselves retain their value.

Poisson's electrical and magnetical investigations were generalized and extended in 1828 by George Green[‡] (*b.* 1793, *d.* 1841). Green's treatment is based on the properties of the function already used by Lagrange, Laplace, and Poisson, which represents the sum of all the electric or magnetic charges in the field, divided by their respective distances from some given point: to this function Green gave the name *potential,* by which it has always since been known.*

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[*] In the present work, vectors will generally be distinguished by heavy type.

[†] This objection was advanced by Maxwell in § 430 of his *Treatise*. An attempt to overcome it was made by Betti: cf. p. 377 of his *Lessons on the Potential.*

[‡] *An essay on the application of mathematical analysis to the theories of electricity and magnetism*, Nottingham, 1828: reprinted in *The Mathematical Papers of the late George Green*, p. 1.

Near the beginning of the memoir is established the celebrated formula connecting surface and volume integrals, which is now generally called *Green's Theorem,* and of which Poisson's result on the equivalent surface- and volume-distributions of magnetization is a particular application. By using this theorem to investigate the properties of the potential, Green arrived at many results of remarkable beauty and interest. We need only mention, as an example of the power of his method, the following:—Suppose that there is a hollow conducting shell, bounded by two closed surfaces, and that a number of electrified bodies are placed, some within and some without it; and let the inner surface and interior bodies be called the interior system, and the outer surface and exterior bodies be called the exterior system. Then all the electrical phenomena of the interior system, relative to attractions, repulsions, and densities, will be the same as if there were no exterior system, and the inner surface were a perfect conductor, put in communication with the earth; and all those of the exterior system will be the same as if the interior system did not exist, and the outer surface were a perfect conductor, containing a quantity of electricity equal to the whole of that originally contained in the shell itself and in all the interior bodies.

It will be evident that electrostatics had by this time attained a state of development in which further progress could be hoped for only in the mathematical superstructure, unless experiment should unexpectedly bring to light phenomena of an entirely new character. This will therefore be a convenient place to pause and consider the rise of another branch of electrical philosophy.

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* Euler in 1744 (*De methodis inveniendi* . . .) had spoken of the *vis potentialis*—what would now be called the potential energy—possessed by an elastic body when bent.

CHAPTER III.

GALVANISM, FROM GALVANI TO OHM.

UNTIL the last decade of the eighteenth century, electricians were occupied solely with statical electricity. Their attention was then turned in a different direction.

In a work entitled *Recherches sur l'origine des sentiments agréables et désagréables,* which was published* in 1752, Johann Georg Sulzer (*b.* 1720, *d.* 1779) had mentioned that, if two pieces of metal, the one of lead and the other of silver, be joined together in such a manner that their edges touch, and if they be placed on the tongue, a taste is perceived "similar to that of vitriol of iron," although neither of these metals applied separately gives any trace of such a taste. "It is not probable," he says, "that this contact of the two metals causes a solution of either of them, liberating particles which might affect the tongue; and we must therefore conclude that the contact sets up a vibration in their particles, which, by affecting the nerves of the tongue, produces the taste in question."

This observation was not suspected to have any connexion with electrical phenomena, and it played no part in the inception of the next discovery, which indeed was suggested by a mere accident.

Luigi Galvani, born at Bologna in 1737, occupied from 1775 onwards a chair of Anatomy in his native city. For many years before the event which made him famous he had been studying the susceptibility of the nerves to irritation; and, having been formerly a pupil of Beccaria, he was also interested in electrical experiments. One day in the latter part of the year 1780 he had, as he tells us,† "dissected and prepared a frog, and laid it on a table, on which, at some distance from the frog, was an electric machine. It happened by chance that one of my assistants touched the inner crural nerve of the frog with the point of a scalpel; whereupon at once the muscles of the limbs were violently convulsed.

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* Mém. de l'Acad. de Berlin, 1752, p. 356.

† Aloysii Galvani, *De Viribus Electricitatis in Motu Musculari*: Commentarii Bononiensi, vii (1791), p. 363.

"Another of those who used to help me in electrical experiments thought he had noticed that at this instant a spark was drawn from the conductor of the machine. I myself was at the time occupied with a totally different matter; but when he drew my attention to this, I greatly desired to try it for myself, and discover its hidden principle. So I, too, touched one or other of the crural nerves with the point of the scalpel, at the same time that one of those present drew a spark; and the same phenomenon was repeated exactly as before."*

After this, Galvani conceived the idea of trying whether the electricity of thunderstorms would induce muscular contractions equally well with the electricity of the machine. Having successfully experimented with lightning, he "wished," as he writes,† "to try the effect of atmospheric electricity in calm weather. My reason for this was an observation I had made, that frogs which had been suitably prepared for these experiments and fastened, by brass hooks in the spinal marrow, to the iron lattice round a certain hanging-garden at my house, exhibited convulsions not only during thunderstorms, but sometimes even when the sky was quite serene. I suspected these effects to be due to the changes which take place during the day in the electric state of the atmosphere; and so, with some degree of confidence, I performed experiments to test the point; and at different hours for many days I watched frogs which I had disposed for the purpose; but could not detect any motion in their muscles. At length, weary of waiting in vain, I pressed the brass hooks, which were driven into the spinal marrow, against the iron lattice, in order to see whether contractions could be excited by varying the incidental circumstances of the experiment. I observed contractions tolerably often, but they did not seem to bear any relation to the changes in the electrical state of the atmosphere.

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* According to a story which has often been repeated, but which rests on no sufficient evidence, the frog was one of a number which had been procured for the Signora Galvani, who, being in poor health, had been recommended to take a soup made of these animals as a restorative.

† Loc. cit., p. 377.

"However, at this time, when as yet I had not tried the experiment except in the open air, I came very near to adopting a theory that the contractions are due to atmospheric electricity, which, having slowly entered the animal and accumulated in it, is suddenly discharged when the hook comes in contact with the iron lattice. For it is easy in experimenting to deceive ourselves, and to imagine we see the things we wish to see.

"But I took the animal into a closed room, and placed it on an iron-plate; and when I pressed the hook which was fixed in the spinal marrow against the plate, behold! the same spasmodic contractions as before. I tried other metals at different hours on various days, in several places, and always with the same result, except that the contractions were more violent with some metals than with others. After this I tried various bodies which are not conductors of electricity, such as glass, gums, resins, stones, and dry wood; but nothing happened. This was somewhat surprising, and led me to suspect that electricity is inherent in the animal itself. This suspicion was strengthened by the observation that a kind of circuit of subtle nervous fluid (resembling the electric circuit which is manifested in the Leyden jar experiment) is completed from the nerves to the muscles when the contractions are produced.

"For, while I with one hand held the prepared frog by the hook fixed in its spinal marrow, so that it stood with its feet on a silver box, and with the other hand touched the lid of the box, or its sides, with any metallic body, I was surprised to see the frog become strongly convulsed every time that I applied this artifice."*

Galvani thus ascertained that the limbs of the frog are convulsed whenever a connexion is made between the nerves and muscles by a metallic arc, generally formed of more than one kind of metal; and he advanced the hypothesis that the convulsions are caused by the transport of a peculiar fluid from the nerves to the muscles, the arc acting as a conductor. To this fluid the names *Galvanism* and *Animal Electricity* were soon generally applied. Galvani himself considered it to be the same as the ordinary electric fluid, and, indeed, regarded the entire phenomenon as similar to the discharge of a Leyden jar.

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* This observation was made in 1786.

The publication of Galvani's views soon engaged the attention of the learned world, and gave rise to an animated controversy between those who supported Galvani's own view, those who believed galvanism to be a fluid distinct from ordinary electricity, and a third school who altogether refused to attribute the effects to a supposed fluid contained in the nervous system. The leader of the last-named party was Alessandro Volta (*b.* 1745, *d.* 1827), Professor of Natural Philosophy in the University of Pavia, who in 1792 put forward the view* that the stimulus in Galvani's experiment is derived essentially from the connexion of two different metals by a moist body. "The metals used in the experiments, being applied to the moist bodies of animals, can by themselves, and of their proper virtue, excite and dislodge the electric fluid from its state of rest; so that the organs of the animal act only passively." At first he inclined to combine this theory of metallic stimulus with a certain degree of belief in such a fluid as Galvani had supposed; but after the end of 1793 he denied the existence of animal electricity altogether.

From this standpoint Volta continued his experiments and worked out his theory. The following quotation from a letter† which he wrote later to Gren, the editor of the *Neues Journal d. Physik,* sets forth his view in a more developed form:—

"The contact of different conductors, particularly the metallic, including pyrites and other minerals, as well as charcoal, which I call dry conductors, or of the *first class,* with moist conductors, or conductors of the *second class,* agitates or disturbs the electric fluid, or gives it a certain impulse. Do not ask in what manner: it is enough that it is a principle, and a general principle. This impulse, whether produced by attraction or any other force, is different or unlike, both in regard to the different metals and to the different moist conductors; so that the direction, or at least the power, with which the electric fluid is impelled or excited, is different when the conductor *A* is applied to the conductor *B*, or to another *C*. In a perfect circle of conductors, where either one of the second class is placed between two different from each other of the first class, or, contrariwise, one of the first class is placed between two of the second class different from each other, an electric stream is occasioned by the predominating force either to the right or to the left—a circulation of this fluid, which ceases only when the circle is broken, and which is renewed when the circle is again rendered complete."

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* Phil. Trans., 1793, pp. 10, 27.

† Phil. Mag. iv (1799), pp. 59, 163, 306.

Another philosopher who, like Volta, denied the existence of a fluid peculiar to animals, but who took a somewhat different view of the origin of the phenomenon, was Giovanni Fabroni, of Florence (*b.* 1752, *d.* 1822), who,* having placed two plates of different metals in water, observed that one of them was partially oxidized when they were put in contact; from which he rightly concluded that some chemical action is inseparably connected with galvanic effects.

The feeble intensity of the phenomena of galvanism, which compared poorly with the striking displays obtained in electrostatics, was responsible for some falling off of interest in them towards the end of the eighteenth century; and the last years of their illustrious discoverer were clouded by misfortune. Being attached to the old order which was overthrown by the armies of the French Revolution, he refused in 1798 to take the oath of allegiance to the newly constituted Cisalpine Republic, and was deposed from his professorial chair. A profound melancholy, which had been induced by domestic bereavement, was aggravated by poverty and disgrace; and, unable to survive the loss of all he held dear, he died broken-hearted before the end of the year.†

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* Phil. Journal, 4to, iii. 308; iv. 120; Journal de Physique, vi. 348.

† A decree of reinstatement had been granted, but had not come into operation at the time of Galvani's death.

Scarcely more than a year after the death of Galvani, the new science suddenly regained the eager attention of philosophers. This renewal of interest was due to the discovery by Volta, in the early spring of 1800, of a means of greatly increasing the intensity of the effects. Hitherto all attempts to magnify the action by enlarging or multiplying the apparatus had ended in failure. If a long chain of different metals was used instead of only two, the convulsions of the frog were no more violent. But Volta now showed* that if any number of couples, each consisting of a zinc disk and a copper disk in contact, were taken, and if each couple was separated from the next by a disk of moistened pasteboard (so that the order was copper, zinc, pasteboard, copper, zinc, pasteboard, &c.), the effect of the *pile* thus formed was much greater than that of any galvanic apparatus previously introduced. When the highest and lowest disks were simultaneously touched by the fingers, a distinct shock was felt; and this could be repeated again and again, the pile apparently possessing within itself an indefinite power of recuperation. It thus resembled a Leyden jar endowed with a power of automatically re-establishing its state of tension after each explosion; with, in fact, "an inexhaustible charge, a perpetual action or impulsion on the electric fluid."

Volta unhesitatingly pronounced the phenomena of the pile to be in their nature electrical. The circumstances of Galvani's original discovery had prepared the minds of philosophers for this belief, which was powerfully supported by the similarity of the physiological effects of the pile to those of the Leyden jar, and by the observation that the galvanic influence was conducted only by those bodies—e.g. the metals—which were already known to be good conductors of static electricity. But Volta now supplied a still more convincing proof. Taking a disk of copper and one of zinc, he held each by an insulating handle and applied them to each other for an instant. After the disks had been separated, they were brought into contact with a delicate electroscope, which indicated by the divergence of its straws that the disks were now electrified—the zinc had, in fact, acquired a positive and the copper a negative electric charge.* Thus the mere contact of two different metals, such as those employed in the pile, was shown to be sufficient for the production of effects undoubtedly electrical in character.

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* Phil. Trans., 1800, p. 403.

On the basis of this result Volta in the same year (1800) put forward a definite theory of the action of the pile. Suppose first that a disk of zinc is laid on a disk of copper, which in turn rests on an insulating support. The experiment just described shows that the electric fluid will be driven from the copper to the zinc. We may then, according to Volta, represent the state or "tension" of the copper by the number $-\frac{1}{2}$, and that of the zinc by the number $+\frac{1}{2}$, the difference being arbitrarily taken as unity, and the sum being (on account of the insulation) zero. It will be seen that Volta's idea of "tension" was a mingling of two ideas, which in modern electric theory are clearly distinguished from each other—namely, electric charge and electric potential.

Now let a disk of moistened pasteboard be laid on the zinc, and a disk of copper on this again. Since the uppermost copper is not in contact with the zinc, the contact-action does not take place between them; but since the moist pasteboard is a conductor, the copper will receive a charge from the zinc. Thus the states will now be represented by $-\frac{2}{3}$ for the lower copper, $+\frac{1}{3}$ for the zinc, and $+\frac{1}{3}$ for the upper copper, giving a zero sum as before.

If, now, another zinc disk is placed on the top, the states will be represented by -1 for the lower copper, 0 for the lower zinc and upper copper, and $+1$ for the upper zinc.

In this way it is evident that the difference between the numbers indicating the tensions of the uppermost and lowest disks in the pile will always be equal to the number of pairs of metallic disks contained in it. If the pile is insulated, the sum of the numbers indicating the states of all the disks must be zero; but if the lowest disk is connected to earth, the tension of this disk will be zero, and the numbers indicating the states of all the other disks will be increased by the same amount, their mutual differences remaining unchanged.

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* Abraham Bennet (*b.* 1750, *d.* 1799) had previously shown (*New Experiments in Electricity*, 1789, pp. 86-102) that many bodies, when separated after contact, are oppositely electrified; he conceived that different bodies have different attractions or capacities for electricity.

The pile as a whole is thus similar to a Leyden jar; when the experimenter touches the uppermost and lowest disks, he receives the shock of its discharge, the intensity being proportional to the number of disks.

The moist layers played no part in Volta's theory beyond that of conductors.* It was soon found that when the moisture is acidified, the pile is more efficient; but this was attributed solely to the superior conducting power of acids.

Volta fully understood and explained the impossibility of constructing a pile from disks of metal alone, without making use of moist substances. As he showed in 1801, if disks of various metals are placed in contact in any order, the extreme metals will be in the same state as if they touched each other directly without the intervention of the others; so that the whole is equivalent merely to a single pair. When the metals are arranged in the order silver, copper, iron, tin, lead, zinc, each of them becomes positive with respect to that which precedes it, and negative with respect to that which follows it; but the moving force from the silver to the zinc is equal to the sum of the moving forces of the metals comprehended between them in the series.

When a connexion was maintained for some time between the extreme disks of a pile by the human body, sensations were experienced which seemed to indicate a continuous activity in the entire system. Volta inferred that the electric current persists during the whole time that communication by conductors exists all round the circuit, and that the current is suspended only when this communication is interrupted. "This endless circulation or perpetual motion of the electric fluid," he says, "may seem paradoxical, and may prove inexplicable; but it is none the less real, and we can, so to speak, touch and handle it."

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* Volta had inclined, in his earlier experiments on galvanism, to locate the seat of power at the interfaces of the metals with the moist conductors. Cf. his letter to Gren, Phil. Mag. iv (1799), p. 62.

Volta announced his discovery in a letter to Sir Joseph Banks, dated from Como, March 20th, 1800. Sir Joseph, who was then President of the Royal Society, communicated the news to William Nicholson (*b.* 1753, *d.* 1815), founder of the *Journal* which is generally known by his name, and his friend Anthony Carlisle (*b.* 1768, *d.* 1840), afterwards a distinguished surgeon. On the 30th of the following month, Nicholson and Carlisle set up the first pile made in England. In repeating Volta's experiments, having made the contact more secure at the upper plate of the pile by placing a drop of water there, they noticed* a disengagement of gas round the conducting wire at this point; whereupon they followed up the matter by introducing a tube of water, into which the wires from the terminals of the pile were plunged. Bubbles of an inflammable gas were liberated at one wire, while the other wire became oxidised; when platinum wires were used, oxygen and hydrogen were evolved in a free state, one at each wire. This effect, which was nothing less than the electric decomposition of water into its constituent gases, was obtained on May 2nd, 1800.†

Although it had long been known that frictional electricity is capable of inducing chemical action,‡ the discovery of Nicholson and Carlisle was of the first magnitude. It was at once extended by William Cruickshank, of Woolwich (*b.* 1745,

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* *Nicholson's Journal* (4to), iv, 179 (1800); Phil. Mag. vii, 337 (1800).

† It was obtained independently four months later by J. W. Ritter.

‡ Beccaria (*Lettere dell' elettricismo*, Bologna, 1758, p. 282) had reduced mercury and other metals from their oxides by discharges of frictional electricity; and Priestley had obtained an inflammable gas from certain organic liquids in the same way. Cavendish in 1781 had established the constitution of water by

d. 1800), who* showed that solutions of metallic salts are also decomposed by the current; and William Hyde Wollaston (*b.* 1766, *d.* 1828) seized on it as a test† of the identity of the electric currents of Volta with those obtained by the discharge of frictional electricity. He found that water could be decomposed by currents of either type, and inferred that all differences between them could be explained by supposing that voltaic electricity as commonly obtained is "less intense, but produced in much larger quantity." Later in the same year (1801), Martin van Marum (*b.* 1750, *d.* 1837) and Christian Heinrich Pfaff (*b.* 1773, *d.* 1852) arrived at the same conclusion by carrying out on a large scale‡ Volta's plan of using the pile to charge batteries of Leyden jars.

The discovery of Nicholson and Carlisle made a great impression on the mind of Humphry Davy (*b.* 1778, *d.* 1829), a young Cornishman who about this time was appointed Professor of Chemistry at the Royal Institution in London. Davy at once began to experiment with Voltaic piles, and in November, 1800,§ showed that they give no current when the water between the pairs of plates is pure, and that their power of action is "in great measure proportional to the power of the conducting fluid substance between the double plates to oxydate the zinc." This result, as he immediately perceived, did not harmonize well with Volta's views on the source of electricity in the pile, but was, on the other hand, in agreement with Fabroni's idea that galvanic effects are always accompanied by chemical action. After a series of experiments he definitely concluded that "the galvanic pile of Volta acts only when the conducting substance between the plates is capable of oxydating the zinc; and that, in proportion as a greater quantity of oxygen enters into combination with the zinc in a given time, so in proportion is the power of the pile to decompose water and to give the shock greater. It seems therefore reasonable to conclude, though with our present quantity of facts we are unable to explain the exact mode of operation, that the oxydation of the zinc in the pile, and the chemical changes connected with it, are *somehow* the cause of the electrical effects it produces." This principle of oxidation guided Davy in designing many new types of pile, with elements chosen from the whole range of the known metals.

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* *Nicholson's Journal* (4to), iv (1800), pp. 187, 245: Phil. Mag., vii (1800), p. 337.

† Phil. Mag., 1801, p. 427.

‡ Phil. Mag., xii (1802), p. 161.

§ *Nicholson's Journal* (4to), iv (1800); Davy's *Works*, ii, p. 155.

Davy's chemical theory of the pile was supported by Wollaston* and by Nicholson,† the latter of whom urged that the existence of piles in which only *one* metal is used (with more than one kind of fluid) is fatal to any theory which places the seat of the activity in the contact of dissimilar metals.

Davy afterwards proposed‡ a theory of the voltaic pile which combines ideas drawn from both the "contact" and "chemical" explanations. He supposed that before the circuit is closed, the copper and zinc disks in each contiguous pair assume opposite electrostatic states, in consequence of inherent "electrical energies" possessed by the metals; and when a communication is made between the extreme disks by a wire, the opposite electricities annihilate each other, as in the discharge of a Leyden jar. If the liquid (which Davy compared to the glass of a Leyden jar) were incapable of decomposition, the current would cease after this discharge. But the liquid in the pile is composed of two elements which have inherent attractions for electrified metallic surfaces: hence arises chemical action, which removes from the disks the outermost layers of molecules, whose energy is exhausted, and exposes new metallic surfaces. The electrical energies of the copper and zinc are consequently again exerted, and the process of electromotion continues. Thus the contact of metals is the cause which *disturbs* the equilibrium, while the chemical changes continually *restore* the conditions under which the contact energy can be exerted.

In this and other memoirs Davy asserted that chemical affinity is essentially of an electrical nature. "Chemical and electrical attractions," he declared,* "are produced by the same cause, acting in one case on particles, in the other on masses, of matter; and the same property, under different modifications, is the cause of all the phenomena exhibited by different voltaic combinations."

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* Phil. Trans., 1801, p. 427.

† *Nicholson's Journal*, i (1802), p. 142.

‡ Phil. Trans., 1807, p. 1.

The further elucidation of this matter came chiefly from researches on electro-chemical decomposition, which we must now consider.

A phenomenon which had greatly surprised Nicholson and Carlisle in their early experiments was the appearance of the products of galvanic decomposition at places remote from each other. The first attempt to account for this was made in 1806 by Theodor von Grothuss† (*b.* 1785, *d.* 1822) and by Davy,‡ who advanced a theory that the terminals at which water is decomposed have attractive and repellent powers; that the pole whence resinous electricity issues has the property of attracting hydrogen and the metals, and of repelling oxygen and acid substances, while the positive terminal has the power of attracting oxygen and repelling hydrogen; and that these forces are sufficiently energetic to destroy or suspend the usual operation of chemical affinity in the water-molecules nearest the terminals. The force due to each terminal was supposed to diminish with the distance from the terminal. When the molecule nearest one of the terminals has been decomposed by the attractive and repellent forces of the terminal, one of its constituents is liberated there, while the other constituent, by virtue of electrical forces (the oxygen and hydrogen being in opposite electrical states), attacks the next molecule, which is then decomposed. The surplus constituent from this attacks the next molecule, and so on. Thus a chain of decompositions and recompositions was supposed to be set up among the molecules intervening between the terminals.

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* Phil. Trans., 1826, p. 383.

† Ann. de Chim., lviii (1806), p. 54.

‡ Bakerian lecture for 1806, Phil. Trans., 1807, p. 1. A theory similar to that of Grothuss and Davy was communicated by Peter Mark Roget (*b.* 1779, *d.* 1869) in 1807 to the Philosophical Society of Manchester: cf. Roget's *Galvanism*, § 106.

The hypothesis of Grothuss and Davy was attacked in 1825 by Auguste De La Rive* (*b.* 1801, *d.* 1873) of Geneva, on the ground of its failure to explain what happens when different liquids are placed in series in the circuit. If, for example, a solution of zinc sulphate is placed in one compartment, and water in another, and if the positive pole is placed in the solution of zinc sulphate, and the negative pole in the water, De La Rive found that oxide of zinc is developed round the latter; although decomposition and recomposition of zinc sulphate could not take place in the water, which contained none of it. Accordingly, he supposed the constituents of the decomposed liquid to be bodily transported across the liquids, in close union with the moving electricity. In the electrolysis of water, one current of electrified hydrogen was supposed to leave the positive pole, and become decomposed into hydrogen and electricity at the negative pole, the hydrogen being there liberated as a gas. Another current in the same way carried electrified oxygen from the negative to the positive pole. In this scheme the chain of successive decompositions imagined by Grothuss does not take place, the only molecules decomposed being those adjacent to the poles.

The appearance of the products of decomposition at the separate poles could be explained either in Grothuss' fashion by assuming dissociations throughout the mass of liquid, or in De La Rive's by supposing particular dissociated atoms to travel considerable distances. Perhaps a preconceived idea of economy in Nature deterred the workers of that time from accepting the two assumptions together, when either of them separately would meet the case. Yet it is to this apparent redundancy that later researches have pointed as the truth. Nature is what she is, and not what we would make her.

De La Rive was one of the most thoroughgoing opponents of Volta's contact theory of the pile; even in the case when two metals are in contact in air only, without the intervention of any liquid, he attributed the electric effect wholly to the chemical affinity of the air for the metals.

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* Annales de Chimie, xxviii, 190.

During the long interval between the publication of the rival hypotheses of Grothuss and De La Rive, little real progress was made with the special problems of the cell; but meanwhile electric theory was developing in other directions. One of these, to which our attention will first be turned, was the electro-chemical theory of the celebrated Swedish chemist, Jöns Jacob Berzelius (*b.* 1779, *d.* 1848).

Berzelius founded his theory,* which had been in one or two of its features anticipated by Davy,† on inferences drawn from Volta's contact effects. "Two bodies," he remarked, "which have affinity for each other, and which have been brought into mutual contact, are found upon separation to be in opposite electrical states. That which has the greatest affinity for oxygen usually becomes positively electrified, and the other negatively."

This seemed to him to indicate that chemical affinity arises from the play of electric forces, which in turn spring from electric charges within the atoms of matter. To be precise, he supposed each atom to possess two poles, which are the seat of opposite electrifications, and whose electrostatic field is the cause of chemical affinity.

By aid of this conception Berzelius drew a simple and vivid picture of chemical combination. Two atoms, which are about to unite, dispose themselves so that the positive pole of one touches the negative pole of the other; the electricities of these two poles then discharge each other, giving rise to the heat and light which are observed to accompany the act of combination.‡ The disappearance of these leaves the compound molecule with the two remaining poles; and it cannot be dissociated into its constituent atoms again until some means is found of restoring to the vanished poles their charges. Such a means is afforded by the action of the galvanic pile in electrolysis: the opposite electricities of the current invade the molecules of the electrolyte, and restore the atoms to their original state of polarization.

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* Memoirs of the Acad. of Stockholm, 1812; Nicholson's Journal of Nat. Phil., xxxiv (1813), 142, 153, 240, 319; xxxv, 38, 118, 159.

† Phil. Trans., 1807.

‡ This idea was Davy's.

If, as Berzelius taught, all chemical compounds are formed by the mutual neutralization of pairs of atoms, it is evident that they must have a binary character. Thus he conceived a salt to be compounded of an acid and an oxide, and each of these to be compounded of two other constituents. Moreover, in any compound the electropositive member would be replaceable only by another electropositive member, and the electronegative member only by another member also electronegative; so that the substitution of, e.g., chlorine for hydrogen in a compound would be impossible—an inference which was overthrown by subsequent discoveries in chemistry.

Berzelius succeeded in bringing the most curiously diverse facts within the scope of his theory. Thus "the combination of polarized atoms requires a motion to turn the opposite poles to each other; and to this circumstance is owing the facility with which combination takes place when one of the two bodies is in the liquid state, or when both are in that state; and the extreme difficulty, or nearly impossibility, of effecting an union between bodies, both of which are solid. And again, since each polarized particle must have an electric atmosphere, and as this atmosphere is the predisposing cause of combination, as we have seen, it follows, that the particles cannot act but at certain distances, proportioned to the intensity of their polarity; and hence it is that bodies, which have affinity for each other, always combine nearly on the instant when mixed in the liquid state, but less easily in the gaseous state, and the union ceases to be possible under a certain degree of dilatation of the gases; as we know by the experiments of Grothuss, that a mixture of oxygen and hydrogen in due proportions, when rarefied to a certain degree, cannot be set on fire at any temperature whatever." And again: "Many bodies require an elevation of temperature to enable them to act upon each other. It appears, therefore, that heat possesses the property of augmenting the polarity of these bodies."

Berzelius accounted for Volta's electromotive series by assuming the electrification at one pole of an atom to be somewhat more or somewhat less than what would be required to neutralize the charge at the other pole. Thus each atom would possess a certain net or residual charge, which might be of either sign; and the order of the elements in Volta's series could be interpreted simply as the order in which they would stand when ranged according to the magnitude of this residual charge. As we shall see, this conception was afterwards overthrown by Faraday.

Berzelius permitted himself to publish some speculations on the nature of heat and electricity, which bring vividly before us the outlook of an able thinker in the first quarter of the nineteenth century. The great question, he says, is whether the electricities and caloric are matter or merely phenomena. If the title of matter is to be granted only to such things as are ponderable, then these problematic entities are certainly not matter; but thus to narrow the application of the term is, he believes, a mistake; and he inclines to the opinion that caloric is truly matter, possessing chemical affinities without obeying the law of gravitation, and that light and all radiations consist in modes of propagating such matter. This conclusion makes it easier to decide regarding electricity. "From the relation which exists between caloric and the electricities," he remarks, "it is clear that what may be true with regard to the materiality of one of them must also be true with regard to that of the other. There are, however, a quantity of phenomena produced by electricity which do not admit of explanation without admitting at the same time that electricity is matter. Electricity, for instance, very often detaches everything which covers the surface of those bodies which conduct it. It, indeed, passes through conductors without leaving any trace of its passage; but it penetrates non-conductors which oppose its course, and makes a perforation precisely of the same description as would have been made by something which had need of place for its passage. We often observe this when electric jars are broken by an over-charge, or when the electric shock is passed through a number of cards, etc. We may therefore, at least with some probability, imagine caloric and the electricities to be matter, destitute of gravitation, but possessing affinity to gravitating bodies. When they are not confined by these affinities, they tend to place themselves in equilibrium in the universe. The suns destroy at every moment this equilibrium, and they send the re-united electricities in the form of luminous rays towards the planetary bodies, upon the surface of which the rays, being arrested, manifest themselves as caloric; and this last in its turn, during the time required to replace it in equilibrium in the universe, supports the chemical activity of organic and inorganic nature."

It was scarcely to be expected that anything so speculative as Berzelius' electric conception of chemical combination would be confirmed in all particulars by subsequent discovery; and, as a matter of fact, it did not as a coherent theory survive the lifetime of its author. But some of its ideas have persisted, and among them the conviction which lies at its foundation, that chemical affinities are, in the last resort, of electrical origin.

While the attention of chemists was for long directed to the theory of Berzelius, the interest of electricians was diverted from it by a discovery of the first magnitude in a different region.

That a relation of some kind subsists between electricity and magnetism had been suspected by the philosophers of the eighteenth century. The suspicion was based in part on some curious effects produced by lightning, of a kind which may be illustrated by a paper published in the *Philosophical Transactions* in 1735.* A tradesman of Wakefield, we are told, "having put up a great number of knives and forks in a large box, and having placed the box in the corner of a large room, there happen'd in July, 1731, a sudden storm of thunder, lightning, etc., by which the corner of the room was damaged, the Box split, and a good many knives and forks melted, the sheaths being untouched. The owner emptying the box upon a Counter where some Nails lay, the Persons who took up the knives, that lay upon the Nails, observed that the knives took up the Nails."

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* Phil. Trans. xxxix (1735), p. 74.

Lightning thus came to be credited with the power of magnetizing steel; and it was doubtless this which led Franklin* in 1751 to attempt to magnetize a sewing-needle by means of the discharge of Leyden jars. The attempt was indeed successful; but, as Van Marum afterwards showed, it was doubtful whether the magnetism was due directly to the current.

More experiments followed.† In 1805 Jean Nicholas Pierre Hachette (*b.* 1769, *d.* 1834) and Charles Bernard Desormes (*b.* 1777, *d.* 1862) attempted to determine whether an insulated voltaic pile, freely suspended, is oriented by terrestrial magnetism; but without positive result. In 1807 Hans Christian Oersted (*b.* 1777, *d.* 1851), Professor of Natural Philosophy in Copenhagen, announced his intention of examining the action of electricity on the magnetic needle; but it was not for some years that his hopes were realized. If one of his pupils is to be believed,‡ he was "a man of genius, but a very unhappy experimenter; he could not manipulate instruments. He must always have an assistant, or one of his auditors who had easy hands, to arrange the experiment."

During a course of lectures which he delivered in the winter of 1819–20 on "Electricity, Galvanism, and Magnetism," the idea occurred to him that the changes observed with the compass-needle during a thunderstorm might give the clue to the effect of which he was in search; and this led him to think that the experiment should be tried with the galvanic circuit

* Letter vi from Franklin to Collinson.

† In 1774 the Electoral Academy of Bavaria proposed the question, "Is there a real and physical analogy between electric and magnetic forces?" as the subject of a prize.

‡ Cf. a letter from Hansteen inserted in Bence Jones' *Life of Faraday*, ii, p. 395.

closed instead of open, and to inquire whether any effect is produced on a magnetic needle when an electric current is passed through a neighbouring wire. At first he placed the wire at right angles to the needle, but observed no result. After the end of a lecture in which this negative experiment had been shown, the idea occurred to him to place the wire parallel to the needle: on trying it, a pronounced deflexion was observed, and the relation between magnetism and the electric current was discovered. After confirmatory experiments with more powerful apparatus, the public announcement was made in July, 1820.*

Oersted did not determine the quantitative laws of the action, but contented himself with a statement of the qualitative effect and some remarks on its cause, which recall the magnetic speculations of Descartes: indeed, Oersted's conceptions may be regarded as linking those of the Cartesian school to those which were introduced subsequently by Faraday. "To the effect which takes place in the conductor and in the surrounding space," he wrote, "we shall give the name of the *conflict of electricity.*" "The electric conflict acts only on the magnetic particles of matter. All non-magnetic bodies appear penetrable by the electric conflict, while magnetic bodies, or rather their magnetic particles, resist the passage of this conflict. Hence they can be moved by the impetus of the contending powers.

"It is sufficiently evident from the preceding facts that the electric conflict is not confined to the conductor, but dispersed pretty widely in the circumjacent space.

"From the preceding facts we may likewise collect, that this conflict performs circles; for without this condition, it seems impossible that the one part of the uniting wire, when placed below the magnetic pole, should drive it toward the east, and when placed above it toward the west; for it is the nature of a circle that the motions in opposite parts should have an opposite direction."

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* Schweigger's Journal für Chemie und Physik, xxix (1820), p. 275; Thomson's Annals of Philosophy, xvi (1820), p. 273; Ostwald's *Klassiker der exakten Wissenschaften*, Nr. 63.

Oersted's discovery was described at the meeting of the French Academy on September 11th, 1820, by an academician (Arago) who had just returned from abroad. Several investigators in France repeated and extended his experiments; and the first precise analysis of the effect was published by two of these, Jean-Baptiste Biot (*b.* 1774, *d.* 1862) and Félix Savart (*b.* 1791, *d.* 1841), who, at a meeting of the Academy of Sciences on October 30th, 1820, announced[*] that the action experienced by a pole of austral or boreal magnetism, when placed at any distance from a straight wire carrying a voltaic current, may be thus expressed: "Draw from the pole a perpendicular to the wire; the force on the pole is at right angles to this line and to the wire, and its intensity is proportional to the reciprocal of the distance." This result was soon further analysed, the attractive force being divided into constituents, each of which was supposed to be due to some particular element of the current; in its new form the law may be stated thus: *the magnetic force due to an element* **ds** *of a circuit, in which a current* i *is flowing, at a point whose vector distance from* **ds** *is* **r**, *is (in suitable units)*

$$\frac{i}{r^3}[\mathbf{ds}\,.\,\mathbf{r}]\dagger \quad \text{or} \quad \operatorname{curl}\frac{i\mathbf{ds}}{r}.\ddagger$$

It was now recognized that a magnetic field may be produced as readily by an electric current as by a magnet; and, as Arago soon showed,[§] this, like any other magnetic field, is capable of inducing magnetization in iron. The question naturally suggested itself as to whether the similarity of properties between currents and magnets extended still further, e.g. whether conductors carrying currents would, like magnets, experience ponderomotive forces when placed in a magnetic field, and whether such conductors would consequently, like magnets, exert ponderomotive forces on each other.

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[*] Annales de Chimie, xv (1820), p. 222; Journal de Phys., xli, p. 51.

[†] If **a** and **b** denote two vectors, the vector whose components are $(a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$ is called the *vector product* of **a** and **b**, and is denoted by [**a**, **b**]. Its direction is at right angles to those of **a** and **b**, and its magnitude is represented by twice the area of the triangle formed by them.

[‡] If **a** denotes any vector, the vector whose components are $\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}$, $\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}$, $\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}$ is denoted by curl **a**.

[§] Annales de Chimie, xv (1820), p. 93.

The first step towards answering these inquiries was taken by Oersted* himself. "As," he said, "a body cannot put another in motion without being moved in its turn, when it possesses the requisite mobility, it is easy to foresee that the galvanic arc must be moved by the magnet"; and this he verified experimentally.

The next step came from André Marie Ampère (*b.* 1775, *d.* 1836), who at the meeting of the Academy on September 18th, exactly a week after the news of Oersted's first discovery had arrived, showed that two parallel wires carrying currents attract each other if the currents are in the same direction, and repel each other if the currents are in opposite directions. During the next three years Ampère continued to prosecute the researches thus inaugurated, and in 1825 published his collected results in one of the most celebrated memoirs† in the history of natural philosophy.

Ampère introduces his work by proclaiming himself a follower of that school which explained all physical phenomena in terms of equal and oppositely directed forces between pairs of particles; and he renounces the attempt to seek more speculative, though possibly more fundamental, explanations in terms of the motions of ultimate fluids and aethers. Nevertheless, he indicates two conceptions of this latter character, on which such explanations might be founded.

In the first‡ he suggests that the ponderomotive forces between circuits carrying electric currents may be due to "the reaction of the elastic fluid which extends throughout all space, whose vibrations produce the phenomena of light," and which is "put in motion by electric currents." This fluid or aether can, he says, "be no other than that which results from the combination of the two electricities."

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* Schweigger's Journal für Chem. u. Phys., xxix (1820), p. 364; Thomson's Annals of Philosophy, xvi (1820), p. 375.

† Mém. de l'Acad., vi, p. 175.

‡ *Recueil d'observations électro-dynamiques*, p. 215; and the memoir just cited, pp. 285, 370.

In the second conception,* Ampère suggests that the interspaces between the metallic molecules of a wire which carries a current may be occupied by a fluid composed of the two electricities, not in the proportions which form the neutral fluid, but with an excess of that one of them which is opposite to the electricity peculiar to the molecules of the metal, and which consequently masks this latter electricity. In this inter-molecular fluid the opposite electricities are continually being dissociated and recombined; a dissociation of the fluid within one inter-molecular interval having taken place, the positive electricity thus produced unites with the negative electricity of the interval next to it in the direction of the current, while the negative electricity of the first interval unites with the positive electricity of the next interval in the other direction. Such interchanges, according to this hypothesis, constitute the electric current.

Ampère's memoir is, however, but little occupied with the more speculative side of the subject. His first aim was to investigate thoroughly by experiment the ponderomotive forces on electric currents.

"When," he remarks, "M. Oersted discovered the action which a current exercises on a magnet, one might certainly have suspected the existence of a mutual action between two circuits carrying currents; but this was not a necessary consequence; for a bar of soft iron also acts on a magnetized needle, although there is no mutual action between two bars of soft iron."

Ampère, therefore, submitted the matter to the test of the laboratory, and discovered that circuits carrying electric currents exert ponderomotive forces on each other, and that ponderomotive forces are exerted on such currents by magnets. To the science which deals with the mutual action of currents he gave the name *electro-dynamics*;[*] and he showed that the action obeys the following laws:—

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* *Recueil d'observations électro-dynamiques*, pp. 297, 300, 371.

(1) The effect of a current is reversed when the direction of the current is reversed.

(2) The effect of a current flowing in a circuit twisted into small sinuosities is the same as if the circuit were smoothed out.

(3) The force exerted by a closed circuit on an element of another circuit is at right angles to the latter.

(4) The force between two elements of circuits is unaffected when all linear dimensions are increased proportionately, the current-strengths remaining unaltered.

From these data, together with his assumption that the force between two elements of circuits acts along the line joining them, Ampère obtained an expression of this force: the deduction may be made in the following way:—

Let **ds**, **ds′** be the elements, **r** the line joining them, and i, i' the current-strengths. From (2) we see that the effect of **ds** on **ds′** is the vector sum of the effects of dx, dy, dz on **ds′**, where these are the three components of **ds**: so the required force must be of the form—

r × a scalar quantity which is linear and homogeneous in **ds**; and it must similarly be linear and homogeneous in **ds′**; so using (1), we see that the force must be of the form

$$\mathbf{F} = ii'\mathbf{r}\{(\mathbf{ds}\,.\,\mathbf{ds}')\,\phi(r) + (\mathbf{ds}\,.\,\mathbf{r})\,(\mathbf{ds}'.\,\mathbf{r})\,\psi(r)\},$$

where ϕ and ψ denote undetermined functions of r.

From (4) it follows that when ds, ds', r are all multiplied by the same number, **F** is unaffected: this shows that

$$\phi(r) = \frac{A}{r^3} \quad\text{and}\quad \psi(r) = \frac{B}{r^5},$$

where A and B denote constants. Thus we have

$$\mathbf{F} = ii'\mathbf{r}\left\{\frac{A(\mathbf{ds}\,.\,\mathbf{ds}')}{r^3} + \frac{B(\mathbf{ds}\,.\,\mathbf{r})\,(\mathbf{ds}'.\,\mathbf{r})}{r^5}\right\}.$$

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[*] Loc. cit., p. 298.

Now, by (3), the resolved part of **F** along $\mathbf{ds}'$ must vanish when integrated round the circuit **s**, i.e. it must be a complete differential when $\mathbf{dr}$ is taken to be equal to $-\mathbf{ds}$. That is to say,

$$\frac{A(\mathbf{ds}\,.\,\mathbf{ds}')(\mathbf{r}\,.\,\mathbf{ds}')}{r^3}+\frac{B(\mathbf{ds}\,.\,\mathbf{r})(\mathbf{ds}'.\,\mathbf{r})^2}{r^5}$$

must be a complete differential; or

$$-\frac{A}{2r^3}d\,.\,(\mathbf{r}\,.\,\mathbf{ds}')^2+\frac{B}{r^5}(\mathbf{ds}\,.\,\mathbf{r})\,.\,(\mathbf{r}\,.\,\mathbf{ds}')^2$$

must be a complete differential; and therefore

$$d\,.\,\frac{A}{2r^3}=-\frac{B}{r^5}(\mathbf{ds}\,.\,\mathbf{r}),$$

or
$$-\frac{3A}{2r^4}dr=\frac{B}{r^4}dr,$$

or
$$B=-\tfrac{3}{2}A.$$

Thus finally we have

$$\mathbf{F}=\text{Constant}\times ii'\mathbf{r}\left\{\frac{2}{r^3}(\mathbf{ds}\,.\,\mathbf{ds}')-\frac{3}{r^5}(\mathbf{ds}\,.\,\mathbf{r})(\mathbf{ds}'.\,\mathbf{r})\right\}.$$

This is Ampère's formula: the multiplicative constant depends of course on the units chosen, and may be taken to be -1.

The weakness of Ampère's work evidently lies in the assumption that the force is directed along the line joining the two elements: for in the analogous case of the action between two magnetic molecules, we know that the force is *not* directed along the line joining the molecules. It is therefore of interest to find the form of **F** when this restriction is removed.

For this purpose we observe that we can add to the expression already found for **F** any term of the form

$$\phi(r)\,.\,(\mathbf{ds}\,.\,\mathbf{r})\,.\,\mathbf{ds}',$$

where $\phi(r)$ denotes any arbitrary function of r; for since

$$(\mathbf{ds}\,.\,\mathbf{r})=-r\,.\,ds\,.\,\frac{dr}{ds},$$

this term vanishes when integrated round the circuit **s**; and it contains $\mathbf{ds}$ and $\mathbf{ds}'$ linearly and homogeneously, as it should. We can also add any terms of the form

$$d\{\mathbf{r} \,.\, (\mathbf{ds}' .\, \mathbf{r}) \,.\, \chi(r)\},$$

where $\chi(r)$ denotes any arbitrary function of r, and d denotes differentiation along the arc $\mathbf{s}$, keeping $\mathbf{ds}'$ fixed (so that $\mathbf{dr} = -\,\mathbf{ds}$); this differential may be written

$$-\,\mathbf{ds} \,.\, (\mathbf{ds}' .\, \mathbf{r}) \,.\, \chi(r) - \mathbf{r}\chi(r)\,(\mathbf{ds}' .\, \mathbf{ds}) - \frac{1}{r}\,\chi'(r)\,\mathbf{r}\,(\mathbf{ds} \,.\, \mathbf{r})\,(\mathbf{ds}' .\, \mathbf{r}).$$

In order that the law of Action and Reaction may not be violated, we must combine this with the former additional term so as to obtain an expression symmetrical in $\mathbf{ds}$ and $\mathbf{ds}'$: and hence we see finally *that the general value of* F *is given by the equation*

$$\begin{aligned} \mathsf{F} = -\,ii'\mathbf{r}\left\{\frac{2}{r^3}\,(\mathbf{ds} \,.\, \mathbf{ds}') - \frac{3}{r^5}(\mathbf{ds} \,.\, \mathbf{r})\,(\mathbf{ds} .\, \mathbf{r})\right\} \\ + \chi(r)\,(\mathbf{ds}' .\, \mathbf{r})\,\mathbf{ds} + \chi(r)\,(\mathbf{ds} \,.\, \mathbf{r}) \,.\, \mathbf{ds}' + \chi(r)\,(\mathbf{ds} \,.\, \mathbf{ds}')\mathbf{r} \\ + \frac{1}{r}\chi'(r)\,(\mathbf{ds} .\, \mathbf{r})\,(\mathbf{ds}' .\, \mathbf{r})\,\mathbf{r}. \end{aligned}$$

The simplest form of this expression is obtained by taking

$$\chi(r) = \frac{ii'}{r^3},$$

when we obtain

$$\mathsf{F} = \frac{ii'}{r^3}\,\{(\mathbf{ds} \,.\, \mathbf{r}) \,.\, \mathbf{ds}' + (\mathbf{ds}' .\, \mathbf{r})\mathbf{ds} - (\mathbf{ds} \,.\, \mathbf{ds}')\mathbf{r}\}.$$

The comparatively simple expression in brackets is the vector part of the quaternion product of the three vectors $\mathbf{ds}$, $\mathbf{r}$, $\mathbf{ds}'$.*

From any of these values of F we can find the ponderomotive force exerted by the whole circuit $\mathbf{s}$ on the element $\mathbf{ds}'$: it is, in fact, from the last expression,

$$ii'\int_s \frac{1}{r^3}\,\{(\mathbf{ds}' .\, \mathbf{r}) \,.\, \mathbf{ds} - (\mathbf{ds} \,.\, \mathbf{ds}')\mathbf{r}\},$$

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* The simpler form of F given in the text is, if the term in $\mathbf{ds}'$ be omitted, the form given by Grassmann, Ann. d. Phys. lxiv (1845), p. 1. For further work on this subject cf. Tait, Proc. R. S. Edin. viii (1873), p. 220, and Korteweg, Journal für Math. xc (1881), p. 45.

or $$ii'\int_s \left[\mathbf{ds}' . \frac{[\mathbf{ds} . \mathbf{r}]}{r^3}\right],$$

or $$i' [\mathbf{ds}' . \mathbf{B}],$$

where $$\mathbf{B} = i\int_s \frac{1}{r^3} [\mathbf{ds} . \mathbf{r}].$$

Now this value of **B** is precisely the value found by Biot and Savart* for the magnetic intensity at $\mathbf{ds}'$ due to the current i in the circuit **s**. Thus we see that the ponderomotive force on a current-element $\mathbf{ds}'$ in a magnetic field **B** is $i' [\mathbf{ds}' . \mathbf{B}]$.

Ampère developed to a considerable extent the theory of the equivalence of magnets with circuits carrying currents; and showed that an electric current is equivalent, in its magnetic effects, to a distribution of magnetism on any surface terminated by the circuit, the axes of the magnetic molecules being everywhere normal to this surface:† such a magnetized surface is called a *magnetic shell*. He preferred, however, to regard the current rather than the magnetic fluid as the fundamental entity, and considered magnetism to be really an electrical phenomenon: each magnetic molecule owes its properties, according to this view, to the presence within it of a small closed circuit in which an electric current is perpetually flowing.

The impression produced by Ampère's memoir was great and lasting. Writing half a century afterwards, Maxwell speaks of it as "one of the most brilliant achievements in science." "The whole," he says, "theory and experiment, seems as if it had leaped, full-grown and full-armed, from the brain of the 'Newton of electricity.' It is perfect in form and unassailable in accuracy; and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electrodynamics."

Not long after the discovery by Oersted of the connexion between galvanism and magnetism, a connexion was discovered between galvanism and heat. In 1822 Thomas Johann Seebeck

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* See ante, p. 86.

† Loc. cit., p. 367.

(*b.* 1770, *d.* 1831), of Berlin discovered* that an electric current can be set up in a circuit of metals, without the interposition of any liquid, merely by disturbing the equilibrium of temperature. Let a ring be formed of copper and bismuth soldered together at the two extremities; to establish a current it is only necessary to heat the ring at one of these junctions. To this new class of circuits the name *thermo-electric* was given.

It was found that the metals can be arranged as a *thermo-electric* series, in the order of their power of generating currents when thus paired, and that this order is quite different from Volta's order of electromotive potency. Indeed antimony and bismuth, which are near each other in the latter series, are at opposite extremities of the former.

The currents generated by thermo-electric means are generally feeble: and the mention of this fact brings us to the question, which was about this time engaging attention, of the efficacy of different voltaic arrangements.

Comparisons of a rough kind had been instituted soon after the discovery of the pile. The French chemists Antoine François de Fourcroy (*b.* 1755, *d.* 1809), Louis Nicolas Vauquelin (*b.* 1763, *d.* 1829), and Louis Jacques Thénard (*b.* 1777, *d.* 1857) found† in 1801, on varying the size of the metallic disks constituting the pile, that the sensations produced on the human frame were unaffected so long as the number of disks remained the same; but that the power of burning finely drawn wire was altered; and that the latter power was proportional to the total surface of the disks employed, whether this were distributed among a small number of large disks, or a large number of small ones. This was explained by supposing that small plates give a small quantity of the electric fluid with a high velocity, while large plates give a larger quantity with no greater velocity. Shocks, which were supposed to depend on the velocity of the fluid alone, would therefore not be intensified by increasing the size of the plates.

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* Abhandl. d. Berlin Akad. 1822–3; Ann. d. Phys. lxxiii (1823), pp. 115, 430; vi (1826), pp. 1, 133, 253.

Volta had previously noticed that a silver plate whose ends were at different temperatures appeared to act like a voltaic cell.

Further experiments were performed by James Cumming (*b.* 1777, *d.* 1861), Professor of Chemistry at Cambridge, Trans. Camb. Phil. Soc. ii (1823), p. 47, and by Antoine César Becquerel (*b.* 1788, *d.* 1878), Annales de Chimie, xxxi (1826), p. 371.

† Ann. de Chimie, xxxix (1801), p. 103.

The effect of varying the conductors which connect the terminals of the pile was also studied. Nicolas Gautherot (*b.* 1753, *d.* 1803) observed* that water contained in tubes which have a narrow opening does not conduct voltaic currents so well as when the opening is more considerable. This experiment is evidently very similar to that which Beccaria had performed half a century previously† with electrostatic discharges.

As we have already seen, Cavendish investigated very completely the power of metals to conduct electrostatic discharges; their power of conducting voltaic currents was now examined by Davy.‡ His method was to connect the terminals of a voltaic battery by a path containing water (which it decomposed), and also by an alternative path consisting of the metallic wire under examination. When the length of the wire was less than a certain quantity, the water ceased to be decomposed; Davy measured the lengths and weights of wires of different materials and cross-sections under these limiting circumstances; and, by comparing them, showed that the conducting power of a wire formed of any one metal is inversely proportional to its length and directly proportional to its sectional area, but independent of the shape of the cross-section.§ The latter fact, as he remarked, showed that voltaic currents pass through the substance of the conductor and not along its surface.

Davy, in the same memoir, compared the conductivities of various metals, and studied the effect of temperature: he found that the conductivity varied with the temperature, being "lower in some inverse ratio as the temperature was higher."

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* Annales de Chim., xxxix (1801), p. 203.

† See p. 53.

‡ Phil. Trans., 1821, p. 433. His results were confirmed afterwards by Becquerel, Annales de Chimie, xxxii (1825), p. 423.

§ These results had been known to Cavendish.

He also observed that the same magnetic power is exhibited by every part of the same circuit, even though it be formed of wires of different conducting powers pieced into a chain, so that "the magnetism seems directly as the quantity of electricity which they transmit."

The current which flows in a given voltaic circuit evidently depends not only on the conductors which form the circuit, but also on the driving-power of the battery. In order to form a complete theory of voltaic circuits, it was therefore necessary to extend Davy's laws by taking the driving-power into account. This advance was effected in 1826 by Georg Simon Ohm* (*b.* 1787, *d.* 1854).

Ohm had already carried out a considerable amount of experimental work on the subject, and had, e.g., discovered that if a number of voltaic cells are placed in series in a circuit, the current is proportional to their number if the external resistance is very large, but is independent of their number if the external resistance is small. He now essayed the task of combining all the known results into a consistent theory.

For this purpose he adopted the idea of comparing the flow of electricity in a current to the flow of heat along a wire, the theory of which had been familiar to all physicists since the publication of Fourier's *Théorie analytique de la chaleur* in 1822. "I have proceeded," he says, "from the supposition that the communication of the electricity from one particle takes place directly only to the one next to it, so that no immediate transition from that particle to any other situate at a greater distance occurs. The magnitude of the flow between two adjacent particles, under otherwise exactly similar circumstances, I have assumed to be proportional to the difference of the electric forces existing in the two particles; just as, in the theory of heat, the flow of caloric between two particles is regarded as proportional to the difference of their temperatures."

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* Ann. d. Phys. vi (1826), p. 459; vii, pp. 45, 117; *Die Galvanische Kette mathematisch bearbeitet:* Berlin, 1827; translated in Taylor's *Scientific Memoirs,* ii (1841), p. 401. Cf. also subsequent papers by Ohm in Kastner's *Archiv für d. ges. Naturlehre,* and Schweigger's Jahrbuch.

The comparison between the flow of electricity and the flow of heat suggested the propriety of introducing a quantity whose behaviour in electrical problems should resemble that of temperature in the theory of heat. The differences in the values of such a quantity at two points of a circuit would provide what was so much needed, namely, a measure of the "driving-power" acting on the electricity between these points. To carry out this idea, Ohm recurred to Volta's theory of the electrostatic condition of the open pile. It was customary to measure the "tension" of a pile by connecting one terminal to earth and testing the other terminal by an electroscope. Accordingly Ohm says: "In order to investigate the changes which occur in the electric condition of a body *A* in a perfectly definite manner, the body is each time brought, under similar circumstances, into relation with a second moveable body of invariable electrical condition, called the *electroscope*; and the force with which the electroscope is repelled or attracted by the body is determined. This force is termed the *electroscopic force* of the body *A*."

"The same body *A* may also serve to determine the electroscopic force in various parts of the same body. For this purpose take the body *A* of very small dimensions, so that when we bring it into contact with the part to be tested of any third body, it may from its smallness be regarded as a substitute for this part: then its electroscopic force, measured in the way described, will, when it happens to be different at the various places, make known the relative differences with regard to electricity between these places."

Ohm assumed, as was customary at that period, that when two metals are placed in contact, "they constantly maintain at the point of contact the same difference between their electroscopic forces." He accordingly supposed that each voltaic cell possesses a definite tension, or discontinuity of electroscopic force, which is to be regarded as its contribution to the driving-force of any circuit in which it may be placed. This assumption confers a definite meaning on his use of the term "electroscopic force"; the force in question is identical with the electrostatic potential. But Ohm and his contemporaries did not correctly understand the relation of galvanic conceptions to the electrostatic functions of Poisson. The electroscopic force in the open pile was generally identified with the thickness of the electrical stratum at the place tested; while Ohm, recognizing that electric currents are not confined to the surface of the conductors, but penetrate their substance, seems to have thought of the electroscopic force at a place in a circuit as being proportional to the volume-density of electricity there—an idea in which he was confirmed by the relation which, in an analogous case, exists between the temperature of a body and the volume-density of heat supposed to be contained in it.

Denoting, then, by S the current which flows in a wire of conductivity γ, when the difference of the electroscopic forces at the terminals is E, Ohm writes

$$S = \gamma E.$$

From this formula it is easy to deduce the laws already given by Davy. Thus, if the area of the cross-section of a wire is A, we can by placing n such wires side by side construct a wire of cross-section nA. If the quantity E is the same for each, equal currents will flow in the wires; and therefore the current in the compound wire will be n times that in the single wire; so when the quantity E is unchanged, the current is proportional to the cross-section; that is, the conductivity of a wire is directly proportional to its cross-section, which is one of Davy's laws.

In spite of the confusion which was attached to the idea of electroscopic force, and which was not dispelled for some years, the publication of Ohm's memoir marked a great advance in electrical philosophy. It was now clearly understood that the current flowing in any conductor depends only on the conductivity inherent in the conductor and on another variable which bears to electricity the same relation that temperature bears to heat; and, moreover, it was realized that this latter variable is the link connecting the theory of currents with the older theory of electrostatics. These principles were a sufficient foundation for future progress; and much of the work which was published in the second quarter of the century was no more than the natural development of the principles laid down by Ohm.*

It is painful to relate that the discoverer had long to wait before the merits of his great achievement were officially recognized. Twenty-two years after the publication of the memoir on the galvanic circuit, he was promoted to a university professorship; this he held for the five years which remained until his death in 1854.

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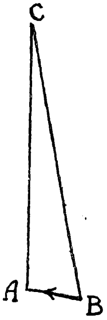
* Ohm's theory was confirmed experimentally by several investigators, among whom may be mentioned Gustav Theodor Fechner (*b.* 1801, *d.* 1887) (*Maassbestimmungen über die Galvanische Kette*, Leipzig, 1831), and Charles Wheatstone (*b.* 1802, *d.* 1875) (Phil. Trans., 1843, p. 303).

CHAPTER IV.

THE LUMINIFEROUS MEDIUM, FROM BRADLEY TO FRESNEL.

ALTHOUGH Newton, as we have seen, refrained from committing himself to any doctrine regarding the ultimate nature of light, the writers of the next generation interpreted his criticism of the wave-theory as equivalent to an acceptance of the corpuscular hypothesis. As it happened, the chief optical discovery of this period tended to support the latter theory, by which it was first and most readily explained. In 1728 James Bradley (*b.* 1692, *d.* 1762), at that time Savilian Professor of Astronomy at Oxford, sent to the Astronomer Royal (Halley) an "Account of a new discovered motion of the Fix'd Stars."* In observing the star γ in the head of the Dragon, he had found that during the winter of 1725–6 the transit across the meridian was continually more southerly, while during the following summer its original position was restored by a motion northwards. Such an effect could not be explained as a result of parallax; and eventually Bradley guessed it to be due to the gradual propagation of light.†

Thus, let *CA* denote a ray of light, falling on the line *BA*; and suppose that the eye of the observer is travelling along *BA*, with a velocity which is to the velocity of light as *BA* is to *CA*. Then the corpuscle of light, by which the object is discernible to the eye at *A*, would have been at *C* when the eye was at *B*. The tube of a telescope must therefore be pointed in the direction *BC*, in order to receive the rays from an object whose light is really propagated in the direction *CA*. The angle *BCA* measures the difference between the real and apparent positions of the object; and it is evident from the figure that the sine of



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* Phil. Trans. xxxv (1728), p. 637.

† Roemer, in a letter to Huygens of date 30th Dec., 1677, mentions a suspected displacement of the apparent position of a star, due to the motion of the earth at right angles to the line of sight. Cf. *Correspondance de Huygens*, viii, p. 53.

this angle is to the sine of the visible inclination of the object to the line in which the eye is moving, as the velocity of the eye is to the velocity of light. Observations such as Bradley's will therefore enable us to deduce the ratio of the mean orbital velocity of the earth to the velocity of light, or, as it is called, *the constant of aberration*; from its value Bradley calculated that light is propagated from the sun to the earth in 8 minutes 12 seconds, which, as he remarked, "is as it were a Mean betwixt what had at different times been determined from the eclipses of Jupiter's satellites."*

With the exception of Bradley's discovery, which was primarily astronomical rather than optical, the eighteenth century was decidedly barren, as regards both the experimental and the theoretical investigation of light; in curious contrast to the brilliance of its record in respect of electrical researches. But some attention must be given to a suggestive study† of the aether, for which the younger John Bernoulli (*b.* 1710, *d.* 1790) was in 1736 awarded the prize of the French Academy. His ideas seem to have been originally suggested by an attempt‡ which his father, the elder John Bernoulli (*b.* 1667, *d.* 1748), had made in 1701 to connect the law of refraction with the mechanical principle of the composition of forces. If two opposed forces whose ratio is μ maintain in equilibrium a particle which is free to move only in a given plane, it follows from the triangle of forces that the directions of the forces must obey the relation

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* Struve in 1845 found for the constant of aberration the value 20″·445, which he afterwards corrected to 20″·463. This was superseded in 1883 by the value 20″·492, determined by M. Nyrén. The observations of both Struve and Nyrén were made with the transit in the prime vertical. The method now generally used depends on the measurement of differences of meridian zenith distances (Talcott's method, as applied by F. Küstner, Beobachtungs-Ergebnisse der kön. Sternwarte zu Berlin, Heft 3, 1888); the value at present favoured for the constant of aberration is 20″·523. Cf. Chandler, Ast. Journal, xxiii, pp. 1, 12 (1903).

The collective translatory motion of the solar system gives rise to aberrational terms in the apparent places of the fixed stars; but the principal term of this character does not vary with the time, and consequently is equivalent to a permanent constant displacement. The second-order terms (i.e. those which involve the ordinary constant of aberration multiplied by the sun's velocity) might be measurable quantities in the case of stars near the Pole; and the same is true of the variations in the first-order terms (i.e. those which involve the sun's velocity not multiplied by the constant of aberration) due to the circumstance that the star's apparent R. A. and Declination, which occur in these terms, are not constant, but are affected by Precession, Nutation, and Aberration. Cf. Seeliger, *Ast. Nach.*, cix., p. 273 (1884).

† Printed in 1752, in the *Recueil des pièces qui ont remportés les prix de l'Acad.*, tome iii.

‡ *Acta eruditorum*, MDCCI, p. 19.

$$\sin i = \mu \sin r,$$

where i and r denote the angles made by these directions with the normals to the plane. This is the same equation as that which expresses the law of refraction, and the elder Bernoulli conjectured that a theory of light might be based on it; but he gave no satisfactory physical reason for the existence of forces along the incident and refracted rays. This defect his son now proceeded to remove.

All space, according to the younger Bernoulli, is permeated by a fluid aether, containing an immense number of excessively small whirlpools. The elasticity which the aether appears to possess, and in virtue of which it is able to transmit vibrations, is really due to the presence of these whirlpools; for, owing to centrifugal force, each whirlpool is continually striving to dilate, and so presses against the neighbouring whirlpools. It will be seen that Bernoulli is a thorough Cartesian in spirit; not only does he reject action at a distance, but he insists that even the elasticity of his aether shall be explicable in terms of matter and motion.

This aggregate of small vortices, or "fine-grained turbulent motion," as it came to be called a century and a half later,* is interspersed with solid corpuscles, whose dimensions are small compared with their distances apart. These are pushed about by the whirlpools whenever the aether is disturbed, but never travel far from their original positions.

A source of light communicates to its surroundings a disturbance which condenses the nearest whirlpools; these by their condensation displace the contiguous corpuscles from their equilibrium position; and these in turn produce condensations in the whirlpools next beyond them, so that vibrations are propagated in every direction from the luminous point. It is curious that Bernoulli speaks of these vibrations as *longitudinal*, and actually contrasts them with those of a stretched cord, which, "when it is slightly displaced from its rectilinear form, and then let go, performs *transverse* vibrations in a direction at right angles to the direction of the cord." When it is remembered that the objection to longitudinal vibrations, on the score of polarization, had already been clearly stated by Newton, and that Bernoulli's aether closely resembles that which Maxwell invented in 1861–2 for the express purpose of securing transversality of vibration, one feels that perhaps no man ever so narrowly missed a great discovery.

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* Cf. Lord Kelvin's vortex-sponge aether, described later in this work.

Bernoulli explained refraction by combining these ideas with those of his father. Within the pores of ponderable bodies the whirlpools are compressed, so the centrifugal force must vary in intensity from one medium to another. Thus a corpuscle situated in the interface between two media is acted on by a greater elastic force from one medium than from the other; and by applying the triangle of forces to find the conditions of its equilibrium, the law of Snell and Descartes may be obtained.

Not long after this, the echoes of the old controversy between Descartes and Fermat about the law of refraction were awakened* by Pierre Louis Moreau de Maupertuis (*b.* 1698, *d.* 1759).

It will be remembered that according to Descartes the velocity of light is greatest in dense media, while according to Fermat the propagation is swiftest in free aether. The arguments of the corpuscular theory convinced Maupertuis that on this particular point Descartes was in the right; but nevertheless he wished to retain for science the beautiful method by which Fermat had derived his result. This he now proposed to do by modifying Fermat's principle so as to make it agree with the corpuscular theory; instead of assuming that light follows the *quickest* path, he supposed that "the path described is that by which the quantity of action is the least"; and this *action* he defined to be proportional to the sum of the spaces described, each multiplied by the velocity with which it is traversed. Thus instead of Fermat's expression

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* Mém. de l'Acad., 1744, p. 417.

$$\int dt \quad \text{or} \quad \int \frac{ds}{v}$$

(where t denotes time, v velocity, and ds an element of the path) Maupertuis introduced

$$\int v\, ds$$

as the quantity which is to assume its minimum value when the path of integration is the actual path of the light. Since Maupertuis' v, which denotes the velocity according to the corpuscular theory, is proportional to the reciprocal of Fermat's v, which denotes the velocity according to the wave-theory, the two expressions are really equivalent, and lead to the same law of refraction. Maupertuis' memoir is, however, of great interest from the point of view of dynamics; for his suggestion was subsequently developed by himself and by Euler and Lagrange into a general principle which covers the whole range of Nature, so far as Nature is a dynamical system.

The natural philosophers of the eighteenth century for the most part, like Maupertuis, accepted the corpuscular hypothesis; but the wave-theory was not without defenders. Franklin* declared for it; and the celebrated mathematician Leonhard Euler (*b.* 1707, *d.* 1783) ranged himself on the same side. In a work entitled *Nova Theoria Lucis et Colorum*, published† while he was living under the patronage of Frederic the Great at Berlin, he insisted strongly on the resemblance between light and sound; "light is in the aether the same thing as sound in air." Accepting Newton's doctrine that colour depends on wave-length, he in this memoir supposed the frequency greatest for red light, and least for violet; but a few years later* he adopted the opposite opinion.

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* Letter xxiii, written in 1752.

† L. Euleri *Opuscula varii argumenti*, Berlin, 1746, p. 169.

The chief novelty of Euler's writings on light is his explanation of the manner in which material bodies appear coloured when viewed by white light; and, in particular, of the way in which the colours of thin plates are produced. He denied that such colours are due to a more copious reflexion of light of certain particular periods, and supposed that they represent vibrations generated within the body itself under the stimulus of the incident light. A coloured surface, according to this hypothesis, contains large numbers of elastic molecules, which, when agitated, emit light of period depending only on their own structure. The colours of thin plates Euler explained in the same way; the elastic response and free period of the plate at any place would, he conceived, depend on its thickness at that place; and in this way the dependence of the colour on the thickness was accounted for, the phenomena as a whole being analogous to well-known effects observed in experiments on sound.

An attempt to improve the corpuscular theory in another direction was made in 1752 by the Marquis de Courtivron,† and independently in the following year by T. Melvill.‡ These writers suggested, as an explanation of the different refrangibility of different colours, that "the differently colour'd rays are projected with different velocities from the luminous body: the red with the greatest, violet with the least, and the intermediate colours with intermediate degrees of velocity." On this supposition, as its authors pointed out, the amount of aberration would be different for every different colour; and the satellites of Jupiter would change colour, from white through green to violet, through an interval of more than half a minute before their immersion into the planet's shadow; while at emersion the contrary succession of colours should be observed, beginning with red and ending in white. The testimony of practical astronomers was soon given that such appearances are not observed; and the hypothesis was accordingly abandoned.

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* Mém. de l'Acad. de Berlin, 1752, p. 262. † Courtivron's *Traité d'optique*, 1752.
‡ Phil. Trans. xlviii (1753), p. 262.

The fortunes of the wave-theory began to brighten at the end of the century, when a new champion arose. Thomas Young, born at Milverton in Somersetshire in 1773, and trained to the practice of medicine, began to write on optical theory in 1799. In his first paper* he remarked that, according to the corpuscular theory, the velocity of emission of a corpuscle must be the same in all cases, whether the projecting force be that of the feeble spark produced by the friction of two pebbles, or the intense heat of the sun itself—a thing almost incredible. This difficulty does not exist in the undulatory theory, since all disturbances are known to be transmitted through an elastic fluid with the same velocity. The reluctance which some philosophers felt to filling all space with an elastic fluid he met with an argument which strangely foreshadows the electric theory of light: "That a medium resembling in many properties that which has been denominated ether does really exist, is undeniably proved by the phenomena of electricity. The rapid transmission of the electrical shock shows that the electric medium is possessed of an elasticity as great as is necessary to be supposed for the propagation of light. Whether the electric ether is to be considered the same with the luminous ether, if such a fluid exists, may perhaps at some future time be discovered by experiment: hitherto I have not been able to observe that the refractive power of a fluid undergoes any change by electricity."

Young then proceeds to show the superior power of the wave-theory to explain reflexion and refraction. In the corpuscular theory it is difficult to see why part of the light should be reflected and another part of the same beam reflected; but in the undulatory theory there is no trouble, as is shown by analogy with the partial reflexion of sound from a cloud or denser stratum of air: "Nothing more is necessary than to suppose all refracting media to retain, by their attraction, a greater or less quantity of the luminous ether, so as to make its density greater than that which it possesses in a vacuum, without increasing its elasticity." This is precisely the hypothesis adopted later by Fresnel and Green.

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* Phil. Trans., 1800, p. 106.

In 1801 Young made a discovery of the first magnitude* when attempting to explain Newton's rings on the principles of the wave-theory. Rejecting Euler's hypothesis of induced vibrations, he assumed that the colours observed all exist in the incident light, and showed that they could be derived from it by a process which was now for the first time recognized in optical science.

The idea of this process was not altogether new, for it had been used by Newton in his theory of the tides. "It may happen," he wrote,† "that the tide may be propagated from the ocean through different channels towards the same port, and may pass in less time through some channels than through others, in which case the same generating tide, being thus divided into two or more succeeding one another, may produce by composition new types of tide." Newton applied this principle to explain the anomalous tides at Batsha in Tonkin, which had previously been described by Halley.‡

Young's own illustration of the principle is evidently suggested by Newton's. "Suppose," he says,§ "a number of equal waves of water to move upon the surface of a stagnant lake, with a certain constant velocity, and to enter a narrow channel leading out of the lake; suppose then another similar cause to have excited another equal series of waves, which arrive at the same channel, with the same velocity, and at the same time with the first. Neither series of waves will destroy the other, but their effects will be combined; if they enter the channel in such a manner that the elevations of one series coincide with those of the other, they must together produce a series of greater joint elevations; but if the elevations of one series are so situated as to correspond to the depressions of the other, they must exactly fill up those depressions, and the surface of the water must remain smooth. Now I maintain that similar effects take place whenever two portions of light are thus mixed; and this I call the general law of the *interference* of light."

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* Phil. Trans., 1802, pp. 12, 387.

† *Principia*, Book III, Prop. 24.

‡ Phil. Trans. xiv (1684), p. 681.

§ Young's *Works*, I, p. 202.

Thus, "whenever two portions of the same light arrive to the eye by different routes, either exactly or very nearly in the same direction, the light becomes most intense when the difference of the routes is any multiple of a certain length, and least intense in the intermediate state of the interfering portions; and this length is different for light of different colours."

Young's explanation of the colours of thin plates as seen by reflexion was, then, that the incident light gives rise to two beams which reach the eye: one of these beams has been reflected at the first surface of the plate, and the other at the second surface; and these two beams produce the colours by their interference.

One difficulty encountered in reconciling this theory with observation arose from the fact that the central spot in Newton's rings (where the thickness of the thin film of air is zero) is black and not white, as it would be if the interfering beams were similar to each other in all respects. To account for this Young showed, by analogy with the impact of elastic bodies, that when light is reflected at the surface of a denser medium, its phase is retarded by half an undulation: so that the interfering beams at the centre of Newton's rings destroy each other. The correctness of this assumption he verified by substituting essence of sassafras (whose refractive index is intermediate between those of crown and flint glass) for air in the space between the lenses; as he anticipated, the centre of the ring-system was now white.

Newton had long before observed that the rings are smaller when the medium producing them is optically more dense. Interpreted by Young's theory, this definitely proved that the wave-length of light is shorter in dense media, and therefore that its velocity is less.

The publication of Young's papers occasioned a fierce attack on him in the *Edinburgh Review*, from the pen of Henry Brougham, afterwards Lord Chancellor of England. Young replied in a pamphlet, of which it is said* that only a single copy was sold; and there can be no doubt that Brougham for the time being achieved his object of discrediting the wave-theory.†

Young now turned his attention to the fringes of shadows. In the corpuscular explanation of these, it was supposed that the attractive forces which operate in refraction extend their influence to some distance from the surfaces of bodies, and inflect such rays as pass close by. If this were the case, the amount of inflexion should obviously depend on the strength of the attractive forces, and consequently on the refractive indices of the bodies—a proposition which had been refuted by the experiments of s'Gravesande. The cause of diffraction effects was thus wholly unknown, until Young, in the Bakerian lecture for 1803,‡ showed that the principle of interference is concerned in their formation; for when a hair is placed in the cone of rays diverging from a luminous point, the internal fringes (i.e. those within the geometrical shadow) disappear when the light passing on one side of the hair is intercepted. His conjecture as to the origin of the interfering rays was not so fortunate; for he attributed the fringes outside the geometrical shadow to interference between the direct rays and rays reflected at the diffracting edge; and supposed the internal fringes of the shadow of a narrow object to be due to the interference of rays inflected by the two edges of the object.

The success of so many developments of the wave-theory led Young to inquire more closely into its capacity for solving the chief outstanding problem of optics—that of the behaviour of light in crystals. The beautiful construction for the extraordinary ray given by Huygens had lain neglected for a century; and the degree of accuracy with which it represented the observations was unknown. At Young's suggestion Wollaston* investigated the matter experimentally, and showed that the agreement between his own measurements and Huygens' rule was remarkably close. "I think," he wrote, "the result must be admitted to be highly favourable to the Huygenian theory; and, although the existence of two refractions at the same time, in the same substance, be not well accounted for, and still less their interchange with each other, when a ray of light is made to pass through a second piece of spar situated transversely to the first, yet the oblique refraction, when considered alone, seems nearly as well explained as any other optical phenomenon."

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* Peacock's *Life of Young*.

† "Strange fellow," wrote Macaulay, when half a century afterwards he found himself sitting beside Brougham in the House of Lords, "his powers gone: his spite immortal."

‡ Phil. Trans., 1804; Young's *Works*, i, p. 179.

Meanwhile the advocates of the corpuscular theory were not idle; and in the next few years a succession of discoveries on their part, both theoretical and experimental, seemed likely to imperil the good position to which Young had advanced the rival hypothesis.

The first of these was a dynamical explanation of the refraction of the extraordinary ray in crystals, which was published in 1808 by Laplace.† His method is an extension of that by which Maupertuis had accounted for the refraction of the ordinary ray, and which since Maupertuis' day had been so developed that it was now possible to apply it to problems of all degrees of complexity. Laplace assumes that the crystalline medium acts on the light-corpuscles of the extraordinary ray so as to modify their velocity, in a ratio which depends on the inclination of the extraordinary ray to the axis of the crystal: so that, in fact, the difference of the squares of the velocities of the ordinary and extraordinary rays is proportional to the square of the sine of the angle which the latter ray makes with the axis. The principle of least action then leads to a law of refraction identical with that found by Huygens' construction with the spheroid; just as Maupertuis' investigation led to a law of refraction for the ordinary ray identical with that found by Huygens' construction with the sphere.

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* Phil. Trans., 1802, p. 381.

† Mém. de l'Inst., 1809, p. 300: Journal de Physique, Jan., 1809; Mém. de la Soc. d'Arcueil, ii.

The law of refraction for the extraordinary ray may also be deduced from Fermat's principle of least time, provided that the velocity is taken inversely proportional to that assumed in the principle of least action; and the velocity appropriate to Fermat's principle agrees with that found by Huygens, being, in fact, proportional to the radius of the spheroid. These results are obvious extensions of those already obtained for ordinary refraction.

Laplace's theory was promptly attacked by Young,* who pointed out the improbability of such a system of forces as would be required to impress the requisite change of velocity on the light-corpuscles. If the aim of controversial matter is to convince the contemporary world, Young's paper must be counted unsuccessful; but it permanently enriched science by proposing a dynamical foundation for double refraction on the principles of the wave-theory. "A solution," he says, "might be deduced upon the Huygenian principles, from the simplest possible supposition, that of a medium more easily compressible in one direction than in any direction perpendicular to it, as if it consisted of an infinite number of parallel plates connected by a substance somewhat less elastic. Such a structure of the elementary atoms of the crystal may be understood by comparing them to a block of wood or of mica. Mr. Chladni found that the mere obliquity of the fibres of a rod of Scotch fir reduced the velocity with which it transmitted sound in the proportion of 4 to 5. It is therefore obvious that a block of such wood must transmit every impulse in spheroidal—that is, oval—undulations; and it may also be demonstrated, as we shall show at the conclusion of this article, that the spheroid will be truly elliptical when the body consists either of plane and parallel strata, or of equidistant fibres, supposing both to be extremely thin, and to be connected by a less highly elastic substance; the spheroid being in the former case oblate and in the latter oblong." Young then proceeds to a formal proof that "an impulse is propagated through every perpendicular section of a lamellar elastic substance in the form of an elliptic undulation." This must be regarded as the beginning of the dynamical theory of light in crystals. It was confirmed in a striking way not long afterwards by Brewster,* who found that compression in one direction causes an isotropic transparent solid to become doubly-refracting.

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* Quarterly Review, Nov., 1809; Young's *Works*, i, p. 220.

Meanwhile, in January, 1808, the French Academy had proposed as the subject for the physical prize in 1810, "To furnish a mathematical theory of double refraction, and to confirm it by experiment." Among those who resolved to compete was Étienne Louis Malus (*b.* 1775, *d.* 1812), a colonel of engineers who had seen service with Napoleon's expedition to Egypt. While conducting experiments towards the end of 1808 in a house in the Rue des Enfers in Paris, Malus happened to analyse with a rhomb of Iceland spar the light of the setting sun reflected from the window of the Luxembourg, and was surprised to notice that the two images were of very different intensities. Following up this observation, he found that light which had been reflected from glass acquires thereby a modification similar to that which Huygens had noticed in rays which have experienced double refraction, and which Newton had explained by supposing rays of light to have "sides." This discovery appeared so important that without waiting for the prize competition he communicated it to the Academy in December, 1808, and published it in the following month.† "I have found," he said, "that this singular disposition, which has hitherto been regarded as one of the peculiar effects of double refraction, can be completely impressed on the luminous molecules by all transparent solids and liquids." "For example, light reflected by the surface of water at an angle of 52°45′ has all the characteristics of one of the beams produced by the double refraction of Iceland spar, whose principal section is parallel to the plane which passes through the incident ray and the reflected ray. If we receive this reflected ray on any doubly-refracting crystal, whose principal section is parallel to the plane of reflexion, it will not be divided into two beams as a ray of ordinary light would be, but will be refracted according to the ordinary law."

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* Phil. Trans., 1815, p. 60.

† Nouveau Bulletin des Sciences, par la Soc. Philomatique, i (1809), p. 266; Mémoires de la Soc. d'Arcueil, ii (1809).

After this Malus found that light which has been refracted at the surface of any transparent substance likewise possesses in some degree this property, to which he gave the name *polarization.* The memoir* which he finally submitted to the Academy, and which contains a rich store of experimental and analytical work on double refraction, obtained the prize in 1810; its immediate effect as regards the rival theories of the ultimate nature of light was to encourage the adherents of the corpuscular doctrine; for it brought into greater prominence the phenomena of polarization, of which the wave-theorists, still misled by the analogy of light with sound, were unable to give any account.

The successful discoverer was elected to the Academy of Sciences, and became a member of the celebrated club of Arcueil.† But his health, which had been undermined by the Egyptian campaign, now broke down completely; and he died, at the age of thirty-six, in the following year.

The polarization of a reflected ray is in general incomplete—i.e. the ray displays only imperfectly the properties of light which has been polarized by double refraction; but for one particular angle of incidence, which depends on the reflecting body, the polarization of the reflected ray is complete. Malus measured with considerable accuracy the polarizing angles for glass and water, and attempted to connect them with the other optical constants of these substances, the refractive indices and dispersive powers, but without success. The matter was afterwards taken up by David Brewster (*b.* 1781, *d.* 1868), who in 1815* showed that there is complete polarization by reflexion when the reflected and refracted rays satisfy the condition of being at right angles to each other.

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* Mém. présentés à l'Inst. par divers Savans, ii (1811), p. 303.

† So called from the village near Paris where Laplace and Berthollet had their country-houses, and where the meetings took place. The club consisted of a dozen of the most celebrated scientific men in France.

Almost at the same time Brewster made another discovery which profoundly affected the theory of double refraction. It had till then been believed that double refraction is always of the type occurring in Iceland spar, to which Huygens' construction is applicable. Brewster now found this belief to be erroneous, and showed that in a large class of crystals there are two axes, instead of one, along which there is no double refraction. Such crystals are called *biaxal*, the simpler type to which Iceland spar belongs being called *uniaxal*.

The wave-theory at this time was still encumbered with difficulties. Diffraction was not satisfactorily explained; for polarization no explanation of any kind was forthcoming; the Huygenian construction appeared to require two different luminiferous media within doubly refracting bodies; and the universality of that construction had been impugned by Brewster's discovery of biaxal crystals.

The upholders of the emission theory, emboldened by the success of Laplace's theory of double refraction, thought the time ripe for their final triumph; and as a step to this, in March, 1817, they proposed Diffraction as the subject of the Academy's prize for 1818. Their expectation was disappointed; and the successful memoir afforded the first of a series of reverses by which, in the short space of seven years, the corpuscular theory was completely overthrown.

The author was Augustin Fresnel (*b.* 1788, *d.* 1827), the son of an architect, and himself a civil engineer in the Government service in Normandy. During the brief dominance of Napoleon after his escape from Elba in 1815, Fresnel fell into trouble for having enlisted in the small army which attempted to bar the exile's return; and it was during a period of enforced idleness following on his arrest that he commenced to study diffraction. In his earliest memoir* he propounded a theory similar to that of Young, which was spoiled like Young's theory by the assumption that the fringes depend on light reflected by the diffracting edge. Observing, however, that the blunt and sharp edges of a knife produce exactly the same fringes, he became dissatisfied with this attempt, and on July 15th, 1816, presented to the Academy a supplement to his paper,† in which, for the first time, diffraction-effects are referred to their true cause—namely, the mutual interference of the secondary waves emitted by those portions of the original wave-front which have not been obstructed by the diffracting screen. Fresnel's method of calculation utilized the principles of both Huygens and Young; he summed the effects due to different portions of the same primary wave-front, with due regard to the differences of phase engendered in propagation.

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* Phil. Trans., 1815, p. 125.

The sketch presented to the Academy in 1816 was during the next two years developed into an exhaustive memoir,‡ which was submitted for the Academy's prize.

It so happened that the earliest memoir, which had been presented to the Academy in the autumn of 1815, had been referred to a Commission of which the reporter was François Arago (*b.* 1786, *d.* 1853); Arago was so much impressed that he sought the friendship of the author, of whom he was later a strenuous champion.

A champion was indeed needed when the larger memoir was submitted; for Laplace, Poisson, and Biot, who constituted a majority of the Commission to which it was referred, were all zealous supporters of the corpuscular theory. During the examination, however, Fresnel was vindicated in a somewhat curious way. He had calculated in the memoir the diffraction-patterns of a straight edge, of a narrow opaque body bounded by parallel sides, and of a narrow opening bounded by parallel edges, and had shown that the results agreed excellently with his experimental measures. Poisson, when reading the manuscript, happened to notice that the analysis could be extended to other cases, and in particular that it would indicate the existence of a bright spot at the centre of the shadow of a circular screen. He suggested to Fresnel that this and some further consequences should be tested experimentally; this was done, and the results were found to confirm the new theory. The concordance of observation and calculation was so admirable in all cases where a comparison was possible that the prize was awarded to Fresnel without further hesitation.

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* Annales de Chimie (2), i (1816), p 239; *Œuvres*, i, p. 89.

† *Œuvres*, i, p. 129.

‡ Mém. de l'Acad., v (1826), p. 339; *Œuvres*, i, p. 247.

In the same year in which the memoir on diffraction was submitted, Fresnel published an investigation* of the influence of the earth's motion on light. We have already seen that aberration was explained by its discoverer in terms of the corpuscular theory; and it was Young who first showed† how it may be explained on the wave-hypothesis. "Upon considering the phenomena of the aberration of the stars," he wrote, "I am disposed to believe that the luminiferous aether pervades the substance of all material bodies with little or no resistance, as freely perhaps as the wind passes through a grove of trees." In fact, if we suppose the aether surrounding the earth to be at rest and unaffected by the earth's motion, the light-waves will not partake of the motion of the telescope, which we may suppose directed to the true place of the star, and the image of the star will therefore be displaced from the central spider-line at the focus by a distance equal to that which the earth describes while the light is travelling through the telescope. This agrees with what is actually observed.

But a host of further questions now suggest themselves. Suppose, for instance, that a slab of glass with a plane face is carried along by the motion of the earth, and it is desired to adjust it so that a ray of light coming from a certain star shall not be bent when it enters the glass: must the surface be placed at right angles to the true direction of the star as freed from aberration, or to its apparent direction as affected by aberration? The question whether rays coming from the stars are refracted differently from rays originating in terrestrial sources had been raised originally by Michell*; and Robison and Wilson† had asserted that the focal length of an achromatic telescope should be increased when it is directed to a star towards which the earth is moving, owing to the change in the relative velocity of light. Arago‡ submitted the matter to the test of experiment, and concluded that the light coming from any star behaves in all cases of reflexion and refraction precisely as it would if the star were situated in the place which it appears to occupy in consequence of aberration, and the earth were at rest; so that the apparent refraction in a moving prism is equal to the absolute refraction in a fixed prism.

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* Annales de Chimie, ix, p. 57 (1818); *Œuvres*, ii, p. 627.

† Phil. Trans., 1804, p. 12; Young's *Works*, i, p. 188.

Fresnel now set out to provide a theory capable of explaining Arago's result. To this end he adopted Young's suggestion, that the refractive powers of transparent bodies depend on the concentration of aether within them; and made it more precise by assuming that the aethereal density in any body is proportional to the square of the refractive index. Thus, if c denote the velocity of light *in vacuo*, and if c_1 denote its velocity in a given material body at rest, so that $\mu = c/c_1$ is the refractive index, then the densities ρ and ρ_1 of the aether in interplanetary space and in the body respectively will be connected by the relation

$$\rho_1 = \mu^2\rho.$$

Fresnel further assumed that, when a body is in motion, part of the aether within it is carried along—namely, that part which constitutes the excess of its density over the density of aether *in vacuo*; while the rest of the aether within the space occupied by the body is stationary. Thus the density of aether carried along is $(\rho_1 - \rho)$ or $(\mu^2 - 1)\rho$, while a quantity of aether of density ρ remains at rest. The velocity with which the centre of gravity of the aether within the body moves forward in the direction of propagation is therefore

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* Phil. Trans., 1784, p. 35.

† Trans. R. S. Edin., i, Hist., p. 30.

‡ Biot, *Astron. Phys.*, 3rd ed., v, p. 364. The accuracy of Arago's experiment can scarcely have been such as to demonstrate absolutely his result.

$$\frac{\mu^2 - 1}{\mu^2} w,$$

where w denotes the component of the velocity of the body in this direction. This is to be added to the velocity of propagation of the light-waves within the body; so that in the moving body the absolute velocity of light is

$$c_1 + \frac{\mu^2 - 1}{\mu^2} w.$$

Many years afterwards Stokes[*] put the same supposition in a slightly different form. Suppose the whole of the aether within the body to move together, the aether entering the body in front, and being immediately condensed, and issuing from it behind, where it is immediately rarefied. On this assumption a mass ρw of aether must pass in unit time across a plane of area unity, drawn anywhere within the body in a direction at right angles to the body's motion; and therefore the aether within the body has a drift-velocity $-w\rho/\rho_1$ relative to the body: so the velocity of light relative to the body will be $c_1 - w\rho/\rho_1$, and the absolute velocity of light in the moving body will be

$$c_1 + w - \frac{w\rho}{\rho_1},$$

or $$c_1 + \frac{\mu^2 - 1}{\mu^2} w,$$ as before.

This formula was experimentally confirmed in 1851 by H. Fizeau,[†] who measured the displacement of interference-fringes formed by light which had passed through a column of moving water.

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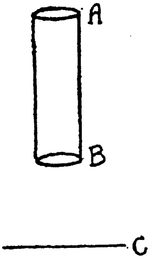
[*] Phil. Mag. xxviii (1846) p. 76.

[†] Annales de Chimie, lvii (1859), p. 385. Also by A. A. Michelson and E. W. Morley, Am. Journ. Science, xxxi (1886), p. 377.



The same result may easily be deduced from an experiment performed by Hoek.* In this a beam of light was divided into two portions, one of which was made to pass through a tube of water *AB* and was then reflected at a mirror *C*, the light being afterwards allowed to return to *A* without passing through the water: while the other portion of the bifurcated beam was made to describe the same path in the reverse order, i.e. passing through the water on its return journey from *C* instead of on the outward journey. On causing the two portions of the beam to interfere, Hoek found that no difference of phase was produced between them when the apparatus was oriented in the direction of the terrestrial motion.

Let w denote the velocity of the earth, supposed to be directed from the tube towards the mirror. Let c/μ denote the velocity of light in the water at rest, and $c/\mu + \phi$ the velocity of light in the water when moving. Let l denote the length of the tube. The magnitude of the distance *BC* does not affect the experiment, so we may suppose it zero.

The time taken by the first portion of the beam to perform its journey is evidently

$$\frac{l}{c/\mu + \phi - w} + \frac{l}{c + w},$$

while the time for the second portion of the beam is

$$\frac{l}{c - w} + \frac{l}{c/\mu - \phi + w}.$$

The equality of these expressions gives at once, when terms of higher orders than the first in w/c are neglected,

$$\phi = (\mu^2 - 1)\, w/\mu^2,$$

which is Fresnel's expression.†

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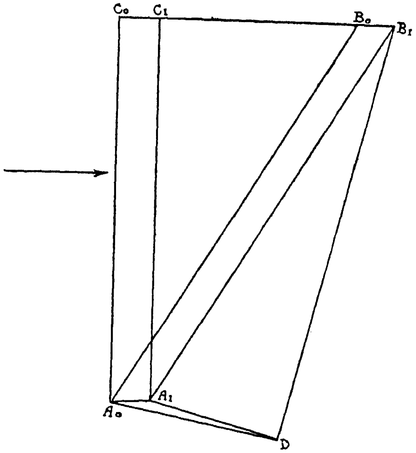
* Archives Néerl. iii, 180 (1868).

† Fresnel's law may also be deduced from the principle that the *amount* of light transmitted by a slab of transparent matter must be the same whether the slab is at rest or in motion: otherwise the equilibrium of exchanges of radiation would be vitiated. Cf. Larmor, Phil. Trans. clxxxv (1893), p. 775.

On the basis of this formula, Fresnel proceeded to solve the problem of refraction in moving bodies. Suppose that a prism $A_0\,C_0\,B_0$ is carried along by the earth's motion *in vacuo*, its face $A_0\,C_0$ being at right angles to the direction of motion; and that light from a star is incident normally on this face. The rays experience no refraction at incidence; and we have only to consider the effect produced by the second surface A_0B_0. Suppose that during an interval τ of time the prism travels from the position $A_0\,C_0\,B_0$ to the position $A_1\,C_1\,B_1$, while the luminous disturbance at C_0 travels to B_1, and the luminous disturbance at A_0 travels to D, so that $B_1\,D$ is the emergent wave-front.



Then we have

$$C_0B_1 = \tau\left(c_1 + \frac{\mu^2 - 1}{\mu^2}\, w\right),$$

$$A_0D = \tau c,$$

$$A_0A_1 = \tau w.$$

If we write $C_1\hat{A_1}B_1 = i$, and denote the total deviation of the wave-front by δ_1, we have

$$A_1D = A_0D - A_1A_0 \cos\delta_1 = \tau c - \tau w \cos\delta_1,$$

$$C_1B_1 = \tau\left(c_1 - \frac{w{c_1}^2}{c^2}\right),$$

and therefore (neglecting second-order terms in w/c)

$$\frac{\sin A_1\hat{B}_1D}{\sin i} = \frac{c - w\cos\delta_1}{c_1 - w\dfrac{c_1^2}{c^2}} = \frac{c}{c_1} + \frac{w}{c} - \frac{w}{c_1}\cos\delta_1.$$

Denoting by δ the value of δ_1 when w is zero, we have

$$\frac{\sin(i-\delta)}{\sin i} = \frac{c}{c_1}.$$

Subtracting this equation from the preceding, we have

$$\frac{\delta - \delta_1}{\sin\delta} = \frac{w}{c}.$$

Now the telescope by which the emergent wave-front B_1D is received is itself being carried forward by the earth's motion; and we must therefore apply the usual correction for aberration in order to find the apparent direction of the emergent ray. But this correction is $w\sin\delta/c$, and precisely counteracts the effect which has been calculated as due to the motion of the prism. So finally we see that the motion of the earth has no first-order influence on the refraction of light from the stars.

Fresnel inferred from his formula that if observations were made with a telescope filled with water, the aberration would be unaffected by the presence of the water—a result which was verified by Airy* in 1871. He showed, moreover, that the apparent positions of terrestrial objects, carried along with the observer, are not displaced by the earth's motion; that experiments in refraction and interference are not influenced by any motion which is common to the source, apparatus, and observer; and that light travels between given points of a moving material system by the path of least time. These predictions have also been confirmed by observation: Respighi† in 1861, and Hoek‡ in 1868, experimenting with a telescope filled with water and a terrestrial source of light, found that no effect was produced on the phenomena of reflexion and refraction by altering the orientation of the apparatus relative to the direction of the earth's motion. E. Mascart* in 1872 discussed experimentally the question of the effect of motion of the source or recipient of light in all its bearings, and showed that the light of the sun and that derived from artificial sources are alike incapable of revealing by diffraction-phenomena the translatory motion of the earth.

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* Proc. Roy. Soc., xx, p. 35.

† Mem. Accad. Sci. Bologna, ii, p. 279.

‡ Ast. Nach., lxxiii, p. 193.

The greatest problem now confronting the investigators of light was to reconcile the facts of polarization with the principles of the wave-theory. Young had long been pondering over this, but had hitherto been baffled by it. In 1816 he received a visit from Arago, who told him of a new experimental result which he and Fresnel had lately obtained†—namely, that two pencils of light, polarized in planes at right angles, do not interfere with each other under circumstances in which ordinary light shows interference-phenomena, but always give by their reunion the same intensity of light, whatever be their difference of path.

Arago had not long left him when Young, reflecting on the new experiment, discovered the long-sought key to the mystery: it consisted in the very alternative which Bernoulli had rejected eighty years before, of supposing that the vibrations of light are executed at right angles to the direction of propagation.

Young's ideas were first embodied in a letter to Arago,‡ dated Jan. 12, 1817. "I have been reflecting," he wrote, "on the possibility of giving an imperfect explanation of the affection of light which constitutes polarization, without departing from the genuine doctrine of undulations. It is a principle in this theory, that all undulations are simply propagated through homogeneous mediums in concentric spherical surfaces like the undulations of sound, consisting simply in the direct and retrograde motions of the particles in the direction of the radius, with their concomitant condensation and rarefactions. And yet it is possible to explain in this theory a transverse vibration, propagated also in the direction of the radius, and with equal velocity, the motions of the particles being in a certain constant direction with respect to that radius; and this is a *polarization.*"

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* Ann. de l'École Noemale, (2) i, p. 157.

† It was not published until 1819, in Annales de Chimie, x; Fresnel's *Œuvres*, i, p. 509. By means of this result, Fresnel was able to give a complete explanation of a class of phenomena which Arago had discovered in 1811, viz. that when polarized light is transmitted through thin plates of sulphate of lime or mica, and afterwards analysed by a prism of Iceland spar, beautiful complementary colours are displayed. Young had shown that these effects are due essentially to interference, but had not made clear the part played by polarization.

‡ Young's *Works*, i., p. 380.

In an article on "Chromatics," which was written in September of the same year* for the supplement to the *Encyclopaedia Britannica,* he says:† "If we assume as a mathematical postulate, on the undulating theory, without attempting to demonstrate its physical foundation, that a transverse motion may be propagated in a direct line, we may derive from this assumption a tolerable illustration of the subdivision of polarized light by reflexion in an oblique plane," by "supposing the polar motion to be resolved" into two constituents, which fare differently at reflexion.

In a further letter to Arago, dated April 29th, 1818, Young recurred to the subject of transverse vibrations, comparing light to the undulations of a cord agitated by one of its extremities.‡ This letter was shown by Arago to Fresnel, who at once saw that it presented the true explanation of the non-interference of beams polarized in perpendicular planes, and that the latter effect could even be made the basis of a proof of the correctness of Young's hypothesis: for if the vibration of each beam be supposed resolved into three components, one along the ray and the other two at right angles to it, it is obvious from the Arago-Fresnel experiment that the components in the direction of the ray must vanish: in other words, that the vibrations which constitute light are executed in the wave-front.

It must be remembered that the theory of the propagation of waves in an elastic solid was as yet unknown, and light was still always interpreted by the analogy with the vibrations of sound in air, for which the direction of vibration is the same as that of propagation. It was therefore necessary to give some justification for the new departure. With wonderful insight Fresnel indicated* the precise direction in which the theory of vibrations in ponderable bodies needed to be extended in order to allow of waves similar to those of light: "the geometers," he wrote, "who have discussed the vibrations of elastic fluids hitherto have taken account of no accelerating forces except those arising from the difference of condensation or dilatation between consecutive layers." He pointed out that if we also suppose the medium to possess a rigidity, or power of resisting distortion, such as is manifested by all actual solid bodies, it will be capable of transverse vibration. The absence of longitudinal waves in the aether he accounted for by supposing that the forces which oppose condensation are far more powerful than those which oppose distortion, and that the velocity with which condensations are propagated is so great compared with the speed of the oscillations of light, that a practical equilibrium of pressure is maintained perpetually.

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* Peacock's *Life of Young,* p. 391.

† Young's *Works,* i., p. 279.

‡ This analogy had been given by Hooke in a communication to the Royal Society on Feb. 15, 1671–2. But there seems no reason to suppose that Hooke appreciated the point now advanced by Young.

The nature of ordinary non-polarized light was next discussed. "If then," Fresnel wrote,† "the polarization of a ray of light consists in this, that all its vibrations are executed in the same direction, it results from any hypothesis on the generation of light-waves, that a ray emanating from a single centre of disturbance will always be polarized in a definite plane at any instant. But an instant afterwards, the direction of the motion changes, and with it the plane of polarization; and these variations follow each other as quickly as the perturbations of the vibrations of the luminous particle: so that even if we could isolate the light of this particular particle from that of other luminous particles, we should doubtless not recognize in it any appearance of polarization. If we consider now the effect produced by the union of all the waves which emanate from the different points of a luminous body, we see that at each instant, at a definite point of the aether, the general resultant of all the motions which commingle there will have a determinate direction, but this direction will vary from one instant to the next. So direct light can be considered as the union, or more exactly as the rapid succession, of systems of waves polarized in all directions. According to this way of looking at the matter, the act of polarization consists not in creating these transverse motions, but in decomposing them in two invariable directions, and separating the components from each other; for then, in each of them, the oscillatory motions take place always in the same plane."

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* Annales de Chimie, xvii (1821), p. 180; *Œuvres*, i, p. 629. Young had already drawn attention to this point. "It is difficult," he says in his *Lectures on Natural Philosophy*, ed. 1807, vol. i, p. 138, "to compare the lateral adhesion, or the force which resists the detrusion of the parts of a solid, with any form of direct cohesion. This force constitutes the rigidity or hardness of a solid body, and is wholly absent from liquids."

† Loc. cit., p. 185.

He then proceeded to consider the relation of the direction of vibration to the plane of polarization. "Apply these ideas to double refraction, and regard a uniaxal crystal as an elastic medium in which the accelerating force which results from the displacement of a row of molecules perpendicular to the axis, relative to contiguous rows, is the same all round the axis; while the displacements parallel to the axis produce accelerating forces of a different intensity, stronger if the crystal is "repulsive," and weaker if it is "attractive." The distinctive character of the rays which are ordinarily refracted being that of propagating themselves with the same velocity in all directions, we must admit that their oscillatory motions are executed at right angles to the plane drawn through these rays and the axis of the crystal; for then the displacements which they occasion, always taking place along directions perpendicular to the axis, will, by hypothesis, always give rise to the same accelerating forces. But, with the conventional meaning which is attached to the expression *plane of polarization*, the plane of polarization of the ordinary rays is the plane through the axis: thus, *in a pencil of polarized light, the* *oscillatory motion is executed at right angles to the plane of polarization.*"

This result afforded Fresnel a foothold in dealing with the problem which occupied the rest of his life: henceforth his aim was to base the theory of light on the dynamical properties of the luminiferous medium.

The first topic which he attacked from this point of view was the propagation of light in crystalline bodies. Since Brewster's discovery that many crystals do not conform to the type to which Huygens' construction is applicable, the wave theory had to some extent lost credit in this region. Fresnel, now, by what was perhaps the most brilliant of all his efforts,* not only reconquered the lost territory, but added a new domain to science.

He had, as he tells us himself, never believed the doctrine that in crystals there are two different luminiferous media, one to transmit the ordinary, and the other the extraordinary waves. The alternative to which he inclined was that the two velocities of propagation were really the two roots of a quadratic equation, derivable in some way from the theory of a single aether. Could this equation be obtained, he was confident of finding the explanation, not only of double refraction, but also of the polarization by which it is always accompanied.

The first step was to take the case of uniaxal crystals, which had been discussed by Huygens, and to see whether Huygens' sphere and spheroid could be replaced by, or made to depend on, a *single* surface.†

Now a wave propagated in any direction through a uniaxal crystal can be resolved into two plane-polarized components; one of these, the "ordinary ray," is polarized in the principal section, and has a velocity v_1, which may be represented by the radius of Huygens' sphere—say,

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* His first memoir on Double Refraction was presented to the Academy on Nov. 19th, 1821, but has not been published except in his collected works: *Œuvres*, ii, p. 261. It was followed by other papers in 1822; and the results were finally collected in a memoir which was printed in 1827, *Mém. de l'Acad.* vii, p. 45, *Œuvres*, ii, p. 479.

† In attempting to reconstruct Fresnel's course of thought at this period, the present writer has derived much help from the Life prefixed to the *Œuvres de Fresnel*. Both Fresnel and Young were singularly fortunate in their biographers: Peacock's *Life of Young*, and this notice of Fresnel, which was the last work of Verdet, are excellent reading.

$$v_1 = b;$$

while the other, the "extraordinary ray," is polarized in a plane at right angles to the principal section, and has a wave-velocity v_2, which may be represented by the perpendicular drawn from the centre of Huygens' spheroid on the tangent-plane parallel to the plane of the wave. If the spheroid be represented by the equation

$$\frac{y^2 + z^2}{a^2} + \frac{x^2}{b^2} = 1,$$

and if (l, m, n) denote the direction-cosines of the normal to the plane of the wave, we have therefore

$$v_2^2 = a^2(m^2 + n^2) + b^2l^2.$$

But the quantities $1/v_1$ and $1/v_2$, as given by these equations, are easily seen to be the lengths of the semi-axes of the ellipse in which the spheroid

$$b^2(y^2 + z^2) + a^2x^2 = 1$$

is intersected by the plane

$$lx + my + nz = 0;$$

and thus the construction in terms of Huygens' sphere and spheroid can be replaced by one which depends only on a single surface, namely the spheroid

$$b^2(y^2 + z^2) + a^2x^2 = 1.$$

Having achieved this reduction, Fresnel guessed that the case of biaxal crystals could be covered by substituting for the latter spheroid an ellipsoid with three unequal axes—say,

$$\frac{x^2}{\epsilon_1} + \frac{y^2}{\epsilon_2} + \frac{z^2}{\epsilon_3} = 1.$$

If $1/v_1$ and $1/v_2$ denote the lengths of the semi-axes of the ellipse in which this ellipsoid is intersected by the plane

$$lx + my + nz = 0,$$

it is well known that v_1 and v_2 are the roots of the equation in v

$$\frac{l^2}{\dfrac{1}{\varepsilon_1} - v^2} + \frac{m^2}{\dfrac{1}{\varepsilon_2} - v^2} + \frac{n^2}{\dfrac{1}{\varepsilon_3} - v^2} = 0;$$

and accordingly Fresnel conjectured that the roots of this equation represent the velocities, in a biaxal crystal, of the two plane-polarized waves whose normals are in the direction (l, m, n).

Having thus arrived at his result by reasoning of a purely geometrical character, he now devised a dynamical scheme to suit it.

The vibrating medium within a crystal he supposed to be ultimately constituted of particles subjected to mutual forces; and on this assumption he showed that the elastic force of restitution when the system is disturbed must depend linearly on the displacement. In this first proposition a difference is apparent between Fresnel's and a true elastic-solid theory; for in actual elastic solids the forces of restitution depend not on the absolute displacement, but on the strains, i.e., the *relative* displacements.

In any crystal there will exist three directions at right angles to each other, for which the force of restitution acts in the same line as the displacement: the directions which possess this property are named *axes of elasticity.* Let these be taken as axes, and suppose that the elastic forces of restitution for unit displacements in these three directions are $1/\varepsilon_1$, $1/\varepsilon_2$, $1/\varepsilon_3$ respectively. That the elasticity should vary with the direction of the molecular displacement seemed to Fresnel to suggest that the molecules of the material body either take part in the luminous vibration, or at any rate influence in some way the elasticity of the aether.

A unit displacement in any arbitrary direction (α, β, γ) can be resolved into component displacements $(\cos \alpha, \cos \beta, \cos \gamma)$ parallel to the axes, and each of these produces its own effect independently; so the components of the force of restitution are

$$\frac{\cos\alpha}{\varepsilon_1}, \quad \frac{\cos\beta}{\varepsilon_2}, \quad \frac{\cos\gamma}{\varepsilon_3}.$$

This resultant force has not in general the same direction as the displacement which produced it; but it may always be decomposed into two other forces, one parallel and the other perpendicular to the direction of the displacement; and the former of these is evidently

$$\frac{\cos^2\alpha}{\varepsilon_1} + \frac{\cos^2\beta}{\varepsilon_2} + \frac{\cos^2\gamma}{\varepsilon_3}.$$

The surface

$$r^4 = \frac{x^2}{\varepsilon_1} + \frac{y^2}{\varepsilon_2} + \frac{z^2}{\varepsilon_3}$$

will therefore have the property that the square of its radius vector in any direction is proportional to the component in that direction of the elastic force due to a unit displacement in that direction: it is called the *surface of elasticity*.

Consider now a displacement along one of the axes of the section on which the surface of elasticity is intersected by the plane of the wave. It is easily seen that in this case the component of the elastic force at right angles to the displacement acts along the normal to the wave-front; and Fresnel assumes that it will be without influence on the propagation of the vibrations, on the ground of his fundamental hypothesis that the vibrations of light are performed solely in the wave-front. This step is evidently open to criticism; for in a dynamical theory everything should be deduced from the laws of motion without special assumptions. But granting his contention, it follows that such a displacement will retain its direction, and will be propagated as a plane-polarized wave with a definite velocity.

Now, in order that a stretched cord may vibrate with unchanged period, when its tension is varied, its length must be increased proportionally to the square root of its tension; and similarly the wave-length of a luminous vibration of given period is proportional to the square root of the elastic force (per unit displacement), which urges the molecules of the medium parallel to the wave-front. Hence the velocity of propagation of a wave, measured at right angles to its front, is proportional to the square root of the component, along the direction of displacement, of the elastic force per unit displacement; and the velocity of propagation of such a plane-polarized wave as we have considered is proportional to the radius vector of the surface of elasticity in the direction of displacement.

Moreover, any displacement in the given wave-front can be resolved into two, which are respectively parallel to the two axes of the diametral section of the surface of elasticity by a plane parallel to this wave-front; and it follows from what has been said that each of these component displacements will be propagated as an independent plane-polarized wave, the velocities of propagation being proportional to the axes of the section,* and therefore inversely proportional to the axes of the section of the inverse surface of this with respect to the origin, which is the ellipsoid

$$\frac{x^2}{\varepsilon_1} + \frac{y^2}{\varepsilon_2} + \frac{z^2}{\varepsilon_3} = 1.$$

But this is precisely the result to which, as we have seen, Fresnel had been led by purely geometrical considerations; and thus his geometrical conjecture could now be regarded as substantiated by a study of the dynamics of the medium.

It is easy to determine the wave-surface or locus at any instant—say, $t = 1$—of a disturbance originated at some previous instant—say, $t = 0$—at some particular point—say, the origin. For this wave-surface will evidently be the envelope of plane waves emitted from the origin at the instant $t = 0$—that is, it will be the envelope of planes

$$lx + my + nz - v = 0,$$

where the constants l, m, n, v are connected by the identical equation

$$l^2 + m^2 + n^2 = 1,$$

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* It is evident from this that the *optic axes*, or lines of single wave-velocity, along which there is no double refraction, will be perpendicular to the two circular sections of the surface of elasticity.

and by the relation previously found—namely,

$$\frac{l^2}{\frac{1}{\varepsilon_1} - v^2} + \frac{m^2}{\frac{1}{\varepsilon_2} - v^2} + \frac{n^2}{\frac{1}{\varepsilon_3} - v^2} = 0.$$

By the usual procedure for determining envelopes, it may be shown that the locus in question is the surface of the fourth degree

$$\frac{x^2}{\varepsilon_1 r^2 - 1} + \frac{y^2}{\varepsilon_2 r^2 - 1} + \frac{z^2}{\varepsilon_3 r^2 - 1} = 0,$$

which is called *Fresnel's wave-surface.** It is a two-sheeted surface, as must evidently be the case from physical considerations. In uniaxal crystals, for which ε_2 and ε_3 are equal, it degenerates into the sphere

$$r^2 = 1/\varepsilon_2,$$

and the spheroid

$$\varepsilon_2 x^2 + \varepsilon_1 (y^2 + z^2) = 1.$$

It is to these two surfaces that tangent-planes are drawn in the construction given by Huygens for the ordinary and extraordinary refracted rays in Iceland spar. As Fresnel observed, exactly the same construction applies to biaxal crystals, when the two sheets of the wave-surface are substituted for Huygens' sphere and spheroid.

"The theory which I have adopted," says Fresnel at the end of this memorable paper, "and the simple constructions which I have deduced from it, have this remarkable character, that all the unknown quantities are determined together by the solution of the problem. We find at the same time the velocities of the ordinary ray and of the extraordinary ray, and their planes of polarization. Physicists who have studied attentively the laws of nature will feel that such simplicity and

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* Another construction for the wave-surface is the following, which is due to MacCullagh, *Coll. Works*, p. 1. Let the ellipsoid

$$\epsilon_1 x^2 + \epsilon_2 y^2 + \epsilon_3 z^2 = 1$$

be intersected by a plane through its centre, and on the perpendicular to that plane take lengths equal to the semi-axes of the section. The locus of these extremities is the wave-surface.

such close relations between the different elements of the phenomenon are conclusive in favour of the hypothesis on which they are based."

The question as to the correctness of Fresnel's construction was discussed for many years afterwards. A striking consequence of it was pointed out in 1832 by William Rowan Hamilton (*b.* 1805, *d.* 1865), Royal Astronomer of Ireland, who remarked* that the surface defined by Fresnel's equation has four conical points, at each of which there is an infinite number of tangent planes; consequently, a single ray, proceeding from a point within the crystal in the direction of one of these points, must be divided on emergence into an infinite number of rays, constituting a conical surface. Hamilton also showed that there are four planes, each of which touches the wave-surface in an infinite number of points, constituting a circle of contact: so that a corresponding ray incident externally should be divided within the crystal into an infinite number of refracted rays, again constituting a conical surface.

These singular and unexpected consequences of the theory were shortly afterwards verified experimentally by Humphrey Lloyd,† and helped greatly to confirm belief in Fresnel's theory. It should, however, be observed that conical refraction only shows his form of the wave-surface to be correct in its general features, and is no test of its accuracy in all details. But it was shown experimentally by Stokes in 1872,‡ Glazebrook in 1879,§ and Hastings in 1887,‖ that the construction of Huygens and Fresnel is certainly correct to a very high degree of approximation; and Fresnel's final formulae have since been regarded as unassailable. The dynamical substructure on which he based them is, as we have seen, open to objection;

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* Trans. Roy. Irish Acad., xvii (1833), p. 1.

† Trans. Roy. Irish Acad., xvii (1833), p. 145. Strictly speaking, the bright cone which is usually observed arises from rays *adjacent* to the singular ray: the latter can, however, be observed, its enfeeblement by dispersion into the conical form causing it to appear dark.

‡ Proc. R. S., xx, p. 443.

§ Phil. Trans., clxxi, p. 421.

‖ Am. Jour. Sci. (3), xxxv, p. 60.

but, as Stokes observed*: "If we reflect on the state of the subject as Fresnel found it, and as he left it, the wonder is, not that he failed to give a rigorous dynamical theory, but that a single mind was capable of effecting so much."

In a second supplement to his first memoir on Double Refraction, presented to the Academy on November 26th, 1821,† Fresnel indicated the lines on which his theory might be extended so as to take account of dispersion. "The molecular groups, or the particles of bodies," he wrote, "may be separated by intervals which, though small, are certainly not altogether insensible relatively to the length of a wave." Such a coarse-grainedness of the medium would, as he foresaw, introduce into the equations terms by which dispersion might be explained; indeed, the theory of dispersion which was afterwards given by Cauchy was actually based on this principle. It seems likely that, towards the close of his life, Fresnel was contemplating a great memoir on dispersion,‡ which was never completed.

Fresnel had reason at first to be pleased with the reception of his work on the optics of crystals: for in August, 1822, Laplace spoke highly of it in public; and when at the end of the year a seat in the Academy became vacant, he was encouraged to hope that the choice would fall on him. In this he was disappointed.§ Meanwhile his researches were steadily continued; and in January, 1823, the very month of his rejection, he presented to the Academy a theory in which reflexion and refraction‖ are referred to the dynamical properties of the luminiferous media.

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* Brit. Assoc. Rep., 1862, p. 254.

† *Œuvres*, ii, p. 438.

‡ Cf. the biography in *Œuvres de Fresnel*, i, p. xcvi.

§ Writing to Young in the spring of 1823, he says: "Tous ces mémoires, que dernièrement j'ai présentés coup sur coup à l'Académie des Sciences, ne m'en ont pas cependant ouvert la porte. C'est M. Dulong qui a été nommé pour remplir la place vacante dans la section de physique. . . Vous voyez, Monsieur, que la théorie des ondulations ne m'a point porté bonheur: mais cela ne m'en dégoûte pas: et je me console de ce malheur en m'occupant d'optique avec une nouvelle ardeur."

‖ The MSS. was for some time believed to be lost, but was ultimately found among the papers of Fourier, and printed in Mém. de l'Acad. xi (1832), p. 393: *Œuvres*, i, p. 767.

As in his previous investigations, he assumes that the vibrations which constitute light are executed at right angles to the plane of polarization. He adopts Young's principle, that reflexion and refraction are due to differences in the inertia of the aether in different material bodies, and supposes (as in his memoir on Aberration) that the inertia is proportional to the inverse square of the velocity of propagation of light in the medium. The conditions which he proposes to satisfy at the interface between two media are that the displacements of the adjacent molecules, resolved parallel to this interface, shall be equal in the two media; and that the energy of the reflected and refracted waves together shall be equal to that of the incident wave.

On these assumptions the intensity of the reflected and refracted light may be obtained in the following way:—

Consider first the case in which the incident light is polarized in the plane of incidence, so that the displacement is at right angles to the plane of incidence; let the amplitude of the displacement at a given point of the interface be f for the incident ray, g for the reflected ray, and h for the refracted ray.

The quantities of energy propagated per second across unit cross-section of the incident, reflected, and refracted beams are proportional respectively to

$$c_1\rho_1 f^2, \quad c_1\rho_1 g^2, \quad c_2\rho_2 h^2,$$

where c_1, c_2, denote the velocities of light, and ρ_1, ρ_2 the densities of aether, in the two media; and the cross-sections of the beams which meet the interface in unit area are

$$\cos i, \quad \cos i, \quad \cos r$$

respectively. The principle of conservation of energy therefore gives

$$c_1\rho_1 \cos i \,.\, f^2 = c_1\rho_1 \cos i \,.\, g^2 + c_2\rho_2 \cos r \,.\, h^2.$$

The equation of continuity of displacement at the interface is

$$f + g = h.$$

Eliminating h between these two equations, and using the formulae

$$\frac{\sin^2 r}{\sin^2 i} = \frac{c_2^2}{c_1^2} = \frac{\rho_1}{\rho_2},$$

we obtain the equation

$$\frac{f}{g} = -\frac{\sin(i-r)}{\sin(i+r)}.$$

Thus when the light is polarized in the plane of reflexion, the amplitude of the reflected wave is

$$\frac{\sin(i-r)}{\sin(i+r)} \times \text{the amplitude of the incident vibration.}$$

Fresnel shows in a similar way that when the light is polarized at right angles to the plane of reflexion, the ratio of the amplitudes of the reflected and incident waves is

$$\frac{\tan(i-r)}{\tan(i+r)}.$$

These formulæ are generally known as *Fresnel's sine-law* and *Fresnel's tangent-law* respectively. They had, however, been discovered experimentally by Brewster some years previously. When the incidence is perpendicular, so that i and r are very small, the ratio of the amplitudes becomes

$$\text{Limit } \frac{i-r}{i+r},$$

or

$$\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1},$$

where μ_2 and μ_1 denote the refractive indices of the media. This formula had been given previously by Young* and Poisson,† on the supposition that the elasticity of the aether is of the same kind as that of air in sound.

When $i + r = 90°$, $\tan(i + r)$ becomes infinite: and thus a theoretical explanation is obtained for Brewster's law, that if the incidence is such as to make the reflected and refracted rays perpendicular to each other, the reflected light will be wholly polarized in the plane of reflexion.

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* Article *Chromatics*, Encycl. Britt. Suppl.

† Mém. Inst. ii. (1817).

Fresnel's investigation can scarcely be called a dynamical theory in the strict sense, as the qualities of the medium are not defined. His method was to work backwards from the known properties of light, in the hope of arriving at a mechanism to which they could be attributed; he succeeded in accounting for the phenomena in terms of a few simple principles, but was not able to specify an aether which would in turn account for these principles. The "displacement" of Fresnel could not be a displacement in an elastic solid of the usual type, since its normal component is not continuous across the interface between two media.*

The theory of ordinary reflexion was completed by a discussion of the case in which light is reflected totally. This had formed the subject of some of Fresnel's experimental researches several years before; and in two papers† presented to the Academy in November, 1817, and January, 1818, he had shown that light polarized in any plane inclined to the plane of reflexion is partly "depolarized" by total reflexion, and that this is due to differences of phase which are introduced between the components polarized in and perpendicular to the plane of reflexion. "When the reflexion is total," he said, "rays polarized in the plane of reflexion are reflected nearer the surface of the glass than those polarized at right angles to the same plane, so that there is a difference in the paths described."

This change of phase he now deduced from the formulae already obtained for ordinary reflexion. Considering light polarized in the plane of reflexion, the ratio of the amplitudes of the reflected and incident light is, as we have seen,

$$-\frac{\sin(i-r)}{\sin(i+r)};$$

when the sine of the angle of incidence is greater than μ_2/μ_1, so that total reflexion takes place, this ratio may be written in the form

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* Fresnel's theory of reflexion can, however, be reconciled with the electromagnetic theory of light, by identifying his "displacement" with the electric force.

† *Œuvres de Fresnel*, i., pp. 441, 487.

$$e^{\theta\sqrt{-1}},$$

where θ denotes a real quantity defined by the equation

$$\tan \tfrac{1}{2}\theta = \frac{(\mu_1{}^2 \sin^2 i - \mu_2{}^2)^{\frac{1}{2}}}{\mu_1 \cos i}.$$

Fresnel interpreted this expression to mean that the amplitude of the reflected light is equal to that of the incident, but that the two waves differ in phase by an amount θ. The case of light polarized at right angles to the plane of reflexion may be treated in the same way, and the resulting formulae are completely confirmed by experiment.

A few months after the memoir on reflexion had been presented, Fresnel was elected to a seat in the Academy; and during the rest of his short life honours came to him both from France and abroad. In 1827 the Royal Society awarded him the Rumford medal; but Arago, to whom Young had confided the mission of conveying the medal, found him dying; and eight days afterwards he breathed his last.

By the genius of Young and Fresnel the wave-theory of light was established in a position which has since remained unquestioned; and it seemed almost a work of supererogation when, in 1850, Foucault* and Fizeau,† carrying out a plan long before imagined by Arago, directly measured the velocity of light in air and in water, and found that on the question so long debated between the rival schools the adherents of the undulatory theory had been in the right.

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* Comptes Rendus, xxx (1850), p. 551.

† *Ibid.*, p. 562.

CHAPTER V.

THE AETHER AS AN ELASTIC SOLID.

WHEN Young and Fresnel put forward the view that the vibrations of light are performed at right angles to its direction of propagation, they at the same time pointed out that this peculiarity might be explained by making a new hypothesis regarding the nature of the luminiferous medium; namely, that it possesses the power of resisting attempts to distort its shape. It is by the possession of such a power that solid bodies are distinguished from fluids, which offer no resistance to distortion; the idea of Young and Fresnel may therefore be expressed by the simple statement that *the aether behaves as an elastic solid.* After the death of Fresnel this conception was developed in a brilliant series of memoirs to which our attention must now be directed.

The elastic-solid theory meets with one obvious difficulty at the outset. If the aether has the qualities of a solid, how is it that the planets in their orbital motions are able to journey through it at immense speeds without encountering any perceptible resistance? This objection was first satisfactorily answered by Sir George Gabriel Stokes* (*b.* 1819, *d.* 1903), who remarked that such substances as pitch and shoemaker's wax, though so rigid as to be capable of elastic vibration, are yet sufficiently plastic to permit other bodies to pass slowly through them. The aether, he suggested, may have this combination of qualities in an extreme degree, behaving like an elastic solid for vibrations so rapid as those of light, but yielding like a fluid to the much slower progressive motions of the planets.

Stokes's explanation harmonizes in a curious way with Fresnel's hypothesis that the velocity of longitudinal waves in

* Trans. Camb. Phil. Soc., viii, p. 287 (1845).

the aether is indefinitely great compared with that of the transverse waves; for it is found by experiment with actual substances that the ratio of the velocity of propagation of longitudinal waves to that of transverse waves increases rapidly as the medium becomes softer and more plastic.

In attempting to set forth a parallel between light and the vibrations of an elastic substance, the investigator is compelled more than once to make a choice between alternatives. He may, for instance, suppose that the vibrations of the aether are executed either parallel to the plane of polarization of the light or at right angles to it; and he may suppose that the different refractive powers of different media are due either to differences in the inertia of the aether within the media, or to differences in its power of resisting distortion, or to both these causes combined. There are, moreover, several distinct methods for avoiding the difficulties caused by the presence of longitudinal vibrations; and as, alas! we shall see, a further source of diversity is to be found in that liability to error from which no man is free. It is therefore not surprising that the list of elastic-solid theories is a long one.

At the time when the transversality of light was discovered, no general method had been developed for investigating mathematically the properties of elastic bodies; but under the stimulus of Fresnel's discoveries, some of the best intellects of the age were attracted to the subject. The volume of Memoirs of the Academy which contains Fresnel's theory of crystal-optics contains also a memoir by Claud Louis Marie Henri Navier* (*b.* 1785, *d.* 1836), at that time Professor of Mechanics in Paris, in which the correct equations of vibratory motion for a particular type of elastic solid were for the first time given. Navier supposed the medium to be ultimately constituted of an immense number of particles, which act on each other with forces directed along the lines joining them, and depending on their distances apart; and showed that if **e** denote the (vector) displacement of the particle whose undisturbed position is (x, y, z), and if ρ denote the density of the medium, the equation of motion is

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* Mém. de l'Acad. vii, p. 375. The memoir was presented in 1821, and published in 1827.

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = -3n \text{ grad div } \mathbf{e} - n \text{ curl curl } \mathbf{e},$$

where n denotes a constant which measures the *rigidity*, or power of resisting distortion, of the medium. All such elastic properties of the body as the velocity of propagation of waves in it must evidently depend on the ratio n/ρ.

Among the referees of one of Navier's papers was Augustine Louis Cauchy (*b.* 1789, *d.* 1857), one of the greatest analysts of the nineteenth century,* who, becoming interested in the question, published in 1828† a discussion of it from an entirely different point of view. Instead of assuming, as Navier had done, that the medium is an aggregate of point-centres of force, and thus involving himself in doubtful molecular hypotheses, he devised a method of directly studying the elastic properties of matter in bulk, and by its means showed that the vibrations of an isotropic solid are determined by the equation

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = -\left(k + \frac{4}{3} n\right) \text{grad div } \mathbf{e} - n \text{ curl curl } \mathbf{e};$$

here n denotes, as before, the constant of rigidity; and the constant k, which is called the *modulus of compression*,‡ denotes the ratio of a pressure to the cubical compression produced by it. Cauchy's equation evidently differs from Navier's in that two constants, k and n, appear instead of one. The reason for this is that a body constituted from point-centres of force in Navier's fashion has its moduli of rigidity and compression connected by the relation*

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* Hamilton's opinion, written in 1833, is worth repeating: "The principal theories of algebraical analysis (under which I include Calculi) require to be entirely remodelled; and Cauchy has done much already for this great object. Poisson also has done much; but he does not seem to me to have nearly so logical a mind as Cauchy, great as his talents and clearness are; and both are in my judgment very far inferior to Fourier, whom I place at the head of the French School of Mathematical Philosophy, even above Lagrange and Laplace, though I rank *their* talents above those of Cauchy and Poisson." (*Life of Sir W. R. Hamilton*, ii, p. 58.)

† Cauchy, *Exercices de Mathématiques* iii, p. 160 (1828).

‡ This notation was introduced at a later period, but is used here in order to avoid subsequent changes.

$$k = \frac{5}{3} n.$$

Actual bodies do not necessarily obey this condition; e.g. for india-rubber, k is much larger than $\frac{5}{3} n$;† and there seems to be no reason why we should impose it on the aether.

In the same year Poisson‡ succeeded in solving the differential equation which had thus been shown to determine the wave-motions possible in an elastic solid. The solution, which is both simple and elegant, may be derived as follows :—Let the displacement vector **e** be resolved into two components, of which one **c** is *circuital*, or satisfies the condition

$$\operatorname{div} \mathbf{c} = 0,$$

while the other **b** is *irrotational*, or satisfies the condition

$$\operatorname{curl} \mathbf{b} = 0.$$

The equation takes the form

$$\rho \frac{\partial^2}{\partial t^2}(\mathbf{b} + \mathbf{c}) - n\nabla^2 \mathbf{c} - \left(k + \frac{4}{3} n\right) \nabla^2 \mathbf{b} = 0.$$

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* In order to construct a body whose elastic properties are not limited by this equation, William John Macquorn Rankine (*b.* 1820, *d.* 1872) considered a continuous fluid in which a number of point-centres of force are situated: the fluid is supposed to be partially condensed round these centres, the elastic atmosphere of each nucleus being retained round it by attraction. An additional volume-elasticity due to the fluid is thus acquired; and no relation between k and n is now necessary. Cf. Rankine's *Miscellaneous Scientific Papers*, pp. 81 sqq.

Sir William Thomson (Lord Kelvin), in 1889, formed a solid not obeying Navier's condition by using pairs of dissimilar atoms. Cf. Thomson's *Papers*, iii, p. 395. Cf. also *Baltimore Lectures*, pp. 123 sqq.

† It may, however, be objected that india-rubber and other bodies which fail to fulfil Navier's relation are not true solids. On this historic controversy, cf. Todhunter and Pearson's *History of Elasticity*, i, p. 496.

‡ Mém. de l'Acad., viii (1828), p. 623. Poisson takes the equation in the restricted form given by Navier; but this does not affect the question of wave-propagation.

The terms which involve **b** and those which involve **c** must be separately zero, since they represent respectively the irrotational and the circuital parts of the equation. Thus, **c** satisfies the pair of equations

$$\rho \frac{\partial^2 \mathbf{c}}{\partial t^2} = n\nabla^2 \mathbf{c}, \quad \operatorname{div} \mathbf{c} = 0;$$

while **b** is to be determined from

$$\rho \frac{\partial^2 \mathbf{b}}{\partial t^2} = \left(k + \frac{4}{3} n\right) \nabla^2 \mathbf{b}, \quad \operatorname{curl} \mathbf{b} = 0.$$

A particular solution of the equations for **c** is easily seen to be

$$c_x = A \sin \lambda \left(z - t \sqrt{\frac{n}{\rho}}\right), \quad c_y = B \sin \lambda \left(z - t \sqrt{\frac{n}{\rho}}\right), \quad c_z = 0,$$

which represents a transverse plane wave propagated with velocity $\sqrt{(n/\rho)}$. It can be shown that the general solution of the differential equations for **c** is formed of such waves as this, travelling in all directions, superposed on each other

A particular solution of the equations for **b** is

$$b_x = 0, \quad b_y = 0, \quad b_z = C \sin \lambda \left(z - t \sqrt{\frac{k + \frac{4}{3}n}{\rho}}\right),$$

which represents a longitudinal wave propagated with velocity

$$\sqrt{(k + \tfrac{4}{3}n)/\rho};$$

the general solution of the differential equation for **b** is formed by the superposition of such waves as this, travelling in all directions.

Poisson thus discovered that the waves in an elastic solid are of two kinds: those in **c** are transverse, and are propagated with velocity $(n/\rho)^{\frac{1}{2}}$; while those in **b** are longitudinal, and are propagated with velocity $\{(k + \frac{4}{3}n)/\rho\}^{\frac{1}{2}}$. The latter are[*] waves of dilatation and condensation, like sound-waves; in the **c**-waves, on the other hand, the medium is not dilated or condensed, but only distorted in a manner consistent with the preservation of a constant density.*

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[*] Cf. Stokes, "On the Dynamical Problem of Diffraction," Camb. Phil. Trans., ix (1849).

The researches which have been mentioned hitherto have all been concerned with isotropic bodies. Cauchy in 1828† extended the equations to the case of crystalline substances. This, however, he accomplished only by reverting to Navier's plan of conceiving an elastic body as a cluster of particles which attract each other with forces depending on their distances apart; the aelotropy he accounted for by supposing the particles to be packed more closely in some directions than in others.

The general equations thus obtained for the vibrations of an elastic solid contain twenty-one constants; six of these depend on the initial stress, so that if the body is initially without stress, only fifteen constants are involved. If, retaining the initial stress, the medium is supposed to be symmetrical with respect to three mutually orthogonal planes, the twenty-one constants reduce to nine, and the equations which determine the vibrations may be written in the form‡

$$\frac{\partial^2 e_x}{\partial t^2} = (a+G)\frac{\partial^2 e_x}{\partial x^2} + (h+H)\frac{\partial^2 e_x}{\partial y^2} + (g+I)\frac{\partial^2 e_x}{\partial z^2} + 2\frac{\partial}{\partial x}\left(a\frac{\partial e_x}{\partial x} + h\frac{\partial e_y}{\partial y} + g\frac{\partial e_z}{\partial z}\right),$$

and two similar equations. The three constants G, H, I represent the stresses across planes parallel to the coordinate planes in the undisturbed state of the aether.§

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* It may easily be shown that any disturbance, in either isotropic or crystalline media, for which the direction of vibration of the molecules lies in the wave-front or surface of constant phase, must satisfy the equation

$$\operatorname{div} \mathbf{e} = 0,$$

where $\mathbf{e}$ denotes the displacement; if, on the other hand, the direction of vibration of the molecules is perpendicular to the wave-front, the disturbance must satisfy the equation

$$\operatorname{curl} \mathbf{e} = 0.$$

These results were proved by M. O'Brien, Trans. Camb. Phil. Soc., 1842.

† *Exercices de Math.*, iii (1828), p. 188.

‡ These are substantially equations (68) on page 208 of the third volume of the *Exercices*.

§ G, H, I are tensions when they are positive, and pressures when they are negative.

On the basis of these equations, Cauchy worked out a theory of light, of which an instalment relating to crystal-optics was presented to the Academy in 1830.* Its characteristic features will now be sketched.

By substitution in the equations last given, it is found that when the wave-front of the vibration is parallel to the plane of yz, the velocity of propagation must be $(h+G)^{\frac{1}{2}}$ if the vibration takes place parallel to the axis of y, and $(g+G)^{\frac{1}{2}}$ if it takes place parallel to the axis of z. Similarly when the wave-front is parallel to the plane of zx, the velocity must be $(h+H)^{\frac{1}{2}}$ if the vibration is parallel to the axis of x, and $(f+H)^{\frac{1}{2}}$ if it is parallel to the axis of z; and when the wave-front is parallel to the plane of xy, the velocity must be $(g+I)^{\frac{1}{2}}$ if the vibration is parallel to the axis of x, and $(f+I)^{\frac{1}{2}}$ if it is parallel to the axis of y.

Now it is known from experiment that the velocity of a ray polarized parallel to one of the planes in question is the same, whether its direction of propagation is along one or the other of the axes in that plane: so, if we assume that the vibrations which constitute light are executed parallel to the plane of polarization, we must have

$$f+H=f+I, \quad g+I=g+G, \quad h+H=h+G;$$

or,

$$G=H=I.$$

This is the assumption made in the memoir of 1830: the theory based on it is generally known as *Cauchy's First Theory*;† the equilibrium pressures G, H, I, being all equal, are taken to be zero.

If, on the other hand, we make the alternative assumption that the vibrations of the aether are executed at right angles to the plane of polarization, we must have

$$h+H=g+I, \quad f+I=h+G, \quad g+G=f+H;$$

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* Mém. de l'Acad., x, p. 293.

In the previous year (Mém. de l'Acad., ix, p. 114) Cauchy had stated that the equations of elasticity lead in the case of uniaxal crystals to a wave-surface of which two sheets are a sphere and spheroid as in Huygens' theory.

† The equations and results of Cauchy's First Theory of crystal-optics were independently obtained shortly afterwards by Franz Ernst Neumann (*b.* 1798, *d.* 1895): cf. Ann. d. Phys. xxv (1832), p. 418, reprinted as No. 76 of Ostwald's *Klassiker der exakten Wissenschaften*, with notes by A. Wangerin.

the theory based on this supposition is known as *Cauchy's Second Theory*: it was published in 1836.*

In both theories, Cauchy imposes the condition that the section of two of the sheets of the wave-surface made by any one of the coordinate planes is to be formed of a circle and an ellipse, as in Fresnel's theory; this yields the three conditions

$$3bc = f(b + c + f); \quad 3ca = g(c + a + g); \quad 3ab = h(a + b + h).$$

Thus in the first theory we have these together with the equations

$$G = 0, \quad H = 0, \quad I = 0,$$

which express the condition that the undisturbed state of the aether is unstressed; and the aethereal vibrations are executed parallel to the plane of polarization. In the second theory we have the three first equations, together with

$$f - G = h - I = g - H;$$

and the plane of polarization is interpreted to be the plane at right angles to the direction of vibration of the aether.

Either of Cauchy's theories accounts tolerably well for the phenomena of crystal-optics; but the wave-surface (or rather the two sheets of it which correspond to nearly transverse waves) is not exactly Fresnel's. In both theories the existence of a third wave, formed of nearly longitudinal vibrations, is a formidable difficulty. Cauchy himself anticipated that the existence of these vibrations would ultimately be demonstrated by experiment, and in one place† conjectured that they might be of a calorific nature. A further objection to Cauchy's theories is that the relations between the constants do not appear to admit of any simple physical interpretation, being evidently assumed for the sole purpose of forcing the formulae into some degree of conformity with the results of experiment. And further difficulties will appear when we proceed subsequently to compare the properties which are assigned to the aether in crystal-optics with those which must be postulated in order to account for reflexion and refraction.

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* Comptes Rendus, ii (1836), p. 341: Mém. de l'Acad. xviii (1839), p. 153.

† Mém. de l'Acad. xviii, p. 161.

To the latter problem Cauchy soon addressed himself, his investigations being in fact published* in the same year (1830) as the first of his theories of crystal-optics.

At the outset of any work on refraction, it is necessary to assign a cause for the existence of refractive indices, i.e. for the variation in the velocity of light from one body to another. Huygens, as we have seen, suggested that transparent bodies consist of hard particles which interact with the aethereal matter, modifying its elasticity. Cauchy in his earlier papers† followed this lead more or less closely, assuming that the density ρ of the aether is the same in all media, but that its rigidity n varies from one medium to another.

Let the axis of x be taken at right angles to the surface of separation of the media, and the axis of z parallel to the intersection of this interface with the incident wave-front; and suppose, first, that the incident vibration is executed at right angles to the plane of incidence, so that it may be represented by

$$e_z = f\left(-x \cos i - y \sin i + \sqrt{\frac{n}{\rho}}\, t\right),$$

where i denotes the angle of incidence; the reflected wave may be represented by

$$e_z = F\left(x \cos i - y \sin i + \sqrt{\frac{n}{\rho}}\, t\right),$$

and the refracted wave by

$$e_z = f_1\left(-x \cos r - y \sin r + \sqrt{\frac{n'}{\rho}}\, t\right),$$

where r denotes the angle of refraction, and n' the rigidity of the second medium.

To obtain the conditions satisfied at the reflecting surface, Cauchy assumed (without assigning reasons) that the x- and y-components of the stress across the xy-plane are equal in the media on either side the interface. This implies in the present case that the quantities

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* Bull. des Sciences Math. xiv. (1830), p. 6.

† As will appear, his views on this subject subsequently changed.

$$n\frac{\partial e_z}{\partial x} \quad \text{and} \quad n\frac{\partial e_z}{\partial y}$$

are to be continuous across the interface: so we have

$$n\cos i'.(f'-F') = n'\cos r.f'_1; \qquad n\sin i.(f'+F') = n'\sin r.f'_1.$$

Eliminating f'_1, we have

$$\frac{F'}{f'} = \frac{\sin(r-i)}{\sin(r+i)}.$$

Now this is Fresnel's sine-law for the ratio of the intensity of the reflected ray to that of the incident ray; and it is known that the light to which it applies is that which is polarized parallel to the plane of incidence. Thus Cauchy was driven to the conclusion that, in order to satisfy the known facts of reflexion and refraction, the vibrations of the aether must be supposed executed at right angles to the plane of polarization of the light.

The case of a vibration performed in the plane of incidence he discussed in the same way. It was found that Fresnel's tangent-law could be obtained by assuming that e_x and the normal pressure across the interface have equal values in the two contiguous media.

The theory thus advanced was encumbered with many difficulties. In the first place, the identification of the plane of polarization with the plane at right angles to the direction of vibration was contrary to the only theory of crystal-optics which Cauchy had as yet published. In the second place, no reasons were given for the choice of the conditions at the interface. Cauchy's motive in selecting these particular conditions was evidently to secure the fulfilment of Fresnel's sine-law and tangent-law; but the results are inconsistent with the true boundary-conditions, which were given later by Green.

It is probable that the results of the theory of reflexion had much to do with the decision, which Cauchy now made,* to reject the first theory of crystal-optics in favour of the second. After 1836 he consistently adhered to the view that the vibrations of the aether are performed at right angles to the plane of polarization. In that year he made another attempt to frame a satisfactory theory of reflexion,* based on the assumption just mentioned, and on the following boundary-conditions:—At the interface between two media curl **e** is to be continuous, and (taking the axis of x normal to the interface) $\partial e_x/\partial x$ is also to be continuous.

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* Comptes Rendus, ii. (1836), p. 341.

Again we find no very satisfactory reasons assigned for the choice of the boundary-conditions; and as the continuity of **e** itself across the interface is not included amongst the conditions chosen, they are obviously open to criticism; but they lead to Fresnel's sine- and tangent-equations, which correctly express the actual behaviour of light.† Cauchy remarks that in order to justify them it is necessary to abandon the assumption of his earlier theory, that the density of the aether is the same in all material bodies.

It may be remarked that neither in this nor in Cauchy's earlier theory of reflexion is any trouble caused by the appearance of longitudinal waves when a transverse wave is reflected, for the simple reason that he assumes the boundary-conditions to be only four in number; and these can all be satisfied without the necessity for introducing any but transverse vibrations.

These features bring out the weakness of Cauchy's method of attacking the problem. His object was to derive the properties of light from a theory of the vibrations of elastic solids. At the outset he had already in his possession the differential equations of motion of the solid, which were to be his starting-point, and the equations of Fresnel, which were to be his goal. It only remained to supply the boundary-conditions at an interface, which are required in the discussion of reflexion, and the relations between the elastic constants of the solid, which are required in the optics of crystals. Cauchy seems to have considered the question from the purely analytical point of view. Given certain differential equations, what supplementary conditions must be adjoined to them in order to produce a given analytical result? The problem when stated in this form admits of more than one solution; and hence it is not surprising that within the space of ten years the great French mathematician produced two distinct theories of crystal-optics and three distinct theories of reflexion,* almost all yielding correct or nearly correct final formulae, and yet mostly irreconcilable with each other, and involving incorrect boundary-conditions and improbable relations between elastic constants.

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* Comptes Rendus, ii. (1836), p. 341: "Mémoire sur la dispersion de la lumière" (*Nouveaux exercices de Math.*, 1836), p. 203.

† These boundary-conditions of Cauchy's are, as a matter of fact, satisfied by the electric force in the electro-magnetic theory of light. The continuity of curl **e** is equivalent to the continuity of the magnetic vector across the interface, and the continuity of $\partial e_x/\partial x$ leads to the same equation as the continuity of the component of electric force in the direction of the intersection of the interface with the plane of incidence.

Cauchy's theories, then, resemble Fresnel's in postulating types of elastic solid which do not exist, and for whose assumed properties no dynamical justification is offered. The same objection applies, though in a less degree, to the original form of a theory of reflexion and refraction which was discovered about this time† almost simultaneously by James MacCullagh (*b.* 1809, *d.* 1847), of Trinity College, Dublin, and Franz Neumann (*b.* 1798, *d.* 1895), of Königsberg. To these authors is due the merit of having extended the laws of reflexion to crystalline media; but the principles of the theory were originally derived in connexion with the simpler case of isotropic media, to which our attention will for the present be confined.

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* One yet remains to be mentioned.

† The outlines of the theory were published by MacCullagh in Brit. Assoc. Rep. 1835; and his results were given in Phil. Mag. x (Jan., 1837), and in Proc. Royal Irish Acad. xviii. (Jan., 1837). Neumann's memoir was presented to the Berlin Academy towards the end of 1835, and published in 1837 in Abh. Berl. Ak. aus dem Jahre 1835, Math. Klasse, p. 1. So far as publication is concerned, the priority would seem to belong to MacCullagh; but there are reasons for believing that the priority of discovery really rests with Neumann, who had arrived at his equations a year before they were communicated to the Berlin Academy.

MacCullagh and Neumann felt that the great objection to Fresnel's theory of reflexion was its failure to provide for the continuity of the normal component of displacement at the interface between two media; it is obvious that a discontinuity in this component could not exist in any true elastic-solid theory, since it would imply that the two media do not remain in contact. Accordingly, they made it a fundamental condition that all three components of the displacement must be continuous at the interface, and found that the sine-law and tangent-law can be reconciled with this condition only by supposing that the aether-vibrations are parallel to the plane of polarization: which supposition they accordingly adopted. In place of the remaining three true boundary-conditions, however, they used only a single equation, derived by assuming that transverse incident waves give rise only to transverse reflected and refracted waves, and that the conservation of energy holds for these—i.e. that the masses of aether put in motion, multiplied by the squares of the amplitudes of vibration, are the same before and after incidence. This is, of course, the same device as had been used previously by Fresnel; it must, however, be remarked that the principle is unsound as applied to an ordinary elastic solid; for in such a body the refracted and reflected energy would in part be carried away by longitudinal waves.

In order to obtain the sine and tangent laws, MacCullagh and Neumann found it necessary to assume that the inertia of the luminiferous medium is everywhere the same, and that the differences in behaviour of this medium in different substances are due to differences in its elasticity. The two laws may then be deduced in much the same way as in the previous investigations of Fresnel and Cauchy.

Although to insist on continuity of displacement at the interface was a decided advance, the theory of MacCullagh and Neumann scarcely showed as yet much superiority over the quasi-mechanical theories of their predecessors. Indeed, MacCullagh himself expressly disavowed any claim to regard his theory, in the form to which it had then been brought, as a final explanation of the properties of light. "If we are asked," he wrote, "what reasons can be assigned for the hypotheses on which the preceding theory is founded, we are far from being able to give a satisfactory answer. We are obliged to confess that, with the exception of the law of *vis viva*, the hypotheses are nothing more than fortunate conjectures. These conjectures are very probably right, since they have led to elegant laws which are fully borne out by experiments; but this is all we can assert respecting them. We cannot attempt to deduce them from first principles; because, in the theory of light, such principles are still to be sought for. It is certain, indeed, that light is produced by undulations, propagated, with transversal vibrations, through a highly elastic aether; but the constitution of this aether, and the laws of its connexion (if it has any connexion) with the particles of bodies, are utterly unknown."

The needful reformation of the elastic-solid theory of reflexion was effected by Green, in a paper* read to the Cambridge Philosophical Society in December, 1837. Green, though inferior to Cauchy as an analyst, was his superior in physical insight; instead of designing boundary-equations for the express purpose of yielding Fresnel's sine and tangent formulae, he set to work to determine the conditions which are actually satisfied at the interfaces of real elastic solids.

These he obtained by means of general dynamical principles. In an isotropic medium which is strained, the potential energy per unit volume due to the state of stress is

$$\phi = \frac{1}{2}\left(k + \frac{4}{3}n\right)\left(\frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} + \frac{\partial e_z}{\partial z}\right)^2 + \frac{1}{2}n\left\{\left(\frac{\partial e_z}{\partial y} + \frac{\partial e_y}{\partial z}\right)^2 + \left(\frac{\partial e_x}{\partial z} + \frac{\partial e_z}{\partial x}\right)^2 + \left(\frac{\partial e_y}{\partial x} + \frac{\partial e_x}{\partial y}\right)^2 - 4\frac{\partial e_y}{\partial y}\frac{\partial e_z}{\partial z} - 4\frac{\partial e_z}{\partial z}\frac{\partial e_x}{\partial x} - 4\frac{\partial e_x}{\partial x}\frac{\partial e_y}{\partial y}\right\},$$

where $\mathbf{e}$ denotes the displacement, and k and n denote the two elastic constants already introduced; by substituting this value of ϕ in the general variational equation

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* Trans. Camb. Phil. Soc., 1838; Green's *Math. Papers*, p. 245.

$$\iiint \rho \left\{ \frac{\partial^2 e_x}{\partial t^2} \delta e_x + \frac{\partial^2 e_y}{\partial t^2} \delta e_y + \frac{\partial^2 e_z}{\partial t^2} \delta e_z \right\} dx\,dy\,dz = -\iiint \delta\phi \, dx\,dy\,dz$$

(where ρ denotes the density), the equation of motion may be deduced.

But this method does more than merely furnish the equation of motion

$$\rho\ddot{\mathbf{e}} = -\left(k + \frac{4}{3} n\right) \text{grad div } \mathbf{e} - n \text{ curl curl } \mathbf{e};$$

or,

$$\rho\ddot{\mathbf{e}} = -\left(k + \frac{1}{3} n\right) \text{grad div } \mathbf{e} + n\nabla^2 \mathbf{e},$$

which had already been obtained by Cauchy; for it also yields the boundary-conditions which must be satisfied at the interface between two elastic media in contact; these are, as might be guessed by physical intuition, that the three components of the displacement* and the three components of stress across the interface are to be equal in the two media. If the axis of x be taken normal to the interface, the latter three quantities are

$$\left(k - \frac{2}{3} n\right) \text{div } \mathbf{e} + 2n \frac{\partial e_x}{\partial x}, \quad n\left(\frac{\partial e_z}{\partial x} + \frac{\partial e_x}{\partial z}\right), \quad \text{and} \quad n\left(\frac{\partial e_x}{\partial y} + \frac{\partial e_y}{\partial x}\right).$$

The correct boundary-conditions being thus obtained, it was a simple matter to discuss the reflexion and refraction of an incident wave by the procedure of Fresnel and Cauchy. The result found by Green was that if the vibration of the aethereal molecules is executed at right angles to the plane of incidence, the intensity of the reflected light obeys Fresnel's sine-law, provided the rigidity n is assumed to be the same for all media, but the inertia ρ to vary from one medium to another. Since the sine-law is known to be true for light polarized in the plane of incidence, Green's conclusion confirmed the hypotheses of

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* These first three conditions are of course not dynamical but geometrical.

Fresnel, that the vibrations are executed at right angles to the plane of polarization, and that the optical differences between media are due to the different densities of aether within them.

It now remained for Green to discuss the case in which the incident light is polarized at right angles to the plane of incidence, so that the motion of the aethereal particles is parallel to the intersection of the plane of incidence with the front of the wave. In this case it is impossible to satisfy all the six boundary-conditions without assuming that longitudinal vibrations are generated by the act of reflexion. Taking the plane of incidence to be the plane of yz, and the interface to be the plane of xy, the incident wave may be represented by the equations

$$e_y = A\frac{\partial}{\partial z}f(t + lz + my)\,; \quad e_z = -A\frac{\partial}{\partial y}f(t + lz + my)\,;$$

where, if i denote the angle of incidence, we have

$$l = \sqrt{\frac{\rho_1}{n}}\cos i, \quad m = -\sqrt{\frac{\rho_1}{n}}\sin i.$$

There will be a transverse reflected wave,

$$e_y = B\frac{\partial}{\partial z}f(t - lz + my)\,; \quad e_z = -B\frac{\partial}{\partial y}f(t - lz + my)\,;$$

and a transverse refracted wave,

$$e_y = C\frac{\partial}{\partial z}f(t + l_1z + my)\,; \quad e_z = -C\frac{\partial}{\partial y}f(t + l_1z + my),$$

where, since the velocity of transverse waves in the second medium is $\sqrt{n/\rho_2}$, we can determine l_1 from the equation

$$l_1^2 + m^2 = \frac{\rho_2}{n};$$

there will also be a longitudinal reflected wave,

$$e_y = D\frac{\partial}{\partial y}f(t - \lambda z + my)\,; \quad e_z = D\frac{\partial}{\partial z}f(t - \lambda z + my),$$

where λ is determined by the equation

$$\lambda^2 + m^2 = \frac{\rho_1}{k_1 + \frac{4}{3}n};$$

and a longitudinal refracted wave,

$$e_y = E\frac{\partial}{\partial y}f(t + \lambda_1 z + my); \quad e_z = E\frac{\partial}{\partial z}f(t + \lambda_1 z + my),$$

where λ_1 is determined by

$$\lambda_1^2 + m^2 = \frac{\rho_2}{k_2 + \frac{4}{3}n}.$$

Substituting these values for the displacement in the boundary-conditions which have been already formulated, we obtain the equations which determine the intensities of the reflected and refracted waves; in particular, it appears that the amplitude of the reflected transverse wave is given by the equation

$$\frac{A - B}{A + B} = \frac{l_1\rho_1}{l\rho_2} + \frac{m^2}{l}\,\frac{(\rho_1 - \rho_2)^2}{\rho_2(\lambda\rho_2 + \lambda_1\rho_1)}.$$

Now if the elastic constants of the media are such that the velocities of propagation of the longitudinal waves are of the same order of magnitude as those of the transverse waves, the direction-cosines of the longitudinal reflected and refracted rays will in general have real values, and these rays will carry away some of the energy which is brought to the interface by the incident wave. Green avoided this difficulty by adopting Fresnel's suggestion that the resistance of the aether to compression may be very large in comparison with the resistance to distortion, as is actually the case with such substances as jelly and caoutchouc: in this case the longitudinal waves are degraded in much the same way as the transverse refracted ray is degraded when there is total reflexion, and so do not carry away energy. Making this supposition, so that k_1 and k_2 are very large, the quantities λ and λ_1 have the values $m\sqrt{-1}$, and we have

$$\frac{A - B}{A + B} = \frac{l_1}{l}\frac{\rho_1}{\rho_2} - \frac{m}{l}\,\frac{(\rho_1 - \rho_2)^2}{\rho_2(\rho_1 + \rho_2)}\sqrt{-1}.$$

Thus if $\overline{B}/\overline{A}$ denote the modulus of B/A, we have

$$\left(\frac{\overline{B}}{\overline{A}}\right)^2 = \frac{\left(\frac{\rho_2}{\rho_1}+1\right)^2\left(\frac{\rho_2}{\rho_1}-\frac{l_1}{l}\right)^2+\left(\frac{\rho_2}{\rho_1}-1\right)^4\frac{m^2}{l^2}}{\left(\frac{\rho_2}{\rho_1}+1\right)^2\left(\frac{\rho_2}{\rho_1}+\frac{l_1}{l}\right)^2+\left(\frac{\rho_2}{\rho_1}-1\right)^4\frac{m^2}{l^2}}.$$

This expression represents the ratio of the intensity of the transverse reflected wave to that of the incident wave. It does not agree with Fresnel's tangent-formula: and both on this account and also because (as we shall see) this theory of reflexion does not harmonize well with the elastic-solid theory of crystal-optics, it must be concluded that the vibrations of a Greenian solid do not furnish an exact parallel to the vibrations which constitute light.

The success of Green's investigation from the standpoint of dynamics, set off by its failure in the details last mentioned, stimulated MacCullagh to fresh exertions. At length he succeeded in placing his own theory, which had all along been free from reproach so far as agreement with optical experiments was concerned, on a sound dynamical basis; thereby effecting that reconciliation of the theories of Light and Dynamics which had been the dream of every physicist since the days of Descartes.

The central feature of MacCullagh's investigation,* which was presented to the Royal Irish Academy in 1839, is the introduction of a new type of elastic solid. He had, in fact, concluded from Green's results that it was impossible to explain optical phenomena satisfactorily by comparing the aether to an elastic solid of the ordinary type, which resists compression and distortion; and he saw that the only hope of the situation was to devise a medium which should be as strictly conformable to dynamical laws as Green's elastic solid, and yet should have its properties specially designed to fulfil the requirements of the theory of light. Such a medium he now described.

If as before we denote by **e** the vector displacement of a point of the medium from its equilibrium position, it is well known that the vector curl **e** denotes twice the rotation of the part of the solid in the neighbourhood of the point (x, y, z) from its equilibrium orientation. In an ordinary elastic solid, the potential energy of strain depends only on the change of size and shape of the volume-elements; on their compression and distortion, in fact. For MacCullagh's new medium, on the other hand, the potential energy depends only on the *rotation* of the volume-elements.

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* Trans. Roy. Irish Acad. xxi.: MacCullagh's *Coll. Works*, p. 145.

Since the medium is not supposed to be in a state of stress in its undisturbed condition, the potential energy per unit volume must be a quadratic function of the derivates of **e**; so that in an isotropic medium this quantity ϕ must be formed from the only invariant which depends solely on the rotation and is quadratic in the derivates, that is from $(\text{curl}\ \mathbf{e})^2$; thus we may write

$$\phi = \tfrac{1}{2}\mu \left\{ \left(\frac{\partial e_z}{\partial y} - \frac{\partial e_y}{\partial z} \right)^2 + \left(\frac{\partial e_x}{\partial z} - \frac{\partial e_z}{\partial x} \right)^2 + \left(\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} \right)^2 \right\}.$$

The equation of motion is now to be determined, as in the case of Green's aether, from the variational equation

$$\iiint \rho \left\{ \frac{\partial^2 e_x}{\partial t^2}\, \delta e_x + \frac{\partial^2 e_y}{\partial t^2}\, \delta e_y + \frac{\partial^2 e_z}{\partial t^2}\, \delta e_z \right\} dx\, dy\, dz = - \iiint \delta\phi\, dx\, dy\, dz;$$

the result is

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = -\mu\ \text{curl curl}\ \mathbf{e}.$$

It is evident from this equation that if div **e** is initially zero it will always be zero: we shall suppose this to be the case, so that no longitudinal waves exist at any time in the medium. One of the greatest difficulties which beset elastic-solid theories is thus completely removed.

The equation of motion may now be written

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = \mu \nabla^2 \mathbf{e},$$

which shows that transverse waves are propagated with velocity $\sqrt{\mu/\rho}$.

From the variational equation we may also determine the boundary-conditions which must be satisfied at the interface between two media; these are, that the three components of **e** are to be continuous across the interface, and that the two components of μ curl **e** parallel to the interface are also to be continuous across it. One of these five conditions, namely, the continuity of the normal component of **e**, is really dependent on the other four; for if we take the axis of x normal to the interface, the equation of motion gives

$$\rho \frac{\partial^2 e_x}{\partial t^2} = -\frac{\partial}{\partial y}(\mu \text{ curl } \mathbf{e})_z + \frac{\partial}{\partial z}(\mu \text{ curl } \mathbf{e})_y,$$

and as the quantities ρ, $(\mu \text{ curl } \mathbf{e})_z$, and $(\mu \text{ curl } \mathbf{e})_y$ are continuous across the interface, the continuity of $\partial^2 e_x/\partial t^2$ follows. Thus the only independent boundary-conditions in MacCullagh's theory are the continuity of the tangential components of **e** and of μ curl **e**.* It is easily seen that these are equivalent to the boundary-conditions used in MacCullagh's earlier paper, namely, the equation of *vis viva* and the continuity of the three components of **e**: and thus the "rotationally elastic" aether of this memoir furnishes a dynamical foundation for the memoir of 1837.

The extension to crystalline media is made by assuming the potential energy per unit volume to have, when referred to the principal axes, the form

$$A\left(\frac{\partial e_z}{\partial y} - \frac{\partial e_y}{\partial z}\right)^2 + B\left(\frac{\partial e_x}{\partial z} - \frac{\partial e_z}{\partial x}\right)^2 + C\left(\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y}\right)^2,$$

where A, B, C denote three constants which determine the optical behaviour of the medium: it is readily seen that the wave-surface is Fresnel's, and that the plane of polarization contains the displacement, and is at right angles to the rotation.

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* MacCullagh's equations may readily be interpreted in the electro-magnetic theory of light: **e** corresponds to the magnetic force, μ curl **e** to the electric force, and curl **e** to the electric displacement.

MacCullagh's work was regarded with doubt by his own and the succeeding generation of mathematical physicists, and can scarcely be said to have been properly appreciated until FitzGerald drew attention to it forty years afterwards. But there can be no doubt that MacCullagh really solved the problem of devising a medium whose vibrations, calculated in accordance with the correct laws of dynamics, should have the same properties as the vibrations of light.

The hesitation which was felt in accepting the rotationally elastic aether arose mainly from the want of any readily conceived example of a body endowed with such a property. This difficulty was removed in 1889 by Sir William Thomson (Lord Kelvin), who designed mechanical models possessed of rotational elasticity. Suppose, for example,* that a structure is formed of spheres, each sphere being in the centre of the tetrahedron formed by its four nearest neighbours. Let each sphere be joined to these four neighbours by rigid bars, which have spherical caps at their ends so as to slide freely on the spheres. Such a structure would, for small deformations, behave like an incompressible perfect fluid. Now attach to each bar a pair of gyroscopically-mounted flywheels, rotating with equal and opposite angular velocities, and having their axes in the line of the bar: a bar thus equipped will require a couple to hold it at rest in any position inclined to its original position, and the structure as a whole will possess that kind of quasi-elasticity which was first imagined by MacCullagh.

This particular representation is not perfect, since a system of forces would be required to hold the model in equilibrium if it were irrotationally distorted. Lord Kelvin subsequently invented another structure free from this defect.†

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* Comptes Rendus, Sept. 16, 1889: Kelvin's *Math. and Phys. Papers*, iii, p. 466.

† Proc. Roy. Soc. Edinb., Mar. 17, 1890: Kelvin's *Math. and Phys. Papers*, iii, p. 468.

The work of Green proved a stimulus not only to MacCullagh but to Cauchy, who now (1839) published yet a third theory of reflexion.* This appears to have owed its origin to a remark of Green's,† that the longitudinal wave might be avoided in either of two ways—namely, by supposing its velocity to be indefinitely great or indefinitely small. Green curtly dismissed the latter alternative and adopted the former, on the ground that the equilibrium of the medium would be unstable if its compressibility were negative (as it must be if the velocity of longitudinal waves is to vanish). Cauchy, without attempting to meet Green's objection, took up the study of a medium whose elastic constants are connected by the equation

$$k + \tfrac{4}{3}n = 0,$$

so that the longitudinal vibrations have zero velocity; and showed that if the aethereal vibrations are supposed to be executed at right angles to the plane of polarization, and if the rigidity of the aether is assumed to be the same in all media, a ray which is reflected will obey the sine-law and tangent-law of Fresnel. The boundary-conditions which he adopted in order to obtain this result were the continuity of the displacement $\mathbf{e}$ and of its derivate $\partial\mathbf{e}/\partial x$, where the axis of x is taken at right angles to the interface.‡ These are not the true boundary-conditions for general elastic solids; but in the particular case now under discussion, where the rigidity is the same in the two media, they yield the same equations as the conditions correctly given by Green.

The aether of Cauchy's third theory of reflexion is well worthy of some further study. It is generally known as the *contractile* or *labile*§ aether, the names being due to William

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* Comptes Rendus, ix, p. 676 (25 Nov., 1839), and p. 726 (2 Dec., 1839).

† Green's *Math. Papers*, p. 246.

‡ Comptes Rendus, x, p. 347 (March 2, 1840): xxvii, p. 621 (1848); xxviii, p. 25 (1849). Mém. de l'Acad., xxii (1848), pp. 17, 29.

§ *Labile* or *neutral* is a term used of such equilibrium as that of a rigid body on a perfectly smooth horizontal plane.

Thomson (Lord Kelvin), who discussed it long afterwards.* It may be defined as an elastic medium of (negative) compressibility such as to make the velocity of the longitudinal wave zero: this implies that no work is required to be done in order to give the medium any small irrotational disturbance. An example is furnished by homogeneous foam free from air and held from collapse by adhesion to a containing vessel.

Cauchy, as we have seen, did not attempt to refute Green's objection that such a medium would be unstable; but, as Thomson remarked, every possible infinitesimal motion of the medium is, in the elementary dynamics of the subject, proved to be resolvable into coexistent wave-motions. If, then, the velocity of propagation for each of the two kinds of wave-motion is real, the equilibrium must be stable, provided the medium either extends through boundless space or has a fixed containing vessel as its boundary.

When the rigidity of the luminiferous medium is supposed to have the same value in all bodies, the conditions to be satisfied at an interface reduce to the continuity of the displacement $\mathbf{e}$, of the tangential components of curl $\mathbf{e}$, and of the scalar quantity $(k + \frac{4}{3}n)$ div $\mathbf{e}$ across the interface.

Now we have seen that when a transverse wave is incident on an interface, it gives rise in general to reflected and refracted waves of both the transverse and the longitudinal species. In the case of the contractile aether, for which the velocity of propagation of the longitudinal waves is very small, the ordinary construction for refracted waves shows that the directions of propagation of the reflected and refracted longitudinal waves will be almost normal to the interface. The longitudinal waves will therefore contribute only to the component of displacement normal to the interface, not to the tangential components: in other words, the only tangential components of displacement at the interface are those due to the three transverse waves—the incident, reflected, and refracted. Moreover, the longitudinal waves do not contribute at all to curl $\mathbf{e}$; and, therefore, in the contractile aether, the conditions that the tangential components of **e** and of n curl **e** shall be continuous across an interface are satisfied by the distortional part of the disturbance taken alone. The condition that the component of **e** normal to the interface is to be continuous is not satisfied by the distortional part of the disturbance taken alone, but is satisfied when the distortional and compressional parts are taken together.

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* Phil. Mag. xxvi (1888), p. 414.

The energy carried away by the longitudinal waves is infinitesimal, as might be expected, since no work is required in order to generate an irrotational displacement. Hence, with this aether, the behaviour of the transverse waves at an interface may be specified without considering the irrotational part of the disturbance at all, by the conditions that the conservation of energy is to hold and that the tangential components of **e** and of n curl **e** are to be continuous. But if we identify these transverse waves with light, assuming that the displacement **e** is at right angles to the plane of polarization of the light, and assuming moreover that the rigidity n is the same in all media* (the differences between media depending on differences in the inertia ρ), we have exactly the assumptions of Fresnel's theory of light: whence it follows that transverse waves in the labile aether must obey in reflexion the sine-law and tangent-law of Fresnel.

The great advantage of the labile aether is that it overcomes the difficulty about securing continuity of the normal component of displacement at an interface between two media: the light-waves taken alone do not satisfy this condition of continuity; but the total disturbance consisting of light-waves and irrotational disturbance taken together does satisfy it; and this is ensured without allowing the irrotational disturbance to carry off any of the energy.†

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* This condition is in any case necessary for stability, as was shown by R. T. Glazebrook: cf. Thomson, Phil. Mag. xxvi, p. 500.

† The labile-aether theory of light may be compared with the electro-magnetic theory, by interpreting the displacement **e** as the electric force, and ρ**e** as the electric displacement.

William Thomson (Lord Kelvin, *b.* 1824, *d.* 1908), who devoted much attention to the labile aether, was at one time led to doubt the validity of this explanation of light* ; for when investigating the radiation of energy from a vibrating rigid globe embedded in an infinite elastic-solid aether, he found that in some cases the irrotational waves would carry away a considerable part of the energy if the aether were of the labile type. This difficulty, however, was removed by the observation† that it is sufficient for the fulfilment of Fresnel's laws if the velocity of the irrotational waves in one of the two media is very small, without regard to the other medium. Following up this idea, Thomson assumed that in space void of ponderable matter the aether is practically incompressible by the forces concerned in light-waves, but that in the space occupied by liquids and solids it has a negative compressibility, so as to give zero velocity for longitudinal aether-waves in these bodies. This assumption was based on the conception that material atoms move through space without displacing the aether: a conception which, as Thomson remarked, contradicts the old scholastic axiom that two different portions of matter cannot simultaneously occupy the same space.‡ He supposed the aether to be attracted and repelled by the atoms, and thereby to be condensed or rarefied.§

The year 1839, which saw the publication of MacCullagh's dynamical theory of light and Cauchy's theory of the labile aether, was memorable also for the appearance of a memoir by Green on crystal-optics.|| This really contains two distinct theories, which respectively resemble Cauchy's First and Second Theories : in one of them, the stresses in the undisturbed state

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* *Baltimore Lectures* (edition 1904), p. 214.

† *Ibid.* (ed. 1904), p. 411.

‡ Michell and Boscovich in the eighteenth century had taught the doctrine of the mutual penetration of matter, i.e. that two substances may be in the same place at the same time without excluding each other : cf. Priestley's *History* i., p. 392.

§ Cf. *Baltimore Lectures* (ed. 1904), pp. 413–14, 463, and Appendices A and E.

|| Cambridge Phil. Trans., 1839 ; Green's *Math. Papers*, p. 293.

of the aether are supposed to vanish, and the vibrations of the aether are supposed to be executed parallel to the plane of polarization of the light; in the other theory, the initial stresses are not supposed to vanish, and the aether-vibrations are at right angles to the plane of polarization. The two investigations are generally known as Green's First and Second Theories of crystal-optics.

The foundations of both theories are, however, the same. Green first of all determined the potential energy of a strained crystalline solid; this in the most general case involves 27 constants, or 21 if there is no initial stress.* If, however, as is here assumed, the medium possesses three planes of symmetry at right angles to each other, the number of constants reduces to 12, or to 9 if there is no initial stress; if **e** denote the displacement, the potential energy per unit volume may be written

$$\begin{aligned}\phi = {} & G\frac{\partial e_x}{\partial x} + H\frac{\partial e_y}{\partial y} + I\frac{\partial e_z}{\partial z} \\ & + \tfrac{1}{2}G\left\{\left(\frac{\partial e_x}{\partial x}\right)^2 + \left(\frac{\partial e_y}{\partial x}\right)^2 + \left(\frac{\partial e_z}{\partial x}\right)^2\right\} + \tfrac{1}{2}H\left\{\left(\frac{\partial e_x}{\partial y}\right)^2 + \left(\frac{\partial e_y}{\partial y}\right)^2 + \left(\frac{\partial e_z}{\partial y}\right)^2\right\} \\ & \qquad + \tfrac{1}{2}I\left\{\left(\frac{\partial e_x}{\partial z}\right)^2 + \left(\frac{\partial e_y}{\partial z}\right)^2 + \left(\frac{\partial e_z}{\partial z}\right)^2\right\} \\ & + \tfrac{3}{2}a\left(\frac{\partial e_x}{\partial x}\right)^2 + \tfrac{3}{2}b\left(\frac{\partial e_y}{\partial y}\right)^2 + \tfrac{3}{2}c\left(\frac{\partial e_z}{\partial z}\right)^2 \\ & + f'\frac{\partial e_y}{\partial y}\frac{\partial e_z}{\partial z} + g'\frac{\partial e_x}{\partial x}\frac{\partial e_z}{\partial z} + h'\frac{\partial e_x}{\partial x}\frac{\partial e_y}{\partial y} \\ & + \tfrac{1}{2}f\left(\frac{\partial e_y}{\partial z} + \frac{\partial e_z}{\partial y}\right)^2 + \tfrac{1}{2}g\left(\frac{\partial e_x}{\partial z} + \frac{\partial e_z}{\partial x}\right)^2 + \tfrac{1}{2}h\left(\frac{\partial e_x}{\partial y} + \frac{\partial e_y}{\partial x}\right)^2.\end{aligned}$$

The usual variational equation

$$\iiint \rho\left\{\frac{\partial^2 e_x}{\partial t^2}\,\delta e_x + \frac{\partial^2 e_y}{\partial t^2}\,\delta e_y + \frac{\partial^2 e_z}{\partial t^2}\,\delta e_z\right\} dx\,dy\,dz = -\iiint \delta\phi\,dx\,dy\,dz,$$

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* For there are 21 terms in a homogeneous function of the second degree in six variables.

then yields the differential equations of motion, namely:

$$\rho \frac{\partial^2 e_x}{\partial t^2} = (a + G) \frac{\partial^2 e_x}{\partial x^2} + (h + H) \frac{\partial^2 e_x}{\partial y^2} + (g + I) \frac{\partial^2 e_x}{\partial z^2}$$

$$+ \frac{\partial}{\partial x}\left(a \frac{\partial e_x}{\partial x} + h \frac{\partial e_y}{\partial y} + g \frac{\partial e_z}{\partial z}\right) + \frac{\partial}{\partial x}\left(a \frac{\partial e_x}{\partial x} + h' \frac{\partial e_y}{\partial y} + g' \frac{\partial e_z}{\partial z}\right),$$

and two similar equations.

These differ from Cauchy's fundamental equations in having greater generality: for Cauchy's medium was supposed to be built up of point-centres of force attracting each other according to some function of the distance; and, as we have seen, there are limitations in this method of construction, which render it incompetent to represent the most general type of elastic solid. Cauchy's equations for crystalline media are, in fact, exactly analogous to the equations originally found by Navier for isotropic media, which contain only one elastic constant instead of two.

The number of constants in the above equations still exceeds the three which are required to specify the properties of a biaxal crystal: and Green now proceeds to consider how the number may be reduced. The condition which he imposes for this purpose is that for two of the three waves whose front is parallel to a given plane, the vibration of the aethereal molecules shall be accurately in the plane of the wave: in other words, that two of the three waves shall be purely distortional, the remaining one being consequently a normal vibration. This condition gives five relations,* which may be written:—

$$a = b = c = \tfrac{1}{3}\mu;$$

$$f' = \mu - 2f \qquad g' = \mu - 2g; \qquad h' = \mu - 2h;$$

where μ denotes a new constant.†

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* As Green showed, the hypothesis of transversality really involves the existence of planes of symmetry, so that it alone is capable of giving 14 relations between the 21 constants: and 3 of the remaining 7 constants may be removed by change of axes, leaving only four.

† It was afterwards shown by Barré de Saint-Venant (*b.* 1797, *d.* 1886), *Journal de Math.*, vii (1863), p. 399, that if the initial stresses be supposed to vanish, the conditions which must be satisfied among the remaining nine constants

Thus the potential energy per unit volume may be written

$$\phi = G\frac{\partial e_x}{\partial x} + H\frac{\partial e_y}{\partial y} + I\frac{\partial e_z}{\partial z}$$

$$+ \tfrac{1}{2}G\left\{\left(\frac{\partial e_x}{\partial x}\right)^2 + \left(\frac{\partial e_y}{\partial x}\right)^2 + \left(\frac{\partial e_z}{\partial x}\right)^2\right\} + \tfrac{1}{2}H\left\{\left(\frac{\partial e_x}{\partial y}\right)^2 + \left(\frac{\partial e_y}{\partial y}\right)^2 + \left(\frac{\partial e_z}{\partial y}\right)^2\right\}$$

$$+ \tfrac{1}{2}I\left\{\left(\frac{\partial e_x}{\partial z}\right)^2 + \left(\frac{\partial e_y}{\partial z}\right)^2 + \left(\frac{\partial e_z}{\partial z}\right)^2\right\}$$

$$+ \tfrac{1}{2}\mu\left(\frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} + \frac{\partial e_z}{\partial z}\right)^2$$

$$+ \tfrac{1}{2}f\left\{\left(\frac{\partial e_y}{\partial z} + \frac{\partial e_z}{\partial y}\right)^2 - 4\frac{\partial e_y}{\partial y}\frac{\partial e_z}{\partial z}\right\} + \tfrac{1}{2}g\left\{\left(\frac{\partial e_z}{\partial x} + \frac{\partial e_x}{\partial z}\right)^2 - 4\frac{\partial e_z}{\partial z}\frac{\partial e_x}{\partial x}\right\}$$

$$+ \tfrac{1}{2}h\left\{\left(\frac{\partial e_x}{\partial y} + \frac{\partial e_y}{\partial x}\right)^2 - 4\frac{\partial e_x}{\partial x}\frac{\partial e_y}{\partial y}\right\}.$$

At this point Green's two theories of crystal-optics diverge from each other. According to the first theory, the initial stresses G, H, I are zero, so that

$$\phi = \tfrac{1}{2}\mu\left(\frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} + \frac{\partial e_z}{\partial z}\right)^2$$

$$+ \tfrac{1}{2}f\left\{\left(\frac{\partial e_y}{\partial z} + \frac{\partial e_z}{\partial y}\right)^2 - 4\frac{\partial e_y}{\partial y}\frac{\partial e_z}{\partial z}\right\} + \tfrac{1}{2}g\left\{\left(\frac{\partial e_z}{\partial x} + \frac{\partial e_x}{\partial z}\right)^2 - 4\frac{\partial e_z}{\partial z}\frac{\partial e_x}{\partial x}\right\}$$

$$+ \tfrac{1}{2}h\left\{\left(\frac{\partial e_x}{\partial y} + \frac{\partial e_y}{\partial x}\right)^2 - 4\frac{\partial e_x}{\partial x}\frac{\partial e_y}{\partial y}\right\}.$$

a, b, c, f, g, h, f' g', h', in order that the wave-surface may be Fresnel's, are the following:—

$$\left\{\begin{aligned} (3b-f)(3c-f) &= (f+f')^2 \\ (3c-g)(3a-g) &= (g+g')^2 \\ (3a-h)(3b-h) &= (h+h')^2 \\ (3a-g)(3b-h)(3c-f) + (3a-h)(3b-f)(3c-g) &= 2(f+f')(g+g')(h+h'). \end{aligned}\right.$$

These reduce to Green's relations when the additional equation $b = c$ is assumed.

Saint-Venant disputed the validity of Green's relations, asserting that they are compatible only with isotropy. On this controversy cf. R. T. Glazebrook, *Brit. Assoc. Report*, 1885, p. 171, and Karl Pearson in Todhunter and Pearson's *History of Elasticity*, ii, § 147.

This expression contains the correct number of constants, namely, four: three of them represent the optical constants of a biaxal crystal, and one (namely, μ) represents the square of the velocity of propagation of longitudinal waves. It is found that the two sheets of the wave-surface which correspond to the two distortional waves form a Fresnel's wave-surface, the third sheet, which corresponds to the longitudinal wave, being an ellipsoid. The directions of polarization and the wave-velocities of the distortional waves are identical with those assigned by Fresnel, provided it is assumed that the direction of vibration of the aether-particles is parallel to the plane of polarization; but this last assumption is of course inconsistent with Green's theory of reflexion and refraction.

In his Second Theory, Green, like Cauchy, used the condition that for the waves whose fronts are parallel to the coordinate planes, the wave-velocity depends only on the plane of polarization, and not on the direction of propagation. He thus obtained the equations already found by Cauchy—

$$G - f = H - g = I - h.$$

The wave-surface in this case also is Fresnel's, provided it is assumed that the vibrations of the aether are executed at right angles to the plane of polarization.

The principle which underlies the Second Theories of Green and Cauchy is that the aether in a crystal resembles an elastic solid which is unequally pressed or pulled in different directions by the unmoved ponderable matter. This idea appealed strongly to W. Thomson (Kelvin), who long afterwards developed it further,* arriving at the following interesting result:—Let an incompressible solid, isotropic when unstrained, be such that its potential energy per unit volume is

$$\tfrac{1}{2}q\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} - 3\right),$$

where q denotes its modulus of rigidity when unstrained, and

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* Proc. R. S. Edin. xv (1887), p. 21: Phil. Mag. xxv (1888) p. 116: *Baltimore Lectures* (ed. 1904), pp. 228–259.

$\alpha^{\frac{1}{2}}$, $\beta^{\frac{1}{2}}$, $\gamma^{\frac{1}{2}}$, denote the proportions in which lines parallel to the axes of strain are altered; then if the solid be initially strained in a way defined by given values of α, β, γ, by forces applied to its surface, and if waves of distortion be superposed on this initial strain, the transmission of these waves will follow exactly the laws of Fresnel's theory of crystal-optics, the wave-surface being

$$\frac{x^2}{\frac{\alpha}{q}r^2 - 1} + \frac{y^2}{\frac{\beta}{q}r^2 - 1} + \frac{z^2}{\frac{\gamma}{q}r^2 - 1} = 0.$$

There is some difficulty in picturing the manner in which the molecules of ponderable matter act upon the aether so as to produce the initial strain required by this theory. Lord Kelvin utilized* the suggestion to which we have already referred, namely, that the aether may pervade the atoms of matter so as to occupy space jointly with them, and that its interaction with them may consist in attractions and repulsions exercised throughout the regions interior to the atoms. These forces may be supposed to be so large in comparison with those called into play in free aether that the resistance to compression may be overcome, and the aether may be (say) condensed in the central region of an isolated atom, and rarefied in its outer parts. A crystal may be supposed to consist of a group of spherical atoms in which neighbouring spheres overlap each other; in the central regions of the spheres the aether will be condensed, and within the lens-shaped regions of overlapping it will be still more rarefied than in the outer parts of a solitary atom, while in the interstices between the atoms its density will be unaffected. In consequence of these rarefactions and condensations, the reaction of the aether on the atoms tends to draw inwards the outermost atoms of the group, which, however, will be maintained in position by repulsions between the atoms themselves; and thus we can account for the pull which, according to the present hypothesis, is exerted on the aether by the ponderable molecules of crystals.

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* *Baltimore Lectures* (ed. 1904), p. 253.

Analysis similar to that of Cauchy's and Green's Second Theory of crystal-optics may be applied to explain the doubly refracting property which is possessed by strained glass; but in this case the formulae derived are found to conflict with the results of experiment. The discordance led Kelvin to doubt the truth of the whole theory. "After earnest and hopeful consideration of the stress theory of double refraction during fourteen years," he said,* "I am unable to see how it can give the true explanation either of the double refraction of natural crystals, or of double refraction induced in isotropic solids by the application of unequal pressures in different directions."

It is impossible to avoid noticing throughout all Kelvin's work evidences of the deep impression which was made upon him by the writings of Green. The same may be said of Kelvin's friend and contemporary Stokes; and, indeed, it is no exaggeration to describe Green as the real founder of that "Cambridge school" of natural philosophers, of which Kelvin, Stokes, Lord Rayleigh, and Clerk Maxwell were the most illustrious members in the latter half of the nineteenth century, and which is now led by Sir Joseph Thomson and Sir Joseph Larmor. In order to understand the peculiar position occupied by Green, it is necessary to recall something of the history of mathematical studies at Cambridge.

The century which elapsed between the death of Newton and the scientific activity of Green was the darkest in the history of the University. It is true that Cavendish and Young were educated at Cambridge; but they, after taking undergraduate courses, removed to London. In the entire period the only natural philosopher of distinction who lived and taught at Cambridge was Michell; and for some reason which at this distance of time it is difficult to understand fully, Michell's researches seem to have attracted little or no attention among his collegiate contemporaries and successors, who silently acquiesced when his discoveries were attributed to others, and allowed his name to perish entirely from Cambridge tradition.

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* *Baltimore Lectures* (ed. 1904), p. 258.

A few years before Green published his first paper, a notable revival of mathematical learning swept over the University; the fluxional symbolism, which since the time of Newton had isolated Cambridge from the continental schools, was abandoned in favour of the differential notation, and the works of the great French analysts were introduced and eagerly read. Green undoubtedly received his own early inspiration from this source; but in clearness of physical insight and conciseness of exposition he far excelled his masters; and the slight volume of his collected papers has to this day a charm which is wanting to the voluminous writings of Cauchy and Poisson. It was natural that such an example should powerfully influence the youthful intellects of Stokes—who was an undergraduate when Green read his memoir on double refraction to the Cambridge Philosophical Society—and of William Thomson (Kelvin), who came into residence two years afterwards.*

In spite of the advances which were made in the great memoirs of the year 1839, the fundamental question as to whether the aether-particles vibrate parallel or at right angles to the plane of polarization was still unanswered. More light was thrown on this problem ten years later by Stokes's investigation of Diffraction.† Stokes showed that on almost any conceivable hypothesis regarding the aether, a disturbance in which the vibrations are executed at right angles to the plane of diffraction must be transmitted round the edge of an opaque body with less diminution of intensity than a disturbance whose vibrations are executed parallel to that plane. It follows that when light, of which the vibrations are oblique to the plane of diffraction, is so transmitted, the plane of vibration will be more nearly at right angles to the plane of diffraction in the diffracted than in the incident light. Stokes himself performed experiments to test the matter, using a grating in order to obtain strong light diffracted at a large angle, and found that when the plane of polarization of the incident light was oblique to the plane of diffraction, the plane of polarization of the diffracted light was more nearly parallel to the plane of diffraction. This result, which was afterwards confirmed by L. Lorenz,* appeared to confirm decisively the hypothesis of Fresnel, that the vibrations of the aethereal particles are executed at right angles to the plane of polarization.

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*It was in the year Thomson took his degree (1845) that he bought, and read with delight, the electrical memoir which Green had published at Nottingham in 1828.

† Trans. Camb. Phil. Soc., ix (1849), p. 1. Stokes's *Math. and Phys. Papers*, ii, p. 243.

Three years afterwards Stokes indicated† a second line of proof leading to the same conclusion. It had long been known that the blue light of the sky, which is due to the scattering of the sun's direct rays by small particles or molecules in the atmosphere, is partly polarized. The polarization is most marked when the light comes from a part of the sky distant 90° from the sun, in which case it must have been scattered in a direction perpendicular to that of the direct sunlight incident on the small particles; and the polarization is in the plane through the sun.

If, then, the axis of y be taken parallel to the light incident on a small particle at the origin, and the scattered light be observed along the axis of x, this scattered light is found to be polarized in the plane xy. Considering the matter from the dynamical point of view, we may suppose the material particle to possess so much inertia (compared to the aether) that it is practically at rest. Its motion relative to the aether, which is the cause of the disturbance it creates in the aether, will therefore be in the same line as the incident aethereal vibration, but in the opposite direction. The disturbance must be transversal, and must therefore be zero in a polar direction and a maximum in an equatorial direction, its amplitude being, in fact, proportional to the sine of the polar distance. The polar line must, by considerations of symmetry, be the line of the incident vibration. Thus we see that none of the light scattered in the x-direction can come from that constituent of the incident light which vibrates parallel to the x-axis; so the light observed in this direction must consist of vibrations parallel to the z-axis. But we have seen that the plane of polarization of the scattered light is the plane of xy; and therefore the vibration is at right angles to the plane of polarization.*

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* Ann. d. Phys. cxi (1860), p. 315. Phil. Mag. xxi (1861), p. 321.

† Phil. Trans., 1852, p. 463. Stokes's *Math. and Phys. Papers*, iii, p. 267. Cf. the foot-note added on p. 361 of the *Math. and Phys. Papers*.

The phenomena of diffraction and of polarization by scattering thus agreed in confirming the result arrived at in Fresnel's and Green's theory of reflexion. The chief difficulty in accepting it arose in connexion with the optics of crystals. As we have seen, Green and Cauchy were unable to reconcile the hypothesis of aethereal vibrations at right angles to the plane of polarization with the correct formulae of crystal-optics, at any rate so long as the aether within crystals was supposed to be free from initial stress. The underlying reason for this can be readily seen. In a crystal, where the elasticity is different in different directions, the resistance to distortion depends solely on the orientation of the plane of distortion, which in the case of light is the plane through the directions of propagation and vibration. Now it is known that for light propagated parallel to one of the axes of elasticity of a crystal, the velocity of propagation depends only on the plane of polarization of the light, being the same whichever of the two axes lying in that plane is the direction of propagation. Comparing these results, we see that the plane of polarization must be the plane of distortion, and therefore the vibrations of the aether-particles must be executed parallel to the plane of polarization.†

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* The theory of polarization by small particles was afterwards investigated by Lord Rayleigh, Phil. Mag. xli (1871).

† In Fresnel's theory of crystal-optics, in which the aether-vibrations are at right angles to the plane of polarization, the velocity of propagation depends only on the direction of vibration, not on the plane through this and the direction of transmission.

A way of escape from this conclusion suggested itself to Stokes,* and later to Rankine† and Lord Rayleigh.‡ What if the aether in a crystal, instead of having its elasticity different in different directions, were to have its rigidity invariable and its *inertia* different in different directions? This would bring the theory of crystal-optics into complete agreement with Fresnel's and Green's theory of reflexion, in which the optical differences between media are attributed to differences of inertia of the aether contained within them. The only difficulty lies in conceiving how aelotropy of inertia can exist; and all three writers overcame this obstacle by pointing out that a solid which is immersed in a fluid may have its effective inertia different in different directions. For instance, a coin immersed in water moves much more readily in its own plane than in the direction at right angles to this.

Suppose then that twice the kinetic energy per unit volume of the aether within a crystal is represented by the expression

$$\rho_1 \left(\frac{\partial e_x}{\partial t}\right)^2 + \rho_2 \left(\frac{\partial e_y}{\partial t}\right)^2 + \rho_3 \left(\frac{\partial e_z}{\partial t}\right)^2,$$

and that the potential energy per unit volume has the same value as in space void of ordinary matter. The aether is assumed to be incompressible, so that div **e** is zero: the potential energy per unit volume is therefore

$$\phi = \tfrac{1}{2}n \left\{ \left(\frac{\partial e_z}{\partial y} + \frac{\partial e_y}{\partial z}\right)^2 + \left(\frac{\partial e_x}{\partial z} + \frac{\partial e_z}{\partial x}\right)^2 + \left(\frac{\partial e_y}{\partial x} + \frac{\partial e_x}{\partial y}\right)^2 - 4\frac{\partial e_y}{\partial y}\frac{\partial e_z}{\partial z} \right.$$
$$\left. - 4\frac{\partial e_z}{\partial z}\frac{\partial e_x}{\partial x} - 4\frac{\partial e_x}{\partial x}\frac{\partial e_y}{\partial y} \right\},$$

where n denotes as usual the rigidity.

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* Stokes, in a letter to Lord Rayleigh, inserted in his *Memoir and Scientific Correspondence*, ii, p. 99, explains that the idea presented itself to him while he was writing the paper on Fluid Motion which appeared in Trans. Camb. Phil. Soc., viii (1843), p. 105. He suggested the wave-surface to which this theory leads in Brit. Assoc. Rep., 1862, p. 269.

† Phil. Mag. (4), i (1851), p. 441.

‡ Phil. Mag. (4), xli (1871), p. 519.

The variational equation of motion is

$$\iiint \left\{\rho_1 \frac{\partial^2 e_x}{\partial t^2} \partial e_x + \rho_2 \frac{\partial^2 e_y}{\partial t^2} \partial e_y + \rho_3 \frac{\partial^2 e_z}{\partial t^2} \delta e_z\right\} dx\, dy\, dz$$

$$= - \iiint \left\{\delta\phi - p\delta\left(\frac{\partial e_x}{\partial x} + \frac{\partial e_y}{\partial y} + \frac{\partial e_z}{\partial z}\right)\right\} dx\, dy\, dz,$$

where p denotes an undetermined function of (x, y, z): the term in p being introduced on account of the kinematical constraint expressed by the equation

$$\operatorname{div} \mathbf{e} = 0.$$

The equations of motion which result from this variational equation are

$$\rho_1 \frac{\partial^2 e_x}{\partial t^2} = - \frac{\partial p}{\partial x} + n\nabla^2 e_x,$$

and two similar equations. It is evident that p resembles a hydrostatic pressure.

Substituting in these equations the analytical expression for a plane wave, we readily find that the velocity V of the wave is connected with the direction-cosines (λ, μ, ν) of its normal by the equation

$$\frac{\lambda^2}{n - \rho_1 V^2} + \frac{\mu^2}{n - \rho_2 V^2} + \frac{\nu^2}{n - \rho_3 V^2} = 0.$$

When this is compared with Fresnel's relation between the velocity and direction of a wave, it is seen that the new formula differs from his only in having the reciprocal of the velocity in place of the velocity. About 1867 Stokes carried out a series of experiments in order to determine which of the two theories was most nearly conformable to the facts: he found the construction of Huygens and Fresnel to be decidedly the more correct, the difference between the results of it and the rival construction being about 100 times the probable error of observation.*

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* Proc. R. S., June, 1872. After these experiments Stokes gave it as his opinion (Phil. Mag. xli (1871), p. 521) that the true theory of crystal-optics was yet to be found. On the accuracy of Fresnel's construction cf. Glazebrook, Phil. Trans. clxxi (1879) p. 421, and Hastings, Am. Journ. Sci. (3) xxxv (1887) p. 60.

The hypothesis that in crystals the inertia depends on direction seemed therefore to be discredited when the theory based on it was compared with the results of observation. But when, in 1888, W. Thomson (Lord Kelvin) revived Cauchy's theory of the labile aether, the question naturally arose as to whether that theory could be extended so as to account for the optical properties of crystals: and it was shown by R. T. Glazebrook[*] that the correct formulae of crystal-optics are obtained when the Cauchy-Thomson hypothesis of zero velocity for the longitudinal wave is combined with the Stokes-Rankine-Rayleigh hypothesis of aelotropic inertia.

For on reference to the formulae which have been already given, it is obvious that the equation of motion of an aether having these properties must be

$$(\rho_1 \ddot{e}_x, \rho_2 \ddot{e}_y, \rho_3 \ddot{e}_z) = - n \text{ curl curl } \mathbf{e},$$

where $\mathbf{e}$ denotes the displacement, n the rigidity, and (ρ_1, ρ_2, ρ_3) the inertia: and this equation leads by the usual analysis to Fresnel's wave-surface. The displacement $\mathbf{e}$ of the aethereal particles is not, however, accurately in the wave-front, as in Fresnel's theory, but is at right angles to the direction of the ray, in the plane passing through the ray and the wave-normal.[†]

Having now traced the progress of the elastic-solid theory so far as it is concerned with the propagation of light in ordinary isotropic media and in crystals, we must consider the attempts which were made about this time to account for the optical properties of a more peculiar class of substances.

It was found by Arago in 1811[‡] that the state of polarization of a beam of light is altered when the beam is passed through a plate of quartz along the optic axis. The phenomenon was studied shortly afterwards by Biot,[*] who showed that the alteration consists in a rotation of the plane of polarization about the direction of propagation: the angle of rotation is proportional to the thickness of the plate and inversely proportional to the square of the wave-length.

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[*] Phil. Mag. xxvi (1888), p. 521; xxviii (1889), p. 110.

[†] This theory of crystal-optics may be assimilated to the electro-magnetic theory by interpreting the elastic displacement $\mathbf{e}$ as electric force, and the vector $(\rho_1 e_x, \rho_2 e_y, \rho_3 e_z)$ as electric displacement.

[‡] Mém. de l'Institut, 1811, Part I, p. 115, sqq.

In some specimens of quartz the rotation is from left to right, in others from right to left. This distinction was shown by Sir John Herschel[†] (*b.* 1792, *d.* 1871) in 1820 to be associated with differences in the crystalline form of the specimens, the two types bearing the same relation to each other as a right-handed and left-handed helix respectively. Fresnel[‡] and W. Thomson[§] proposed the term *helical* to denote the property of rotating the plane of polarization, exhibited by such bodies as quartz: the less appropriate term *natural rotatory polarization* is, however, generally used.[||]

Biot showed that many liquid organic bodies, e.g. turpentine and sugar solutions, possess the natural rotatory property: we might be led to infer the presence of a helical structure in the molecules of such substances; and this inference is supported by the study of their chemical constitution; for they are invariably of the "mirror-image" or "enantiomorphous" type, in which one of the atoms (generally carbon) is asymmetrically linked to other atoms.

The next advance in the subject was due to Fresnel,[¶] who showed that in naturally active bodies the velocity of propagation of circularly polarized light is different according as the polarization is right-handed or left-handed. From this property the rotation of the plane of polarization of a plane-polarized ray may be immediately deduced; for the plane-polarized ray may be resolved into two rays circularly polarized in opposite senses, and these advance in phase by different amounts in passing through a given thickness of the substance: at any stage they may be recompounded into a plane-polarized ray, the azimuth of whose plane of polarization varies with the length of path traversed.

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[*] Mém. de l'Institut, 1812, Part I, p. 218, sqq.; Annales de Chim., ix (1818), p. 372; x (1819), p. 63.

[†] Camb. Phil. Soc. Trans. i, p. 43.

[‡] Mém. de l'Inst. vii, p. 73.

[§] *Baltimore Lectures* (ed. 1904), p. 31.

[||] The term *rotatory* may be applied with propriety to the property discovered by Faraday, which will be discussed later.

[¶] Annales de Chim. xxviii (1825), p. 147.

It is readily seen from this that a ray of light incident on a crystal of quartz will in general bifurcate into two refracted rays, each of which will be elliptically polarized, i.e. will be capable of resolution into two plane-polarized components which differ in phase by a definite amount. The directions of these refracted rays may be determined by Huygens' construction, provided the wave-surface is supposed to consist of a sphere and spheroid which *do not touch.*

The first attempt to frame a theory of naturally active bodies was made by MacCullagh in 1836.* Suppose a plane wave of light to be propagated within a crystal of quartz. Let (x, y, z) denote the coordinates of a vibrating molecule, when the axis of x is taken at right angles to the plane of the wave, and the axis of z at right angles to the axis of the crystal. Using Y and Z to denote the displacements parallel to the axes of y and z respectively at any time t, MacCullagh assumed that the differential equations which determine Y and Z are

$$\frac{\partial^2 Y}{\partial t^2} = c_1^2 \frac{\partial^2 Y}{\partial x^2} + \mu \frac{\partial^3 Z}{\partial x^3}$$

$$\frac{\partial^2 Z}{\partial t^2} = c_2^2 \frac{\partial^2 Z}{\partial x^2} - \mu \frac{\partial^3 Y}{\partial x^3}$$

where μ denotes a constant on which the natural rotatory property of the crystal depends. In order to avoid complications arising from the ordinary crystalline properties of quartz, we shall suppose that the light is propagated parallel to the optic axis, so that we can take c_1 equal to c_2.

Assuming first that the beam is circularly polarized, let it be represented by

$$Y = A \sin \frac{2\pi}{\tau}(lx - t), \quad Z = \pm A \cos \frac{2\pi}{\tau}(lx - t),$$

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* Trans. Royal Irish Acad., xvii.; MacCullagh's *Coll. Works*, p. 63.

the ambiguous sign being determined according as the circular polarization is right-handed or left-handed.

Substituting in the above differential equations, we have

$$1 = c_1^2 l^2 \mp \mu \, . \, \frac{2\pi}{\tau} \, . \, l^3,$$

or

$$l = \frac{1}{c_1} \pm \frac{\pi\mu}{\tau c_1^4}.$$

Since $1/l$ denotes the velocity of propagation, it is evident that the reciprocals of the velocities of propagation of a right-handed and left-handed beam differ by the quantity

$$\frac{2\pi\mu}{\tau c_1^4};$$

from which it is easily shown that the angle through which the plane of polarization of a plane-polarized beam rotates in unit length of path is

$$\frac{2\pi^2\mu}{\tau^2 c_1^4}.$$

If we neglect the variation of c_1 with the period of the light, this expression satisfies Biot's law that the angle of rotation in unit length of path is proportional to the inverse square of the wave-length.

MacCullagh's investigation can be scarcely called a theory, for it amounts only to a reduction of the phenomena to empirical, though mathematical, laws; but it was on this foundation that later workers built the theory which is now accepted.*

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* The later developments of this theory will be discussed in a subsequent chapter; but mention may here be made of an attempt which was made in 1856 by Carl Neumann, then a very young man, to provide a rational basis for MacCullagh's equations. Neumann showed that the equations may be derived from the hypothesis that the relative displacement of one aethereal particle with respect to another acts on the latter according to the same law as an element of an electric current acts on a magnetic pole. Cf. the preface to C. Neumann's *Die magnetische Drehung der Polarisationsebene des Lichtes*, Halle, 1863.

The great investigators who developed the theory of light after the death of Fresnel devoted considerable attention to the optical properties of metals. Their researches in this direction must now be reviewed.

The most striking properties of metals are the power of brilliantly reflecting light at all angles of incidence, which is so well shown by the mirrors of reflecting telescopes, and the opacity, which causes a train of waves to be extinguished before it has proceeded many wave-lengths into a metallic medium. That these two attributes are connected appears probable from the fact that certain non-metallic bodies—e.g., aniline dyes—which strongly absorb the rays in certain parts of the spectrum, reflect those rays with almost metallic brilliance. A third quality in which metals differ from transparent bodies, and which, as we shall see, is again closely related to the other two, is in regard to the polarization of the light reflected from them. This was first noticed by Malus; and in 1830 Sir David Brewster* showed that plane-polarized light incident on a metallic surface remains polarized in the same plane after reflexion if its polarization is either parallel or perpendicular to the plane of reflexion, but that in other cases the reflected light is polarized elliptically.

It was this discovery of Brewster's which suggested to the mathematicians a theory of metallic reflexion. For, as we have seen, elliptic polarization is obtained when plane-polarized light is totally reflected at the surface of a transparent body; and this analogy between the effects of total reflexion and metallic reflexion led to the surmise that the latter phenomenon might be treated in the same way as Fresnel had treated the former, namely, by introducing imaginary quantities into the formulae of ordinary reflexion. On these principles mathematical formulae were devised by MacCullagh† and Cauchy.‡

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* Phil. Trans., 1830.

† Proc. Roy. Irish Acad., i (1836), p. 2; ii (1843), p. 376: Trans. Roy. Irish Acad., xviii (1837), p. 71: MacCullagh's *Coll. Works*, pp. 58, 132, 230.

‡ Comptes Rendus, vii (1838), p. 953; viii (1839), pp. 553, 658, 961; xxvi (1848), p. 86.

To explain their method, we shall suppose the incident light to be polarized in the plane of incidence. According to Fresnel's sine-law, the amplitude of the light (polarized in this way) reflected from a transparent body is to the amplitude of the incident light in the ratio

$$J = \frac{\sin (i - r)}{\sin (i + r)},$$

where i denotes the angle of incidence and r is determined from the equation

$$\sin i = \mu \sin r.$$

MacCullagh and Cauchy assumed that these equations hold good also for reflexion at a metallic surface, provided the refractive index μ is replaced by a complex quantity

$$\mu = \nu (1 - \kappa \sqrt{-1}) \qquad \text{say,}$$

where ν and κ are to be regarded as two constants characteristic of the metal. We have therefore

$$J = \frac{\tan i - \tan r}{\tan i + \tan r} = \frac{(\mu^2 - \sin^2 i)^{\frac{1}{2}} - \cos i}{(\mu^2 - \sin^2 i)^{\frac{1}{2}} + \cos i}$$

If then we write

$$\nu^2 (1 - \kappa \sqrt{-1})^2 - \sin^2 i = U^2 e^{2v\sqrt{-1}},$$

so that equations defining U and v are obtained by equating separately the real and the imaginary parts of this equation, we have

$$J = \frac{Ue^{v\sqrt{-1}} - \cos i}{Ue^{v\sqrt{-1}} + \cos i}$$

and this may be written in the form

$$\overline{J}e^{\delta\sqrt{-1}}$$

where

$$\begin{cases} \overline{J}^2 = \dfrac{U^2 + \cos^2 i - 2U \cos v \cos i}{U^2 + \cos^2 i + 2U \cos v \cos i} \\ \tan \delta = \dfrac{2U \cos i \sin v}{U^2 - \cos^2 i}. \end{cases}$$

The quantities $\bar{J}$ and δ are interpreted in the same way as in Fresnel's theory of total reflexion: that is, we take $\bar{J}^2$ to mean the ratio of the intensities of the reflected and incident light, while δ measures the change of phase experienced by the light in reflexion.

The case of light polarized at right angles to the plane of incidence may be treated in the same way.

When the incidence is perpendicular, U evidently reduces to $\nu\,(1+\kappa^2)^{\frac{1}{2}}$, and υ reduces to $-\tan^{-1}\kappa$. For silver at perpendicular incidence almost all the light is reflected, so $\bar{J}^2$ is nearly unity: this requires $\cos\upsilon$ to be small, and κ to be very large. The extreme case in which κ is indefinitely great but ν indefinitely small, so that the quasi-index of refraction is a pure imaginary, is generally known as the case of *ideal silver.*

The physical significance of the two constants ν and κ was more or less distinctly indicated by Cauchy; in fact, as the difference between metals and transparent bodies depends on the constant κ, it is evident that κ must in some way measure the opacity of the substance. This will be more clearly seen if we inquire how the elastic-solid theory of light can be extended so as to provide a physical basis for the formulae of MacCullagh and Cauchy.* The sine-formula of Fresnel, which was the starting-point of our investigation of metallic reflexion, is a consequence of Green's elastic-solid theory: and the differences between Green's results and those which we have derived arise solely from the complex value which we have assumed for μ. We have therefore to modify Green's theory in such a way as to obtain a complex value for the index of refraction.

Take the plane of incidence as plane of xy, and the metallic surface as plane of yz. If the light is polarized in the plane of incidence, so that the light-vector is parallel to the axis of z, the incident light may be taken to be a function of the argument

$$ax + by + ct,$$

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* This was done by Lord Rayleigh, Phil. Mag. xliii (1872), p. 321.

where

$$\frac{a}{c} = -\left(\frac{\rho}{n}\right)^{\frac{1}{2}} \cos i, \qquad \frac{b}{c} = -\left(\frac{\rho}{n}\right)^{\frac{1}{2}} \sin i;$$

here i denotes the angle of incidence, ρ the inertia of the aether, and n its rigidity.

Let the reflected light be a function of the argument

$$a_1x + by + ct,$$

where, in order to secure continuity at the boundary, b and c must have the same values as before. Since Green's formulae are to be still applicable, we must have

$$\frac{a_1}{b} = \cot r,$$

where $\sin i = \mu \sin r$, but μ has now a complex value. This equation may be written in the form

$$a_1^2 + b^2 = \frac{\mu^2 \rho c^2}{n}.$$

Let the complex value of μ^2 be written

$$\mu^2 = \frac{\rho_1}{\rho} - A\sqrt{-1},$$

the real part being written ρ_1/ρ in order to exhibit the analogy with Green's theory of transparent media: then we have

$$a_1^2 + b^2 = \frac{\rho_1}{n} c^2 - \frac{\rho c^2}{n} A\sqrt{-1}.$$

But an equation of this kind must (as in Green's theory) represent the condition to be satisfied in order that the quantity

$$e^{(a_1x + by + ct)\sqrt{-1}}$$

may satisfy the differential equation of motion of the aether; from which we see that the equation of motion of the aether in the metallic medium is probably of the form

$$\rho_1 \frac{\partial^2 e_z}{\partial t^2} + \rho c A \frac{\partial e_z}{\partial t} = n\left(\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2}\right).$$

This equation of motion differs from that of a Greenian elastic solid by reason of the occurrence of the term in $\partial e_z/\partial t$. But this is evidently a " viscous " term, representing something like a frictional dissipation of the energy of luminous vibrations: a dissipation which, in fact, occasions the opacity of the metal. Thus the term which expresses opacity in the equation of motion of the luminiferous medium appears as the origin of the peculiarities of metallic reflexion.* It is curious to notice how closely this accords with the idea of Huygens, that metals are characterized by the presence of soft particles which damp the vibrations of light.

There is, however, one great difficulty attending this explanation of metallic reflexion, which was first pointed out by Lord Rayleigh.† We have seen that for ideal silver μ^2 is real and negative: and therefore A must be zero and ρ_1 negative; that is to say, the inertia of the luminiferous medium in the metal must be negative. This seems to destroy entirely the physical intelligibility of the theory as applied to the case of ideal silver.

The difficulty is a deep-seated one, and was not overcome for many years. The direction in which the true solution lies will suggest itself when we consider the resemblance which has already been noticed between metals and those substances which show "surface colour"—e.g. the aniline dyes. In the case of the latter substances, the light which is so copiously reflected from them lies within a restricted part of the spectrum; and it therefore seems probable that the phenomenon is not to be attributed to the existence of dissipative terms, but that it belongs rather to the same class of effects as dispersion, and is to be referred to the same causes. In fact, dispersion means that the value of the refractive index of a substance with respect to any kind of light depends on the period of the light; and we have only to suppose that the physical causes which operate in dispersion cause the refractive index to become imaginary for certain kinds of light, in order to explain satisfactorily both the surface colours of the aniline dyes and the strong reflecting powers of the metals.

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* It is easily seen that the amplitude is reduced by the factor $e^{-2\pi\kappa}$ when light travels one wave-length in the metal: κ is generally called the *coefficient of absorption.*

† Loc. cit.

Dispersion was the subject of several memoirs by the founders of the elastic-solid theory. So early as 1830 Cauchy's attention was directed* to the possibility of constructing a mathematical theory of this phenomenon on the basis of Fresnel's "Hypothesis of Finite Impacts"†—i.e. the assumption that the radius of action of one particle of the luminiferous medium on its neighbours is so large as to be comparable with the wave-length of light. Cauchy supposed the medium to be formed, as in Navier's theory of elastic solids, of a system of point-centres of force: the force between two of these point-centres, m at (x, y, z), and μ at $(x + \Delta x, y + \Delta y, z + \Delta z)$, may be denoted by $m\mu f(r)$, where r denotes the distance between m and μ. When this medium is disturbed by light-waves propagated parallel to the z-axis, the displacement being parallel to the x-axis, the equation of motion of m is evidently

$$\frac{\partial^2\xi}{\partial t^2} = \sum_{\mu} \mu f(r + \rho)\,\frac{\Delta x + \Delta\xi}{r + \rho},$$

where ξ denotes the displacement of m, $(\xi + \Delta\xi)$ the displacement of μ, and $(r + \rho)$ the new value of r. Substituting for ρ its value, and retaining only terms of the first degree in $\Delta\xi$, this equation becomes

$$\frac{\partial^2\xi}{\partial t^2} = \Sigma\mu\frac{f(r)}{r}\,\Delta\xi + \Sigma\mu\,\frac{d}{dr}\left\{\frac{f(r)}{r}\right\}\frac{(\Delta x)^2}{r}\,\Delta\xi.$$

Now, by Taylor's theorem, since ξ depends only on z, we have

$$\Delta\xi = \frac{\partial\xi}{\partial z}\,\Delta z + \frac{1}{2\,!}\frac{\partial^2\xi}{\partial z^2}\,(\Delta z)^2 + \frac{1}{3\,!}\frac{\partial^3\xi}{\partial z^3}\,(\Delta z)^3 \ldots$$

Substituting, and remembering that summations which involve odd powers of Δz must vanish when taken over all the point-centres within the sphere of influence of m, we obtain an equation of the form

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* Bull. des Sc. Math. xiv (1830), p. 9: "Sur la dispersion de la lumière," *Nouv. Exercices de Math.*, 1836.

† Cf. p. 132.

$$\frac{\partial^2\xi}{\partial t^2} = \alpha\frac{\partial^2\xi}{\partial z^2} + \beta\frac{\partial^4\xi}{\partial z^4} + \gamma\frac{\partial^6\xi}{\partial z^6} + \ldots,$$

where α, β, γ . . . denote constants.

Each successive term on the right-hand side of this equation involves an additional factor $(\Delta z)^2/\lambda^2$ as compared with the preceding term, where λ denotes the wave-length of the light: so if the radii of influence of the point-centres were indefinitely small in comparison with the wave-length of the light, the equation would reduce to

$$\frac{\partial^2\xi}{\partial t^2} = \alpha\frac{\partial^2\xi}{\partial z^2},$$

which is the ordinary equation of wave-propagation in one dimension in non-dispersive media. But if the medium is so coarse-grained that λ is not large compared with the radii of influence, we must retain the higher derivates of ξ. Substituting

$$\xi = e^{\frac{2\pi i}{\lambda}(z - c_1 t)}$$

in the differential equation with these higher derivates retained, we have

$$c_1^2 = \alpha - \beta\left(\frac{2\pi}{\lambda}\right)^2 + \gamma\left(\frac{2\pi}{\lambda}\right)^4 \ldots,$$

which shows that c_1, the velocity of the light in the medium, depends on the wave-length λ; as it should do in order to explain dispersion.

Dispersion is, then, according to the view of Fresnel and Cauchy, a consequence of the coarse-grainedness of the medium. Since the luminiferous medium was found to be dispersive only within material bodies, it seemed natural to suppose that in these bodies the aether is *loaded* by the molecules of matter, and that dispersion depends essentially on the ratio of the wave-length to the distance between adjacent material molecules.

This theory, in one modification or another, held its ground until forty years later it was overthrown by the facts of anomalous dispersion.

The distinction between aether and ponderable matter was more definitely drawn in memoirs which were published independently in 1841–2 by F. E. Neumann* and Matthew O'Brien.† These authors supposed the ponderable particles to remain sensibly at rest while the aether surges round them, and is acted on by them with forces which are proportional to its displacement. Thus‡ the equation of motion of the aether becomes

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = -(k + \tfrac{4}{3}n) \operatorname{grad} \operatorname{div} \mathbf{e} - n \operatorname{curl} \operatorname{curl} \mathbf{e} - C\mathbf{e},$$

where C denotes a constant on which the phenomena of dispersion depend. For polarized plane waves propagated parallel to the axis of x, this equation becomes

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = n \frac{\partial^2 \mathbf{e}}{\partial x^2} - C\mathbf{e};$$

and substituting

$$\mathbf{e} = e^{\frac{2\pi\sqrt{-1}}{\tau}\left(t - \frac{x}{V}\right)},$$

where τ denotes the period and V the velocity of the light, we have

$$\frac{n}{V^2} = \rho - \frac{C}{4\pi^2}\tau^2,$$

an equation which expresses the dependence of the velocity on the period.

The attempt to represent the properties of the aether by those of an elastic solid lost some of its interest after the rise of the electromagnetic theory of light. But in 1867, before the electromagnetic hypothesis had attracted much attention, an elastic-solid theory in many respects preferable to its predecessors was presented to the French Academy* by Joseph Boussinesq (*b.* 1842). Until this time, as we have seen, investigators had been divided into two parties, according as they attributed the optical properties of different bodies to variations in the inertia of the luminiferous medium, or to variations in its elastic properties. Boussinesq, taking up a position apart from both these schools, assumed that the aether is exactly the same in all material bodies as in interplanetary space, in regard both to inertia and to rigidity, and that the optical properties of matter are due to interaction between the aether and the material particles, as had been imagined more or less by Neumann and O'Brien. These material particles he supposed to be disseminated in the aether, in much the same way as dust-particles floating in the air.

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* Berlin Abhandlungen aus dem Jahre 1841, Zweiter Teil, p. 1: Berlin, 1843.

† Trans. Camb. Phil. Soc. vii (1842), p. 397.

‡ O'Brien, loc. cit., §§ 15, 28.

If $\mathbf{e}$ denote the displacement at the point (x, y, z) in the aether, and $\mathbf{e}'$ the displacement of the ponderable particles at the same place, the equation of motion of the aether is

$$\rho \frac{\partial^2 \mathbf{e}}{\partial t^2} = -(k + \tfrac{1}{3}n) \text{ grad div } \mathbf{e} + n\nabla^2 \mathbf{e} - \rho_1 \frac{\partial^2 \mathbf{e}'}{\partial t^2}, \qquad (1)$$

where ρ and ρ_1 denote the densities of the aether and matter respectively, and k and n denote as usual the elastic constants of the aether. This differs from the ordinary Cauchy-Green equation only in the presence of the term $\rho_1 \partial^2 \mathbf{e}'/\partial t^2$, which represents the effect of the inertia of the matter. To this equation we must adjoin another expressing the connexion between the displacements of the matter and of the aether: if we assume that these are simply proportional to each other—say,

$$\mathbf{e}' = A\mathbf{e}, \qquad (2)$$

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* Journal de Math. (2) xiii (1868), pp. 313, 425: cf. also Comptes Rendus, cxvii (1893), pp. 80, 139, 193. Equations kindred to some of those of Boussinesq were afterwards deduced by Karl Pearson, Proc. Lond. Math. Soc , xx (1889), p. 297, from the hypothesis that the strain-energy involves the velocities.

where the constant A depends on the nature of the ponderable body—our equation becomes

$$(\rho + \rho_1 A)\frac{\partial^2 \mathbf{e}}{\partial t^2} = -(k + \tfrac{1}{3}n)\,\text{grad div}\,\mathbf{e} + n\nabla^2\mathbf{e},$$

which is essentially the same equation as is obtained in those older theories which suppose the inertia of the luminiferous medium to vary from one medium to another. So far there would seem to be nothing very new in Boussinesq's work. But when we proceed to consider crystal-optics, dispersion, and rotatory polarization, the advantage of his method becomes evident: he retains equation (1) as a formula universally true—at any rate for bodies at rest—while equation (2) is varied to suit the circumstances of the case. Thus dispersion can be explained if, instead of equation (2), we take the relation

$$\mathbf{e}' = A\mathbf{e} - D\nabla^2\mathbf{e},$$

where D is a constant which measures the dispersive power of the substance: the rotation of the plane of polarization of sugar solutions can be explained if we suppose that in these bodies equation (2) is replaced by

$$\mathbf{e}' = A\mathbf{e} + B\,\text{curl}\,\mathbf{e},$$

where B is a constant which measures the rotatory power; and the optical properties of crystals can be explained if we suppose that for them equation (2) is to be replaced by the equations

$$e_x' = A_1 e_x, \qquad e_y' = A_2 e_y, \qquad e_z' = A_3 e_z$$

When these values for the components of $\mathbf{e}'$ are substituted in equation (1), we evidently obtain the same formulae as were derived from the Stokes-Rankine-Rayleigh hypothesis of inertia different in different directions in a crystal; to which Boussinesq's theory of crystal-optics is practically equivalent.

The optical properties of bodies in motion may be accounted for by modifying equation (1), so that it takes the form

$$\rho\frac{\partial^2 \mathbf{e}}{\partial t^2} = -(k + \tfrac{1}{3}n)\,\text{grad div}\,\mathbf{e} + n\nabla^2\mathbf{e} - \rho_1\left(\frac{\partial}{\partial t} + w_x\frac{\partial}{\partial x} + w_y\frac{\partial}{\partial y} + w_z\frac{\partial}{\partial z}\right)^2\mathbf{e}',$$

where **w** denotes the velocity of the ponderable body. If the body is an ordinary isotropic one, and if we consider light propagated parallel to the axis of z, in a medium moving in that direction, the light-vector being parallel to the axis of x, the equation reduces to

$$\rho \frac{\partial^2 e_x}{\partial t^2} = n \frac{\partial^2 e_x}{\partial z^2} - \rho_1 A \left(\frac{\partial}{\partial t} + w \frac{\partial}{\partial z}\right)^2 e_x ;$$

substituting

$$e_x = f(z - Vt),$$

where V denotes the velocity of propagation of light in the medium estimated with reference to the fixed aether, we obtain for V the value

$$\left(\frac{n}{\rho + \rho_1 A}\right)^{\frac{1}{2}} + \frac{\rho_1 A}{\rho + \rho_1 A} w.$$

The absolute velocity of light is therefore increased by the amount $\rho_1 A w/(\rho + \rho_1 A)$ owing to the motion of the medium; and this may be written $(\mu^2 - 1) w/\mu^2$, where μ denotes the refractive index; so that Boussinesq's theory leads to the same formula as had been given half a century previously by Fresnel.[*]

It is Boussinesq's merit to have clearly asserted that all space, both within and without ponderable bodies, is occupied by one identical aether, the same everywhere both in inertia and elasticity; and that all aethereal processes are to be represented by two kinds of equations, of which one kind expresses the invariable equations of motion of the aether, while the other kind expresses the interaction between aether and matter. Many years afterwards these ideas were revived in connexion with the electromagnetic theory, in the modern forms of which they are indeed of fundamental importance.

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[*] Cf. p. 115 sqq.

CHAPTER VI.

FARADAY.

TOWARDS the end of the year 1812, Davy received a letter in which the writer, a bookbinder's journeyman named Michael Faraday, expressed a desire to escape from trade, and obtain employment in a scientific laboratory. With the letter was enclosed a neatly written copy of notes which the young man —he was twenty-one years of age—had made of Davy's own public lectures. The great chemist replied courteously, and arranged an interview; at which he learnt that his correspondent had educated himself by reading the volumes which came into his hands for binding. "There were two," Faraday wrote later, "that especially helped me, the 'Encyclopaedia Britannica,' from which I gained my first notions of electricity, and Mrs. Marcet's 'Conversations on Chemistry,' which gave me my foundation in that science." Already, before his application to Davy, he had performed a number of chemical experiments, and had made for himself a voltaic pile, with which he had decomposed several compound bodies.

At Davy's recommendation Faraday was in the following spring appointed to a post in the laboratory of the Royal Institution, which had been established at the close of the eighteenth century under the auspices of Count Rumford; and here he remained for the whole of his active life, first as assistant, then as director of the laboratory, and from 1833 onwards as the occupant of a chair of chemistry which was founded for his benefit.

For many years Faraday was directly under Davy's influence, and was occupied chiefly in chemical investigations. But in 1821, when the new field of inquiry opened by Oersted's discovery was attracting attention, he wrote an *Historical Sketch of Electro-Magnetism,*[*] as a preparation for which he carefully repeated the experiments described by the writers he was reviewing; and this seems to have been the beginning of the researches to which his fame is chiefly due.

The memoir which stands first in the published volumes of Faraday's electrical work[†] was communicated to the Royal Society on November 24th, 1831. The investigation was inspired, as he tells us, by the hope of discovering analogies between the behaviour of electricity as observed in motion in currents, and the behaviour of electricity at rest on conductors. Static electricity was known to possess the power of "induction" —i.e., of causing an opposite electrical state on bodies in its neighbourhood; was it not possible that electric currents might show a similar property? The idea at first was that if in any circuit a current were made to flow, any adjacent circuit would be traversed by an induced current, which would persist exactly as long as the inducing current. Faraday found that this was not the case; a current was indeed induced, but it lasted only for an instant, being in fact perceived only when the primary current was started or stopped. It depended, as he soon convinced himself, not on the mere existence of the inducing current, but on its variation.

Faraday now set himself to determine the laws of induction of currents, and for this purpose devised a new way of representing the state of a magnetic field. Philosophers had been long accustomed[‡] to illustrate magnetic power by strewing iron filings on a sheet of paper, and observing the curves in which they dispose themselves when a magnet is brought underneath.

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[*] Published in *Annals of Philosophy*, ii (1821), pp. 195, 274; iii (1822), p. 107.

[†] *Experimental Researches in Electricity*, by Michael Faraday: 3 vols.

[‡] The practice goes back at least as far as Niccolo Cabeo; indeed the curves traced by Petrus Peregrinus on his globular lodestone (cf. p. 8) were projections of lines of force. Among eighteenth-century writers La Hire mentions the use of iron filings, Mém. de l'Acad., 1717. Faraday had referred to them in his electromagnetic paper of 1821, *Exp. Res.* ii, p. 127.

These curves suggested to Faraday* the idea of *lines of magnetic force*, or curves whose direction at every point coincides with the direction of the magnetic intensity at that point; the curves in which the iron filings arrange themselves on the paper resemble these curves so far as is possible subject to the condition of not leaving the plane of the paper.

With these lines of magnetic force Faraday conceived all space to be filled. Every line of force is a closed curve, which in some part of its course passes through the magnet to which it belongs.† Hence if any small closed curve be taken in space, the lines of force which intersect this curve must form a tubular surface returning into itself; such a surface is called a *tube of force*. From a tube of force we may derive information not only regarding the direction of the magnetic intensity, but also regarding its magnitude; for the product of this magnitude‡ and the cross-section of any tube is constant along the entire length of the tube.§ On the basis of this result, Faraday conceived the idea of partitioning all space into compartments by tubes, each tube being such that this product has the same definite value. For simplicity, each of these tubes may be called a "unit line of force"; the strength of the field is then indicated by the separation or concentration of the unit lines of force,‖ so that the number of them which intersect a unit area placed at right angles to their direction at any point measures the intensity of the magnetic field at that point.

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* They were first defined in *Exp. Res.*, § 114: "By magnetic curves, I mean the lines of magnetic forces, however modified by the juxtaposition of poles, which could be depicted by iron filings; or those to which a very small magnetic needle would form a tangent."

† *Exp. Res.* iii, p. 405.

‡ Within the substance of magnetized bodies we must in this connexion understand the magnetic intensity to be that experienced in a crevice whose sides are perpendicular to the lines of magnetization: in other words, we must take it to be what since Maxwell's time has been called the magnetic induction.

§ *Exp. Res.*, § 3073. This theorem was first proved by the French geometer Michel Chasles, in his memoir on the attraction of an ellipsoidal sheet, Journal de l'École Polyt. xv (1837), p. 266.

‖ *Ibid.*, § 3122. "The relative amount of force, or of lines of force, in a given space is indicated by their concentration or separation—i.e., by their number in that space."

Faraday constantly thought in terms of lines of force. "I cannot refrain," he wrote, in 1851,* "from again expressing my conviction of the truthfulness of the representation, which the idea of lines of force affords in regard to magnetic action. All the points which are experimentally established in regard to that action—i.e. all that is not hypothetical—appear to be well and truly represented by it."†

Faraday found that a current is induced in a circuit either when the strength of an adjacent current is altered, or when a magnet is brought near to the circuit, or when the circuit itself is moved about in presence of another current or a magnet. He saw from the first‡ that in all cases the induction depends on the relative motion of the circuit and the lines of magnetic force in its vicinity. The precise nature of this dependence was the subject of long-continued further experiments. In 1832 he found§ that the currents produced by induction under the same circumstances in different wires are proportional to the conducting powers of the wires—a result which showed that the induction consists in the production of a definite electromotive force, independent of the nature of the wire, and dependent only on the intersections of the wire and the magnetic curves. This electromotive force is produced whether the wire forms a closed circuit (so that a current flows) or is open (so that electric tension results).

All that now remained was to inquire in what way the electromotive force depends on the relative motion of the wire and the lines of force. The answer to this inquiry is, in

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* *Exp. Res.*, § 3174.

† Some of Faraday's most distinguished contemporaries were far from sharing this conviction. "I declare," wrote Sir George Airy in 1855, "that I can hardly imagine anyone who practically and numerically knows this agreement" between observation and the results of calculation based on action at a distance, "to hesitate an instant in the choice between this simple and precise action, on the one hand, and anything so vague and varying as lines of force, on the other hand." Cf. Bence Jones's *Life of Faraday*, ii, p. 353.

‡ *Exp. Res.*, § 116.

§ *Ibid.*, § 213.

Faraday's own words,* that "whether the wire moves directly or obliquely across the lines of force, in one direction or another, it sums up the amount of the forces represented by the lines it has crossed," so that "the quantity of electricity thrown into a current is directly as the number of curves intersected."† The induced electromotive force is, in fact, simply proportional to the number of the unit lines of magnetic force intersected by the wire per second.

This is the fundamental principle of the induction of currents. Faraday is undoubtedly entitled to the full honour of its discovery; but for a right understanding of the progress of electrical theory at this period, it is necessary to remember that many years elapsed before all the conceptions involved in Faraday's principle became clear and familiar to his contemporaries; and that in the meantime the problem of formulating the laws of induced currents was approached with success from other points of view. There were indeed many obstacles to the direct appropriation of Faraday's work by the mathematical physicists of his own generation; not being himself a mathematician, he was unable to address them in their own language; and his favourite mode of representation by moving lines of force repelled analysts who had been trained in the school of Laplace and Poisson. Moreover, the idea of electromotive force itself, which had been applied to currents a few years previously in Ohm's memoir, was, as we have seen, still involved in obscurity and misapprehension.

A curious question which arose out of Faraday's theory was whether a bar-magnet which is rotated on its own axis carries its lines of magnetic force in rotation with it. Faraday himself believed that the lines of force do not rotate‡: on this view a revolving magnet like the earth is to be regarded as moving through its own lines of force, so that it must become charged at the equator and poles with electricity of opposite signs; and if a wire not partaking in the earth's rotation were to have sliding contact with the earth at a pole and at the equator, a current would steadily flow through it. Experiments confirmatory of these views were made by Faraday himself;[*] but they do not strictly prove his hypothesis that the lines of force remain at rest; for it is easily seen[†] that, if they were to rotate, that part of the electromotive force which would be produced by their rotation would be derivable from a potential, and so would produce no effect in closed circuits such as Faraday used.

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* *Exp. Res.*, § 3082.

† *Ibid.*, § 3115.

‡ *Ibid.*, § 3090.

Three years after the commencement of Faraday's researches on induced currents he was led to an important extension of them by an observation which was communicated to him by another worker. William Jenkin had noticed that an electric shock may be obtained with no more powerful source of electricity than a single cell, provided the wire through which the current passes is long and coiled; the shock being felt when contact is broken.[‡] As Jenkin did not choose to investigate the matter further, Faraday took it up, and showed[§] that the powerful momentary current, which was observed when the circuit was interrupted, was really an induced current governed by the same laws as all other induced currents, but with this peculiarity, that the induced and inducing currents now flowed in the same circuit. In fact, the current in its steady state establishes in the surrounding region a magnetic field, whose lines of force are linked with the circuit; and the removal of these lines of force when the circuit is broken originates an induced current, which greatly reinforces the primary current just before its final extinction. To this phenomenon the name of *self-induction* has been given.

The circumstances attending the discovery of self-induction occasioned a comment from Faraday on the number of suggestions which were continually being laid before him. He remarked that although at different times a large number of authors had presented him with their ideas, this case of Jenkin was the only one in which any result had followed. "The volunteers are serious embarrassments generally to the experienced philosopher."*

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[*] *Exp. Res.*, §§ 218, 3109, &c.

[†] Cf. W. Weber, Ann. d. Phys. lii (1841); S. Tolver Preston, Phil. Mag. xix (1885), p. 131. In 1891 S. T. Preston, Phil. Mag. xxxi, p. 100, designed a crucial experiment to test the question; but it was not tried for want of a sufficiently delicate electrometer.

[‡] A similar observation had been made by Henry, and published in the Amer. Jour. Sci. xxii (1832), p. 408. The spark at the rupture of a spirally-wound circuit had been often observed, e.g., by Pouillet and Nobili.

[§] *Exp. Res.*, § 1048.

The discoveries of Oersted, Ampère, and Faraday had shown the close connexion of magnetic with electric science. But the connexion of the different branches of electric science with each other was still not altogether clear. Although Wollaston's experiments of 1801 had in effect proved the identity in kind of the currents derived from frictional and voltaic sources, the question was still regarded as open thirty years afterwards,† no satisfactory explanation being forthcoming of the fact that frictional electricity appeared to be a surface-phenomenon, whereas voltaic electricity was conducted within the interior substance of bodies. To this question Faraday now applied himself; and in 1833 he succeeded‡ in showing that every known effect of electricity—physiological, magnetic, luminous, calorific, chemical, and mechanical—may be obtained indifferently either with the electricity which is obtained by friction or with that obtained from a voltaic battery. Henceforth the identity of the two was beyond dispute.

Some misapprehension, however, has existed among later writers as to the conclusions which may be drawn from this identification. What Faraday proved is that the process which goes on in a wire connecting the terminals of a voltaic cell is of the same nature as the process which for a short time goes on in a wire by which a condenser is discharged. He did not prove, and did not profess to have proved, that this process consists in the actual movement of a quasi-substance, electricity, from one plate of the condenser to the other, or of two quasi-substances, the resinous and vitreous electricities, in opposite directions. The process had been pictured in this way by many of his predecessors, notably by Volta; and it has since been so pictured by most of his successors: but from such assumptions Faraday himself carefully abstained.

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* Bence Jones's *Life of Faraday*, ii, p. 45.

† Cf. John Davy, Phil. Trans., 1832, p. 259; W. Ritchie, *ibid.*, p. 279. Davy suggested that the electrical power, "according to the analogy of the solar ray," might be "not a simple power, but a combination of powers, which may occur variously associated, and produce all the varieties of electricity with which we are acquainted."

‡ *Exp. Res.*, Series iii.

What is common to all theories, and is universally conceded, is that the rate of increase in the total quantity of electrostatic charge within any volume-element is equal to the excess of the influx over the efflux of current from it. This statement may be represented by the equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \mathbf{i} = 0, \qquad (1)$$

where ρ denotes the volume-density of electrostatic charge, and $\mathbf{i}$ the current, at the place (x, y, z) at the time t. Volta's assumption is really one way of interpreting this equation physically: it presents itself when we compare equation (1) with the equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div} (\rho \mathbf{v}) = 0,$$

which is the equation of continuity for a fluid of density ρ and velocity $\mathbf{v}$: we may identify the two equations by supposing $\mathbf{i}$ to be of the same physical nature as the product $\rho \mathbf{v}$; and this is precisely what is done by those who accept Volta's assumption.

But other assumptions might be made which would equally well furnish physical interpretations to equation (1). For instance, if we suppose ρ to be the convergence of *any* vector of which $\mathbf{i}$ is the time-flux,* equation (1) is satisfied automatically;

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* In symbols,

$$\operatorname{div} \mathbf{s} = -\rho,$$
$$\frac{\partial \mathbf{s}}{\partial t} = \mathbf{i},$$

where $\mathbf{s}$ denotes the vector in question.

we can picture this vector as being of the nature of a displacement. By such an assumption we should avoid altogether the necessity for regarding the conduction-current as an actual flow of electric charges, or for speculating whether the drifting charges are positive or negative; and there would be no longer anything surprising in the production of a null effect by the coalescence of electric charges of opposite signs.

Faraday himself wished to leave the matter open, and to avoid any definite assumption.* Perhaps the best indication of his views is afforded by a laboratory note† of date 1837:—

"After much consideration of the manner in which the electric forces are arranged in the various phenomena generally, I have come to certain conclusions which I will endeavour to note down without committing myself to any opinion as to the cause of electricity, i.e., as to the nature of the power. If electricity exist independently of matter, then I think that the hypothesis of one fluid will not stand against that of two fluids. There are, I think, evidently what I may call two elements of power, of equal force and acting toward each other. But these powers may be distinguished only by *direction*, and may be no more separate than the north and south forces in the elements of a magnetic needle. They may be the polar points of the forces originally placed in the particles of matter."

It may be remarked that since the rise of the mathematical theory of electrostatics, the controversy between the supporters of the one-fluid and the two-fluid theories had become manifestly barren. The analytical equations, in which interest was now largely centred, could be interpreted equally well on either hypothesis; and there seemed to be little prospect of discriminating between them by any new experimental discovery. But a problem does not lose its fascination because it appears insoluble. "I said once to Faraday," wrote Stokes to his father-in-law in 1879, "as I sat beside him at a British Association dinner, that I thought a great step would be made when we should be able to say of electricity that which we say of light, in saying that it consists of undulations. He said to me he thought we were a long way off that yet."*

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* "His principal aim," said Helmholtz in the Faraday Lecture of 1881, "was to express in his new conceptions only facts, with the least possible use of hypothetical substances and forces. This was really a progress in general scientific method, destined to purify science from the last remains of metaphysics."

† Bence Jones's *Life of Faraday*, ii, p. 77.

For his next series of researches,† Faraday reverted to subjects which had been among the first to attract him as an apprentice attending Davy's lectures: the voltaic pile, and the relations of electricity to chemistry.

It was at this time generally supposed that the decomposition of a solution, through which an electric current is passed, is due primarily to attractive and repellent forces exercised on its molecules by the metallic terminals at which the current enters and leaves the solution. Such forces had been assumed both in the hypothesis of Grothuss and Davy, and in the rival hypothesis of De La Rive;‡ the chief difference between these being that whereas Grothuss and Davy supposed a chain of decompositions and recompositions in the liquid, De La Rive supposed the molecules adjacent to the terminals to be the only ones decomposed, and attributed to their fragments the power of travelling through the liquid from one terminal to the other.

To test this doctrine of the influence of terminals, Faraday moistened a piece of paper in a saline solution, and supported it in the air on wax, so as to occupy part of the interval between two needle-points which were connected with an electric machine. When the machine was worked, the current was conveyed between the needle-points by way of the moistened paper and the two air-intervals on either side of it; and under these circumstances it was found that the salt underwent decomposition. Since in this case no metallic terminals of any kind were in contact with the solution, it was evident that all hypotheses which attributed decomposition to the action of the terminals were untenable.

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* Stokes's *Scientific Correspondence*, vol. i, p. 353.

† *Exp. Res.*, § 450 (1833).

‡ Cf. pp. 78–9.

The ground being thus cleared by the demolition of previous theories, Faraday was at liberty to construct a theory of his own. He retained one of the ideas of Grothuss' and Davy's doctrine, namely, that a chain of decompositions and recombinations takes place in the liquid; but these molecular processes he attributed not to any action of the terminals, but to a power possessed by the electric current itself, at all places in its course through the solution. If as an example we consider neighbouring molecules *A, B, C, D*, ... of the compound—say water, which was at that time believed to be directly decomposed by the current—Faraday supposed that before the passage of the current the hydrogen of *A* would be in close union with the oxygen of *A*, and also in a less close relation with the oxygen atoms of *B, C, D*, . . . : these latter relations being conjectured to be the cause of the attraction of aggregation in solids and fluids.* When an electric current is sent through the liquid, the affinity of the hydrogen of *A* for the oxygen of *B* is strengthened, if *A* and *B* lie along the direction of the current; while the hydrogen of *A* withdraws some of its bonds from the oxygen of *A*, with which it is at the moment combined. So long as the hydrogen and oxygen of *A* remain in association, the state thus induced is merely one of polarization; but the compound molecule is unable to stand the strain thus imposed on it, and the hydrogen and oxygen of *A* part company from each other. Thus decompositions take place, followed by recombinations: with the result that after each exchange an oxygen atom associates itself with a partner nearer to the positive terminal, while a hydrogen atom associates with a partner nearer to the negative terminal.

This theory explains why, in all ordinary cases, the evolved substances appear only at the terminals; for the terminals are the limiting surfaces of the decomposing substance; and, except at them, every particle finds other particles having a contrary tendency with which it can combine. It also explains why, in numerous cases, the atoms of the evolved substances are not retained by the terminals (an obvious difficulty in the way of all theories which suppose the terminals to attract the atoms): for the evolved substances are expelled from the liquid, not drawn out by an attraction.

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* *Exp. Res.*, § 523.

Many of the perplexities which had harassed the older theories were at once removed when the phenomena were regarded from Faraday's point of view. Thus, mere mixtures (as opposed to chemical compounds) are not separated into their constituents by the electric current; although there would seem to be no reason why the Grothuss-Davy polar attraction should not operate as well on elements contained in mixtures as on elements contained in compounds.

In the latter part of the same year (1833) Faraday took up the subject again.* It was at this time that he introduced the terms which have ever since been generally used to describe the phenomena of electro-chemical decomposition. To the terminals by which the electric current passes into or out of the decomposing body he gave the name *electrodes.* The electrode of high potential, at which oxygen, chlorine, acids, &c., are evolved, he called the *anode,* and the electrode of low potential, at which metals, alkalis, and bases are evolved, the *cathode.* Those bodies which are decomposed directly by the current he named *electrolytes*; the parts into which they are decomposed, *ions*; the acid ions, which travel to the anode, he named *anions*; and the metallic ions, which pass to the cathode, *cations.*

Faraday now proceeded to test the truth of a supposition which he had published rather more than a year previously,† and which indeed had apparently been suspected by Gay-Lussac and Thénard‡ so early as 1811; namely, that the rate at which an electrolyte is decomposed depends solely on the intensity of the electric current passing through it, and not at all on the size of the electrodes or the strength of the solution. Having established the accuracy of this law,* he found by a comparison of different electrolytes that the mass of any ion liberated by a given quantity of electricity is proportional to its *chemical equivalent*, i.e. to the amount of it required to combine with some standard mass of some standard element. If an element is n-valent, so that one of its atoms can hold in combination n atoms of hydrogen, the chemical equivalent of this element may be taken to be $1/n$ of its atomic weight; and therefore Faraday's result may be expressed by saying that an electric current will liberate exactly one atom of the element in question in the time which it would take to liberate n atoms of hydrogen.†

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* *Exp. Res.*, § 661. † *Ibid.*, § 377 (Dec. 1832).

‡ *Recherches physico-chimiques faites sur la pile*; Paris, 1811, p. 12.

The quantitative law seemed to Faraday‡ to indicate that "the atoms of matter are in some way endowed or associated with electrical powers, to which they owe their most striking qualities, and amongst them their mutual chemical affinity." Looking at the facts of electrolytic decomposition from this point of view, he showed how natural it is to suppose that the electricity which passes through the electrolyte is the exact equivalent of that which is *possessed* by the atoms separated at the electrodes; which implies that *there is a certain absolute quantity of the electric power associated with each atom of matter.*

The claims of this splendid speculation he advocated with conviction. "The harmony," he wrote,§ "which it introduces into the associated theories of definite proportions and electrochemical affinity is very great. According to it, the equivalent weights of bodies are simply those quantities of them which contain equal quantities of electricity, or have naturally equal electric powers; it being the ELECTRICITY which *determines* the equivalent number, *because* it determines the combining force. Or, if we adopt the atomic theory or phraseology, then the atoms of bodies which are equivalent to each other in their ordinary chemical action, have equal quantities of electricity naturally associated with them. "But," he added, "I must confess I am jealous of the term *atom*: for though it is very easy to talk of atoms, it is very difficult to form a clear idea of their nature, especially when compound bodies are under consideration."

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* *Exp. Res.*, §§ 713–821.

† In the modern units, 96580 coulombs of electricity must pass round the circuit in order to liberate of each ion a number of grams equal to the quotient of the atomic weight by the valency.

‡ *Exp. Res.*, § 852.

§ *Ibid.*, § 869.

These discoveries and ideas tended to confirm Faraday in preferring, among the rival theories of the voltaic cell, that one to which all his antecedents and connexions predisposed him. The controversy between the supporters of Volta's contact hypothesis on the one hand, and the chemical hypothesis of Davy and Wollaston on the other, had now been carried on for a generation without any very decisive result. In Germany and Italy the contact explanation was generally accepted, under the influence of Christian Heinrich Pfaff, of Kiel (*b.* 1773, *d.* 1852), and of Ohm, and, among the younger men, of Gustav Theodor Fechner (*b.* 1801, *d.* 1887), of Leipzig,* and Stefano Marianini (*b.* 1790, *d.* 1866), of Modena. Among French writers De La Rive, of Geneva, was, as we have seen, active in support of the chemical hypothesis; and this side in the dispute had always been favoured by the English philosophers.

There is no doubt that when two different metals are put in contact, a difference of potential is set up between them without any apparent chemical action; but while the contact party regarded this as a direct manifestation of a "contact-force" distinct in kind from all other known forces of nature, the chemical party explained it as a consequence of chemical affinity or incipient chemical action between the metals and the surrounding air or moisture. There is also no doubt that the continued activity of a voltaic cell is always accompanied by chemical unions or decompositions; but while the chemical party asserted that these constitute the efficient source of the current, the contact party regarded them as secondary actions, and attributed the continual circulation of electricity to the perpetual tendency of the electromotive force of contact to transfer charge from one substance to another.

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* Johann Christian Poggendorff (*b.* 1796, *d.* 1877), of Berlin, for long the editor of the Annalen der Physik, leaned originally to the chemical side, but in 1838 became convinced of the truth of the contact theory, which he afterwards actively defended. Moritz Hermann Jacobi (*b.* 1801, *d.* 1874), of Dorpat, is also to be mentioned among its advocates.

Faraday's first series of investigations on this subject were made in 1834: *Exp. Res.*, series viii. In 1836 De La Rive followed on the same side with his *Recherches sur la Cause de l'Electr. Voltaique.* The views of Faraday and De La Rive were criticized by Pfaff, *Revision der Lehre vom Galvanismus*, Kiel, 1837, and by Fechner, Ann. d. Phys., xlii (1837), p. 481, and xliii (1838), p. 433: translated Phil. Mag., xiii (1838), pp. 205, 367. Faraday returned to the question in 1840, *Exp. Res.*, series xvi and xvii.

One of the most active supporters of the chemical theory among the English physicists immediately preceding Faraday was Peter Mark Roget (*b.* 1779, *d.* 1869), to whom are due two of the strongest arguments in its favour. In the first place, carefully distinguishing between the *quantity* of electricity put into circulation by a cell and the *tension* at which this electricity is furnished, he showed that the latter quantity depends on the "energy of the chemical action"*—a fact which, when taken together with Faraday's discovery that the quantity of electricity put into circulation depends on the amount of chemicals consumed, places the origin of voltaic activity beyond all question. Roget's principle was afterwards verified by Faraday† and by De La Rive‡; "the electricity of the voltaic pile is proportionate in its intensity to the intensity of the affinities concerned in its production," said the former in 1834; while De La Rive wrote in 1836, "The intensity of the currents developed in combinations and in decompositions is exactly proportional to the degree of affinity which subsists between the atoms whose combination or separation has given rise to these currents."

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* "The absolute quantity of electricity which is thus developed, and made to circulate, will depend upon a variety of circumstances, such as the extent of the surfaces in chemical action, the facilities afforded to its transmission, &c. But its degree of intensity, or *tension*, as it is often termed, will be regulated by other causes, and more especially by the energy of the chemical action." Roget's *Galvanism* (1832), § 70.

† *Exp. Res.*, §§ 908, 909, 916, 988, 1958.

‡ Annales de Chim., lxi (1836), p. 38.

Not resting here, however, Roget brought up another argument of far-reaching significance. "If," he wrote,* "there could exist a power having the property ascribed to it by the [contact] hypothesis, namely, that of giving continual impulse to a fluid in one constant direction, without being exhausted by its own action, it would differ essentially from all the other known powers in nature. All the powers and sources of motion, with the operation of which we are acquainted, when producing their peculiar effects, are expended in the same proportion as those effects are produced; and hence arises the impossibility of obtaining by their agency a perpetual effect; or, in other words, a perpetual motion. But the electro-motive force ascribed by Volta to the metals when in contact is a force which, as long as a free course is allowed to the electricity it sets in motion, is never expended, and continues to be exerted with undiminished power, in the production of a never-ceasing effect. Against the truth of such a supposition the probabilities are all but infinite."

This principle, which is little less than the doctrine of conservation of energy applied to a voltaic cell, was reasserted by Faraday. The process imagined by the contact school "would," he wrote, "indeed be a *creation of power*, like no other force in nature." In all known cases energy is not generated, but only transformed. There is no such thing in the world as "a pure creation of force; a production of power without a corresponding exhaustion of something to supply it."†

As time went on, each of the rival theories of the cell became modified in the direction of the other. The contact party admitted the importance of the surfaces at which the metals are in contact with the liquid, where of course the chief chemical action takes place; and the chemical party confessed their inability to explain the state of tension which subsists before the circuit is closed, without introducing hypotheses just as uncertain as that of contact force.

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* Roget's *Galvanism* (1832), § 113.

† *Exp. Res.*, § 2071 (1840).

Faraday's own view on this point* was that a plate of amalgamated zinc, when placed in dilute sulphuric acid, "has power so far to act, by its attraction for the oxygen of the particles in contact with it, as to place the similar forces already active between these and the other particles of oxygen and the particles of hydrogen in the water, in a peculiar state of tension or polarity, and probably also at the same time to throw those of its own particles which are in contact with the water into a similar but opposed state. Whilst this state is retained, no further change occurs: but when it is relieved by completion of the circuit, in which case the forces determined in opposite directions, with respect to the zinc and the electrolyte, are found exactly competent to neutralize each other, then a series of decompositions and recompositions takes place amongst the particles of oxygen and hydrogen which constitute the water, between the place of contact with the platina and the place where the zinc is active: these intervening particles being evidently in close dependence upon and relation to each other. The zinc forms a direct compound with those particles of oxygen which were, previously, in divided relation to both it and the hydrogen: the oxide is removed by the acid, and a fresh surface of zinc is presented to the water, to renew and repeat the action."

These ideas were developed further by the later adherents of the chemical theory, especially by Faraday's friend Christian Friedrich Schönbein,† of Basle (*b.* 1799, *d.* 1868), the discoverer of ozone. Schönbein made the hypothesis more definite by assuming that when the circuit is open, the molecules of water adjacent to the zinc plate are electrically polarized, the oxygen side of each molecule being turned towards the zinc and being negatively charged, while the hydrogen side is turned away from the zinc and is positively charged. In the third quarter of the nineteenth century, the general opinion was in favour of some such conception as this. Helmholtz* attempted to grasp the molecular processes more intimately by assuming that the different chemical elements have different attractive powers (exerted only at small distances) for the vitreous and resinous electricities: thus potassium and zinc have strong attractions for positive charges, while oxygen, chlorine, and bromine have strong attractions for negative electricity. This differs from Volta's original hypothesis in little else but in assuming two electric fluids where Volta assumed only one. It is evident that the contact difference of potential between two metals may be at once explained by Helmholtz's hypothesis, as it was by Volta's; and the activity of the voltaic cell may be referred to the same principles: for the two ions of which the liquid molecules are composed will also possess different attractive powers for the electricities, and may be supposed to be united respectively with vitreous and resinous charges. Thus when two metals are immersed in the liquid, the circuit being open, the positive ions are attracted to the negative metal and the negative ions to the positive metal, thereby causing a polarized arrangement of the liquid molecules near the metals. When the circuit is closed, the positively charged surface of the positive metal is dissolved into the fluid; and as the atoms carry their charge with them, the positive charge on the immersed surface of this metal must be perpetually renewed by a current flowing in the outer circuit.

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* *Exp. Res.*, § 949.

† Ann. d. Phys., lxxviii (1849), p. 289, translated Archives des sc. phys., xiii (1850), p. 192. Faraday and Schönbein for many years carried on a correspondence, which has been edited by G. W. A. Kahlbaum and F. V. Darbishire: London, Williams and Norgate.

It will be seen that Helmholtz did not adhere to Davy's doctrine of the electrical nature of chemical affinity quite as simply or closely as Faraday, who preferred it in its most direct and uncompromising form. "All the facts show us," he wrote,† "that that power commonly called chemical affinity can be communicated to a distance through the metals and certain forms of carbon; that the electric current is only another form of the forces of chemical affinity; that its power is in proportion to the chemical affinities producing it; that when it is deficient in force it may be helped by calling in chemical aid, the want in the former being made up by an equivalent of the latter; that, in other words, *the forces termed chemical affinity and electricity are one and the same.*"

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* In his celebrated memoir of 1847 on the Conservation of Energy.

† *Exp. Res.*, § 918.

In the interval between Faraday's earlier and later papers on the cell, some important results on the same subject were published by Frederic Daniell (*b.* 1790, *d.* 1845), Professor of Chemistry in King's College, London.* Daniell showed that when a current is passed through a solution of a salt in water, the ions which carry the current are those derived from the salt, and not the oxygen and hydrogen ions derived from the water; this follows since a current divides itself between different mixed electrolytes according to the difficulty of decomposing each, and it is known that pure water can be electrolysed only with great difficulty. Daniell further showed that the ions arising from (say) sodium sulphate are not represented by Na_2O and SO_3, but by Na and SO_4; and that in such a case as this, sulphuric acid is formed at the anode and soda at the cathode by secondary action, giving rise to the observed evolution of oxygen and hydrogen respectively at these terminals.

The researches of Faraday on the decomposition of chemical compounds placed between electrodes maintained at different potentials led him in 1837 to reflect on the behaviour of such substances as oil of turpentine or sulphur, when placed in the same situation. These bodies do not conduct electricity, and are not decomposed; but if the metallic faces of a condenser are maintained at a definite potential difference, and if the space between them is occupied by one of these insulating substances, it is found that the charge on either face depends on the nature of the insulating substance. If for any particular insulator the charge has a value ε times the value which it would have if the intervening body were air, the number ε may be regarded as a measure of the influence which the insulator exerts on the propagation of electrostatic action through it: it was called by Faraday the *specific inductive capacity* of the insulator.[*]

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* Phil. Trans., 1839, p. 97.

The discovery of this property of insulating substances or *dielectrics* raised the question as to whether it could be harmonized with the old ideas of electrostatic action. Consider, for example, the force of attraction or repulsion between two small electrically-charged bodies. So long as they are in air, the force is proportional to the inverse square of the distance; but if the medium in which they are immersed be partly changed—e.g., if a globe of sulphur be inserted in the intervening space—this law is no longer valid: the change in the dielectric affects the distribution of electric intensity throughout the entire field.

The problem could be satisfactorily solved only by forming a physical conception of the action of dielectrics: and such a conception Faraday now put forward.

The original idea had been promulgated long before by his master Davy. Davy, it will be remembered,[†] in his explanation of the voltaic pile, had supposed that at first, before chemical decompositions take place, the liquid plays a part analogous to that of the glass in a Leyden jar, and that in this is involved an electric polarization of the liquid molecules.[‡] This hypothesis was now developed by Faraday. Referring first to his own work on electrolysis, he asserted[§] that the behaviour of a dielectric is exactly the same as that of an electrolyte, up to the point at which the electrolyte breaks down under the electric stress; a dielectric being, in fact, a body which is capable of sustaining the stress without suffering decomposition.

"For," he argued,[||] "let the electrolyte be water, a plate of ice being coated with platina foil on its two surfaces, and these coatings connected with any continued source of the two electrical powers, the ice will charge like a Leyden arrangement, presenting a case of common induction, but no current will pass. If the ice be liquefied, the induction will now fall to a certain degree, because a current can now pass; but its passing is dependent upon a *peculiar molecular arrangement* of the particles consistent with the transfer of the elements of the electrolyte in opposite directions . . . As, therefore, in the electrolytic action, *induction* appeared to be the *first* step, and *decomposition* the *second* (the power of separating these steps from each other by giving the solid or fluid condition to the electrolyte being in our hands); as the induction was the same in its nature as that through air, glass, wax, &c., produced by any of the ordinary means; and as the whole effect in the electrolyte appeared to be an action of the particles thrown into a peculiar or polarized state, I was glad to suspect that common induction itself was in all cases an *action of contiguous particles*, and that electrical action at a distance (i.e., ordinary inductive action) never occurred except through the influence of the intervening matter."

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[*] *Exp. Res.*, § 1252 (1837). Cavendish had discovered specific inductive capacity long before, but his papers were still unpublished.

[†] Cf. p. 77.

[‡] This is expressly stated in Davy's *Elements of Chemical Philosophy* (1812), Div. i, § 7, where he lays it down that an essential "property of non-conductors" is "to receive electrical polarities."

[§] *Exp. Res.*, §§ 1164, 1338, 1343, 1621.

[||] *Exp. Res.*, § 1164.

Thus at the root of Faraday's conception of electrostatic induction lay this idea that the whole of the insulating medium through which the action takes place is in a state of polarization similar to that which precedes decomposition in an electrolyte. "Insulators," he wrote,* "may be said to be bodies whose particles can retain the polarized state, whilst conductors are those whose particles cannot be permanently polarized."

The conception which he at this time entertained of the polarization may be reconstructed from what he had already written concerning electrolytes. He supposed† that in the ordinary or unpolarized condition of a body, the molecules consist of atoms which are bound to each other by the forces of chemical affinity, these forces being really electrical in their nature; and that the same forces are exerted, though to a less

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[*] *Exp. Res.*, § 1338.

[†] This must not be taken to be more than an idea which Faraday mentioned as present to his mind. He declined as yet to formulate a definite hypothesis.

degree, between atoms which belong to different molecules, thus producing the phenomena of cohesion. When an electric field is set up, a change takes place in the distribution of these forces; some are strengthened and some are weakened, the effect being symmetrical about the direction of the applied electric force.

Such a polarized condition acquired by a dielectric when placed in an electric field presents an evident analogy to the condition of magnetic polarization which is acquired by a mass of soft iron when placed in a magnetic field; and it was therefore natural that in discussing the matter Faraday should introduce *lines of electric force*, similar to the lines of magnetic force which he had employed so successfully in his previous researches. A line of electric force he defined to be a curve whose tangent at every point has the same direction as the electric intensity.

The changes which take place in an electric field when the dielectric is varied may be very simply described in terms of lines of force. Thus if a mass of sulphur, or other substance of high specific inductive capacity, is introduced into the field, the effect is as if the lines of force tend to crowd into it: as W. Thomson (Kelvin) showed later, they are altered in the same way as the lines of flow of heat, in a case of steady conduction of heat, would be altered by introducing a body of greater conducting power for heat. By studying the figures of the lines of force in a great number of individual cases, Faraday was led to notice that they always dispose themselves as if they were subject to a mutual repulsion, or as if the tubes of force had an inherent tendency to dilate.*

It is interesting to interpret by aid of these conceptions the law of Priestley and Coulomb regarding the attraction between two oppositely-charged spheres. In Faraday's view, the medium intervening between the spheres is the seat of a system of stresses, which may be represented by an attraction or tension along the lines of electric force at every point, together with a mutual repulsion of these lines, or pressure laterally. Where a line of force ends on one of the spheres, its tension is exercised on the sphere: in this way, every surface-element of each sphere is pulled outwards. If the spheres were entirely removed from each other's influence, the state of stress would be uniform round each sphere, and the pulls on its surface-elements would balance, giving no resultant force on the sphere. But when the two spheres are brought into each other's presence, the unit lines of force become somewhat more crowded together on the sides of the spheres which face than on the remote sides, and thus the resultant pull on either sphere tends to draw it toward the other. When the spheres are at distances great compared with their radii, the attraction is nearly proportional to the inverse square of the distance, which is Priestley's law.

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* *Exp. Res.*, §§ 1224, 1297 (1837).

In the following year (1838) Faraday amplified* his theory of electrostatic induction, by making further use of the analogy with the induction of magnetism. Fourteen years previously Poisson had imagined† an admirable model of the molecular processes which accompany magnetization; and this was now applied with very little change by Faraday to the case of induction in dielectrics. "The particles of an insulating dielectric," he suggested,‡ "whilst under induction may be compared to a series of small magnetic needles, or, more correctly still, to a series of small insulated conductors. If the space round a charged globe were filled with a mixture of an insulating dielectric, as oil of turpentine or air, and small globular conductors, as shot, the latter being at a little distance from each other so as to be insulated, then these would in their condition and action exactly resemble what I consider to be the condition and action of the particles of the insulating dielectric itself. If the globe were charged, these little conductors would all be polar; if the globe were discharged, they would all return to their normal state, to be polarized again upon the recharging of the globe."

That this explanation accounts for the phenomena of specific inductive capacity may be seen by what follows, which is substantially a translation into electrostatical language of Poisson's theory of induced magnetism.[*]

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* *Exp. Res.*, Series xiv. † Cf. p. 65. ‡ *Exp. Res.*, § 1679.

Let ρ denote volume-density of electric charge. For each of Faraday's "small shot" the integral

$$\iiint \rho dx\, dy\, dz,$$

integrated throughout the shot, will vanish, since the total charge of the shot is zero: but if $\mathbf{r}$ denote the vector (x, y, z), the integral

$$\iiint \rho\, \mathbf{r}\, dx\, dy\, dz$$

will not be zero, since it represents the electric polarization of the shot: if there are N shot per unit volume, the quantity

$$\mathbf{P} = N \iiint \rho\, \mathbf{r}\, dx\, dy\, dz$$

will represent the total polarization per unit volume. If $\mathbf{d}$ denote the electric force, and $\mathbf{E}$ the average value of $\mathbf{d}$, $\mathbf{P}$ will be proportional to $\mathbf{E}$, say

$$\mathbf{P} = (\epsilon - 1)\, \mathbf{E}.$$

By integration by parts, assuming all the quantities concerned to vary continuously and to vanish at infinity, we have

$$\iiint \left(P_x \frac{\partial}{\partial x} + P_y \frac{\partial}{\partial y} + P_z \frac{\partial}{\partial z} \right) \phi\, (x, y, z)\, dx\, dy\, dz = - \iiint \phi \operatorname{div} \mathbf{P}\, dx\, dy\, dz,$$

where ϕ denotes an arbitrary function, and the volume-integrals are taken throughout infinite space. This equation shows that the polar-distribution of electric charge on the shot is equivalent to a volume-distribution throughout space, of density

$$\bar{\rho} = - \operatorname{div} \mathbf{P}.$$

Now the fundamental equation of electrostatics may in suitable units be written,

$$\operatorname{div} \mathbf{d} = \rho;$$

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[*] W. Thomson (Kelvin), Camb. and Dub. Math. Journal, November, 1845; W. Thomson's *Papers on Electrostatics and Magnetism*, § 43 sqq.; F. O. Mossotti, Arch. des sc. phys. (Geneva) vi (1847), p. 193; Mem. della Soc. Ital. Modena, (2) xiv (1850), p. 49.

and this gives on averaging

$$\operatorname{div} \mathbf{E} = \rho_1 + \bar{\rho},$$

where ρ_1 denotes the volume-density of free electric charge, i.e. excluding that in the doublets; or

$$\operatorname{div} (\mathbf{E} + \mathbf{P}) = \rho_1,$$

or

$$\operatorname{div} (\varepsilon \mathbf{E}) = \rho_1.$$

This is the fundamental equation of electrostatics, as modified in order to take into account the effect of the specific inductive capacity ε.

The conception of action propagated step by step through a medium by the influence of contiguous particles had a firm hold on Faraday's mind, and was applied by him in almost every part of physics. "It appears to me possible," he wrote in 1838,[*] "and even probable, that magnetic action may be communicated to a distance by the action of the intervening particles, in a manner having a relation to the way in which the inductive forces of static electricity are transferred to a distance; the intervening particles assuming for the time more or less of a peculiar condition, which (though with a very imperfect idea) I have several times expressed by the term *electro-tonic state*."[†]

The same set of ideas sufficed to explain electric currents. Conduction, Faraday suggested,[‡] might be "an action of contiguous particles, dependent on the forces developed in electrical excitement; these forces bring the particles into a state of tension or polarity;[§] and being in this state the contiguous particles have a power or capability of communicating these forces, one to the other, by which they are lowered and discharge occurs."

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[*] *Exp Res.*, § 1729.

[†] This name had been devised in 1831 to express the state of matter subject to magneto-electric induction; cf. *Exp. Res.*, § 60.

[‡] *Exp. Res.* iii, p. 513.

[§] As in electrostatic induction in dielectrics.

After working strenuously for the ten years which followed the discovery of induced currents, Faraday found in 1841 that his health was affected; and for four years he rested. A second period of brilliant discoveries began in 1845.

Many experiments had been made at different times by various investigators* with the purpose of discovering a connexion between magnetism and light. These had generally taken the form of attempts to magnetize bodies by exposure in particular ways to particular kinds of radiation; and a successful issue had been more than once reported, only to be negatived on re-examination.

The true path was first indicated by Sir John Herschel. After his discovery of the connexion between the outward form of quartz crystals and their property of rotating the plane of polarization of light, Herschel remarked that a rectilinear electric current, deflecting a needle to right and left all round it, possesses a helicoidal dissymmetry similar to that displayed by the crystals. "Therefore," he wrote,† "induction led me to conclude that a similar connexion exists, and must turn up somehow or other, between the electric current and polarized light, and that the plane of polarization would be deflected by magneto-electricity."

The nature of this connexion was discovered by Faraday, who so far back as 1834‡ had transmitted polarized light through an electrolytic solution during the passage of the current, in the hope of observing a change of polarization. This early attempt failed; but in September, 1845, he varied the experiment by placing a piece of heavy glass between the poles of an excited electro-magnet; and found that the plane of polarization of a beam of light was rotated when the beam travelled through the glass parallel to the lines of force of the magnetic field.§

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* e.g. by Morichini, of Rome, in 1813, Quart. Journ. Sci. xix, p. 338; by Samuel Hunter Christie, of Cambridge, in 1825, Phil. Trans., 1826, p. 219; and by Mary Somerville in the same year, Phil. Trans., 1826, p. 132.

† Sir. J. Herschel in Bence Jones's *Life of Faraday*, p. 205.

‡ *Exp. Res.*, § 951.

§ *Ib.*, § 2152.

In the year following Faraday's discovery, Airy* suggested a way of representing the effect analytically; as might have been expected, this was by modifying the equations which had been already introduced by Mac Cullagh for the case of naturally active bodies. In Mac Cullagh's equations

$$\begin{cases} \dfrac{\partial^2 Y}{\partial t^2} = c_1{}^2 \dfrac{\partial^2 Y}{\partial x^2} + \mu \dfrac{\partial^3 Z}{\partial x^3} \\ \dfrac{\partial^2 Z}{\partial t^2} = c_1{}^2 \dfrac{\partial^2 Z}{\partial x^2} - \mu \dfrac{\partial^3 Y}{\partial x^3}, \end{cases}$$

the terms $\partial^3 Z/\partial x^3$ and $\partial^3 Y/\partial x^3$ change sign with x, so that the rotation of the plane of polarization is always right-handed or always left-handed with respect to the direction of the beam. This is the case in naturally-active bodies; but the rotation due to a magnetic field is in the same absolute direction whichever way the light is travelling, so that the derivations with respect to x must be of even order. Airy proposed the equations

$$\begin{cases} \dfrac{\partial^2 Y}{\partial t^2} = c_1{}^2 \dfrac{\partial^2 Y}{\partial x^2} + \mu \dfrac{\partial Z}{\partial t} \\ \dfrac{\partial^2 Z}{\partial t^2} = c_1{}^2 \dfrac{\partial^2 Z}{\partial x^2} - \mu \dfrac{\partial Y}{\partial t}, \end{cases}$$

where μ denotes a constant, proportional to the strength of the magnetic field which is used to produce the effect. He remarked, however, that instead of taking $\mu\, \partial Z/\partial t$ and $\mu\, \partial Y/\partial t$ as the additional terms, it would be possible to take $\mu\, \partial^3 Z/\partial t^3$ and $\mu\, \partial^3 Y/\partial t^3$, or $\mu\, \partial^3 Z/\partial x^2 \partial t$ and $\mu\, \partial^3 Y/\partial x^2 \partial t$, or any other derivates in which the number of differentiations is odd with respect to t and even with respect to x. It may, in fact, be shown by the method previously applied to Mac Cullagh's formulae that, if the equations are

$$\begin{cases} \dfrac{\partial^2 Y}{\partial t^2} = c_1{}^2 \dfrac{\partial^2 Y}{\partial x^2} + \mu \dfrac{\partial^{r+s} Z}{\partial x^r \partial t^s} \\ \dfrac{\partial^2 Z}{\partial t^2} = c_1{}^2 \dfrac{\partial^2 Z}{\partial x^2} - \mu \dfrac{\partial^{r+s} Y}{\partial x^r \partial t^s}, \end{cases}$$

where $(r + s)$ is an odd number, the angle through which the plane of polarization rotates in unit length of path is a numerical multiple of

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* Phil. Mag. xxviii (1846) p. 469.

$$\frac{\mu}{\tau^{r+s-1}c_1^{r+1}},$$

where τ denotes the period of the light. Now it was shown by Verdet* that the magnetic rotation is approximately proportional to the inverse square of the wave-length; and hence we must have

$$r + s = 3;$$

so that the only equations capable of correctly representing Faraday's effect are either

$$\begin{cases} \dfrac{\partial^2 Y}{\partial t^2} = c_1^2 \dfrac{\partial^2 Y}{\partial x^2} + \mu \dfrac{\partial^3 Z}{\partial x^2 \partial t} \\ \dfrac{\partial^2 Z}{\partial t^2} = c_1^2 \dfrac{\partial^2 Z}{\partial x^2} - \mu \dfrac{\partial^3 Y}{\partial x^2 \partial t} \end{cases}$$

or

$$\begin{cases} \dfrac{\partial^2 Y}{\partial t^2} = c_1^2 \dfrac{\partial^2 Y}{\partial x^2} + \mu \dfrac{\partial^3 Z}{\partial t^3} \\ \dfrac{\partial^2 Z}{\partial t^2} = c_1^2 \dfrac{\partial^2 Z}{\partial x^2} - \mu \dfrac{\partial^3 Y}{\partial t^3}. \end{cases}$$

The former pair arise, as will appear later, in Maxwell's theory of rotatory polarization: the latter pair, which were suggested in 1868 by Boussinesq,† follow from that physical theory of the phenomenon which is generally accepted at the present time.‡

Airy's work on the magnetic rotation of light was limited in the same way as MacCullagh's work on the rotatory power of quartz; it furnished only an analytical representation of the effect, without attempting to justify the equations. The earliest endeavour to provide a physical theory seems to have been made in 1858, in the inaugural dissertation of Carl Neumann, of Halle.* Neumann assumed that every element of an electric current exerts force on the particles of the aether; and in particular that this is true of the molecular currents which constitute magnetization, although in this case the force vanishes except when the aethereal particle is already in motion. If $\mathbf{e}$ denote the displacement of the aethereal particle m, the force in question may be represented by the term

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* Comptes Rendus, lvi (1863), p. 630.

† *Journal de Math.*, xiii (1868), p. 430.

‡ Y and Z being interpreted as components of electric force.

$$km\,[\,\dot{\mathbf{e}}\,.\,\mathbf{K}]$$

where $\mathbf{K}$ denotes the imposed magnetic field, and k denotes a magneto-optic constant characteristic of the body. When this term is introduced into the equations of motion of the aether, they take the form which had been suggested by Airy; whence Neumann's hypothesis is seen to lead to the incorrect conclusion that the rotation is independent of the wave-length.

The rotation of plane-polarized light depends, as Fresnel had shown,† on a difference between the velocities of propagation of the right-handed and left-handed circularly polarized waves into which plane-polarized light may be resolved. In the case of magnetic rotation, this difference was shown by Verdet to be proportional to the component of the magnetic force in the direction of propagation of the light; and Cornu‡ showed further that the mean of the velocities of the right-handed and left-handed waves is equal to the velocity of light in the medium when there is no magnetic field. From these data, by Fresnel's geometrical method, the wave-surface in the medium may be obtained; it is found to consist of two spheres (one relating to the right-handed and one to the left-handed light), each identical with the spherical wave-surface of the unmagnetized medium, displaced from each other along the lines of magnetic force.§

The discovery of the connexion between magnetism and light gave interest to a short paper of a speculative character which Faraday published[*] in 1846, under the title "Thoughts on Ray-Vibrations." In this it is possible to trace the progress of Faraday's thought towards something like an electro-magnetic theory of light.

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* *Explicare tentatur, quomodo fiat, ut lucis planum polarisationis per vires el. vel mag. declinetur. Halis Saxonum,* 1858. The results were republished in a tract *Die magnetische Drehung der Polarisationsebene des Lichtes.* Halle, 1863.

† Cf. p. 174.

‡ Comptes Rendus, xcii (1881), p. 1368.

§ Cornu, Comptes Rendus, xcix (1884), p. 1045.

Considering first the nature of ponderable matter, he suggests that an ultimate atom may be nothing else than a field of force—electric, magnetic, and gravitational—surrounding a point-centre; on this view, which is substantially that of Michell and Boscovich, an atom would have no definite size, but ought rather to be conceived of as completely penetrable, and extending throughout all space; and the molecule of a chemical compound would consist not of atoms side by side, but of "spheres of power mutually penetrated, and the centres even coinciding."[†]

All space being thus permeated by lines of force, Faraday suggested that light and radiant heat might be transverse vibrations propagated along these lines of force. In this way he proposed to "dismiss the aether," or rather to replace it by lines of force between centres, the centres together with their lines of force constituting the particles of material substances.

If the existence of a luminiferous aether were to be admitted, Faraday suggested that it might be the vehicle of magnetic force; "for," he wrote in 1851,[‡] "it is not at all unlikely that if there be an aether, it should have other uses than simply the conveyance of radiations." This sentence may be regarded as the origin of the electro-magnetic theory of light.

At the time when the "Thoughts on Ray-Vibrations" were published, Faraday was evidently trying to comprehend everything in terms of lines of force; his confidence in which had been recently justified by another discovery. A few weeks after the first observation of the magnetic rotation of light, he noticed[§] that a bar of the heavy glass which had been used in this investigation, when suspended between the poles of an electro-magnet, set itself *across* the line joining the poles: thus behaving in the contrary way to a bar of an ordinary magnetic substance, which would tend to set itself *along* this line. A simpler manifestation of the effect was obtained when a cube or sphere of the substance was used; in such forms it showed a disposition to move from the stronger to the weaker places of the magnetic field. The pointing of the bar was then seen to be merely the resultant of the tendencies of each of its particles to move outwards into the positions of weakest magnetic action.

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[*] Phil. Mag. (3), xxviii (1846): *Exp. Res.*, iii, p. 447.

[†] Cf. Bence Jones's *Life of Faraday*, ii, p. 178.

[‡] *Exp. Res.*, § 3075.

[§] Phil. Trans., 1846, p. 21: *Exp. Res.*, § 2253.

Many other bodies besides heavy glass were found to display the same property; in particular, bismuth.[*] The name *diamagnetic* was given to them.

"Theoretically," remarked Faraday, "an explanation of the movements of the diamagnetic bodies might be offered in the supposition that magnetic induction caused in them a contrary state to that which it produced in magnetic matter; i.e. that if a particle of each kind of matter were placed in the magnetic field, both would become magnetic, and each would have its axis parallel to the resultant of magnetic force passing through it; but the particle of magnetic matter would have its north and south poles opposite, or facing toward the contrary poles of the inducing magnet, whereas with the diamagnetic particles the reverse would be the case; and hence would result approximation in the one substance, recession in the other. Upon Ampère's theory, this view would be equivalent to the supposition that, as currents are induced in iron and magnetics parallel to those existing in the inducing magnet or battery wire, so in bismuth, heavy glass, and diamagnetic bodies, the currents induced are in the contrary direction."

This explanation became generally known as the "hypothesis of diamagnetic polarity"; it represents diamagnetism as similar to ordinary induced magnetism in all respects, except that the direction of the induced polarity is reversed. It was accepted by other investigators, notably by W. Weber, Plücker, Reich, and Tyndall; but was afterwards displaced from the favour of its inventor by another conception, more agreeable to his peculiar views on the nature of the magnetic field. In this second hypothesis, Faraday supposed an ordinary magnetic or *paramagnetic*[*] body to be one which offers a specially easy passage to lines of magnetic force, so that they tend to crowd into it in preference to other bodies; while he supposed a diamagnetic body to have a low degree of conducting power for the lines of force, so that they tend to avoid it. "If, then," he reasoned,[†] "a medium having a certain conducting power occupy the magnetic field, and then a portion of another medium or substance be placed in the field having a greater conducting power, the latter will tend to draw up towards the place of greatest force, displacing the former." There is an electrostatic effect to which this is quite analogous; a charged body attracts a body whose specific inductive capacity is greater than that of the surrounding medium, and repels a body whose specific inductive capacity is less; in either case the tendency is to afford the path of best conductance to the lines of force.[‡]

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[*] The repulsion of bismuth in the magnetic field had been previously observed by A. Brugmans in 1778; *Antonii Brugmans Magnetismus*, Lugd. Bat., 1778.

[†] *Exp. Res.*, § 2429.

For some time the advocates of the "polarity" and "conduction" theories of diamagnetism carried on a controversy which, indeed, like the controversy between the adherents of the one-fluid and two-fluid theories of electricity, persisted after it had been shown that the rival hypotheses were mathematically equivalent, and that no experiment could be suggested which would distinguish between them.

Meanwhile new properties of magnetizable bodies were being discovered. In 1847 Julius Plücker (*b.* 1801, *d.* 1868), Professor of Natural Philosophy in the University of Bonn, while repeating and extending Faraday's magnetic experiments, observed* that certain uniaxal crystals, when placed between the two poles of a magnet, tend to set themselves so that the optic axis has the equatorial position. At this time Faraday was continuing his researches; and, while investigating the diamagnetic properties of bismuth, was frequently embarrassed by the occurrence of anomalous results. In 1848 he ascertained that these were in some way connected with the crystalline form of the substance, and showed† that when a crystal of bismuth is placed in a field of uniform magnetic force (so that no tendency to motion arises from its diamagnetism) it sets itself so as to have one of its crystalline axes directed along the lines of force.

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[*] This term was introduced by Faraday, *Exp. Res.*, § 2790.

[†] *Exp. Res.*, § 2798.

[‡] The mathematical theory of the motion of a magnetizable body in a non-uniform field of force was discussed by W. Thomson (Kelvin) in 1847.

At first he supposed this effect to be distinct from that which had been discovered shortly before by Plücker. "The results," he wrote,‡ "are altogether very different from those produced by diamagnetic action. They are equally distinct from those discovered and described by Plücker, in his beautiful researches into the relation of the optic axis to magnetic action; for there the force is equatorial, whereas here it is axial. So they appear to present to us a new force, or a new form of force, in the molecules of matter, which, for convenience sake, I will conventionally designate by a new word, as the *magnecrystallic* force." Later in the same year, however, he recognized§ that "the phaenomena discovered by Plücker and those of which I have given an account have one common origin and cause."

The idea of the "conduction" of lines of magnetic force by different substances, by which Faraday had so successfully explained the phenomena of diamagnetism, he now applied to the study of the magnetic behaviour of crystals. "If," he wrote,‖ "the idea of conduction be applied to these magnecrystallic bodies, it would seem to satisfy all that requires explanation in their special results. A magnecrystallic substance would then be one which in the crystallized state could conduct onwards, or permit the exertion of the magnetic force with more facility in one direction than another; and that direction would be the magnecrystallic axis. Hence, when in the magnetic field, the magnecrystallic axis would be urged into a position coincident with the magnetic axis, by a force correspondent to that difference, just as if two different bodies were taken, when the one with the greater conducting power displaces that which is weaker."

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* Ann. d. Phys. lxxii (1847), p. 315; Taylor's *Scientific Memoirs*, v, p. 353.

† Phil. Trans., 1849, p. 1; *Exp. Res.*, § 2454.

‡ *Exp. Res.*, § 2469.

§ *Ibid.*, § 2605.

‖ *Ibid.*, § 2837.

This hypothesis led Faraday to predict the existence of another type of magnecrystallic effect, as yet unobserved. "If such a view were correct," he wrote,* "it would appear to follow that a diamagnetic body like bismuth ought to be less diamagnetic when its magnecrystallic axis is parallel to the magnetic axis than when it is perpendicular to it. In the two positions it should be equivalent to two substances having different conducting powers for magnetism, and therefore if submitted to the differential balance ought to present differential phaenomena." This expectation was realized when the matter was subjected to the test of experiment.†

The series of Faraday's "Experimental Researches in Electricity" end in the year 1855. The closing period of his life was quietly spent at Hampton Court, in a house placed at his disposal by the kindness of the Queen; and here on August 25th, 1867, he passed away.

Among experimental philosophers Faraday holds by universal consent the foremost place. The memoirs in which his discoveries are enshrined will never cease to be read with admiration and delight; and future generations will preserve with an affection not less enduring the personal records and familiar letters, which recall the memory of his humble and unselfish spirit.

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* *Exp. Res.*, § 2839.

† *Ibid.*, § 2841.

CHAPTER VII.

THE MATHEMATICAL ELECTRICIANS OF THE MIDDLE OF THE NINETEENTH CENTURY.

WHILE Faraday was engaged in discovering the laws of induced currents in his own way, by use of the conception of lines of force, his contemporary Franz Neumann was attacking the same problem from a different point of view. Neumann preferred to take Ampère as his model; and in 1845 published a memoir,* in which the laws of induction of currents were deduced by the help of Ampère's analysis.

Among the assumptions on which Neumann based his work was a rule which had been formulated, not long after Faraday's original discovery, by Emil Lenz,† and which may be enunciated as follows: when a conducting circuit is moved in a magnetic field, the induced current flows in such a direction that the ponderomotive forces on it tend to oppose the motion.

Let ds denote an element of the circuit which is in motion, and let Cds denote the component, taken in the direction of motion, of the ponderomotive force exerted by the inducing current on ds, when the latter is carrying unit current; so that the value of C is known from Ampère's theory. Then Lenz's rule requires that the product of C into the strength of the induced current should be negative. Neumann assumed that this is because it consists of a negative coefficient multiplying the square of C; that is, he assumed the induced electromotive force to be proportional to C. He further assumed it to be proportional to the velocity v of the motion; and thus obtained for the electromotive force induced in ds the expression

$$-\epsilon vCds,$$

where ϵ denotes a constant coefficient. By aid of this formula, in the earlier part* of the memoir, he calculated the induced currents in various particular cases.

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* Berlin Abhandlungen, 1845, p. 1; 1848, p. 1; reprinted as No. 10 and No. 36 of Ostwald's *Klassiker*; translated Journal de Math. xiii (1848), p. 113.

† Ann. d. Phys. xxxi (1834), p. 483.

But having arrived at the formulae in this way, Neumann noticed† a peculiarity in them which suggested a totally different method of treating the subject. In fact, on examining the expression for the current induced in a circuit which is in motion in the field due to a magnet, it appeared that this induced current depends only on the alteration caused by the motion in the value of a certain function; and, moreover, that this function is no other than the potential of the ponderomotive forces which, according to Ampère's theory, act between the circuit, supposed traversed by unit current, and the magnet.

Accordingly, Neumann now proposed to reconstruct his theory by taking this potential function as the foundation.

The nature of Neumann's potential, and its connexion with Faraday's theory, will be understood from the following considerations:—

The potential energy of a magnetic molecule **M** in a field of magnetic intensity **B** is (**B**.**M**); and therefore the potential energy of a current i flowing in a circuit s in this field is

$$i \iint_S (\mathbf{B} \cdot \mathbf{dS}),$$

where S denotes a diaphragm bounded by the circuit s, as is seen at once on replacing the circuit by its equivalent magnetic shell S. If the field **B** be produced by a current i' flowing in a circuit s', we have, by the formula of Biot and Savart,

$$\mathbf{B} = i' \int_{s'} \frac{[\mathbf{ds}' \cdot \mathbf{r}]}{r^3}$$

$$= i' \int_{s'} \operatorname{curl} \frac{\mathbf{ds}'}{r}.$$

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* §§ 1–8. It may be remarked that Neumann, in making use of Ohm's law, was (like everyone else at this time) unaware of the identity of electroscopic force with electrostatic potential.

† § 9.

Hence, the mutual potential energy of the two currents is

$$ii' \iiint_{s' S} \left(\operatorname{curl} \frac{\mathbf{ds}'}{r} \cdot \mathbf{dS}\right),$$

which by Stokes's transformation may be written in the form

$$ii' \int_s \int_{s'} \frac{(\mathbf{ds} \cdot \mathbf{ds}')}{r}.$$

This expression represents the amount of mechanical work which must be performed against the electro-dynamic ponderomotive forces, in order to separate the two circuits to an infinite distance apart, when the current-strengths are maintained unaltered.

The above potential function has been obtained by considering the ponderomotive forces; but it can now be connected with Faraday's theory of induction of currents. For by interpreting the expression

$$\iint_S (\mathbf{B} \cdot \mathbf{dS})$$

in terms of lines of force, we see that the potential function represents the product of i into the number of unit-lines of magnetic force due to s', which pass through the gap formed by the circuit s; and since by Faraday's law the currents induced in s depend entirely on the variation in the number of these lines, it is evident that the potential function supplies all that is needed for the analytical treatment of the induced currents. This was Neumann's discovery.

The electromotive force induced in a circuit s by the motion of other circuits s', carrying currents i', is thus proportional to the time-rate of variation of the potential

$$i' \int_s \int_{s'} \frac{(\mathbf{ds} \cdot \mathbf{ds}')}{r};$$

so that if we denote by **a** the vector

$$i' \int_{s'} \frac{\mathbf{ds}'}{r},$$

which, of course, is a function of the position of the element ds from which r is measured, then the electromotive force induced in any circuit-element ds by any alteration in the currents which give rise to $\mathbf{a}$ is

$$(\dot{\mathbf{a}} \, . \, \mathbf{ds}).$$

The induction of currents is therefore governed by the vector $\mathbf{a}$; this, which is generally known as the *vector-potential,* has from Neumann's time onwards played a great part in electrical theory. It may be readily interpreted in terms of Faraday's conceptions; for $(\mathbf{a} \, . \, \mathbf{ds})$ represents the total number of unit lines of magnetic force which have passed across the line-element $\mathbf{ds}$ prior to the instant t. The vector-potential may in fact be regarded as the analytical measure of Faraday's *electrotonic state.*[*]

While Neumann was endeavouring to comprehend the laws of induced currents in an extended form of Ampère's theory, another investigator was attempting a still more ambitious project: no less than that of uniting electrodynamics into a coherent whole with electrostatics.

Wilhelm Weber (*b.* 1804, *d.* 1890) was in the earlier part of his scientific career a friend and colleague of Gauss at Göttingen. In 1837, however, he became involved in political trouble. The union of Hanover with the British Empire, which had subsisted since the accession of the Hanoverian dynasty to the British throne, was in that year dissolved by the operation of the Salic law; the Princess Victoria succeeded to the crown of England, and her uncle Ernest-Augustus to that of Hanover. The new king, who was a pronounced reactionary, revoked the free constitution which the Hanoverians had for some time enjoyed; and Weber, who took a prominent part in opposing this action, was deprived of his professorship. From 1843 to 1849, when his principal theoretical researches in electricity were made, he occupied a chair in the University of Leipzig.

The theory of Weber was in its origin closely connected with the work of another Leipzig professor, Fechner, who in 1845[†] introduced certain assumptions regarding the nature of electric currents. Fechner supposed every current to consist in a streaming of electric charges, the vitreous charges travelling in one direction, and the resinous charges, equal to them in magnitude and number, travelling in the opposite direction with equal velocity. He further supposed that like charges attract each other when they are moving parallel to the same direction, while unlike charges attract when they are moving in opposite directions. On these assumptions he succeeded in bringing Faraday's induction effects into connexion with Ampère's laws of electrodynamics.

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[*] Cf. pp. 212, 272.

[†] Ann. d. Phys. lxiv (1845), p. 337.

In 1846 Weber,* adopting the same assumptions as Fechner, analysed the phenomena in the following way:—

The formula of Ampère for the ponderomotive force between two elements ds, ds' of currents i, i', may be written

$$F = ii' \, ds \, ds' \left(\frac{2}{r} \frac{d^2 r}{ds \, ds'} - \frac{1}{r^2} \frac{dr}{ds} \frac{dr}{ds'} \right).$$

Suppose now that λ units of vitreous electricity are contained in unit length of the wire s, and are moving with velocity u; and that an equal quantity of resinous electricity is moving with velocity u in the opposite direction; so that

$$i = 2\lambda u.$$

Let λ', u', denote the corresponding quantities for the other current; and let the suffix $_1$ be taken to refer to the action between the positive charges in the two wires, the suffix $_2$ to the action between the positive charge in s and the negative charge in s', the suffix $_3$ to the action between the negative charge in s and the positive charge in s', and the suffix$_4$ to the action between the negative charges in the two wires. Then we have

$$\left(\frac{dr}{dt} \right)_1 = u \frac{dr}{ds} + u' \frac{dr}{ds'},$$

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* *Elektrodynamische Maassbestimmungen,* Leipzig Abhandl., 1846: Ann. d. Phys. lxxiii (1848), p. 193: English translation in Taylor's *Scientific Memoirs,* v (1852), p. 489.

and

$$\left(\frac{d^2r}{dt^2}\right)_1 = u^2\frac{d^2r}{ds^2} + 2uu'\frac{d^2r}{ds\,ds'} + u'^2\frac{d^2r}{ds'^2}.$$

By aid of these and the similar equations with the suffixes $_2$, $_3$, $_4$, the equation for the ponderomotive force may be transformed into the equation

$$F = \frac{\lambda\lambda'\,ds\,ds'}{r^2}\left\{\begin{array}{l}\left(r\frac{d^2r}{dt^2}\right)_1 - \left(r\frac{d^2r}{dt^2}\right)_2 - \left(r\frac{d^2r}{dt^2}\right)_3 + \left(r\frac{d^2r}{dt^2}\right)_4 \\ -\frac{1}{2}\left(\frac{dr}{dt}\right)_1^2 + \frac{1}{2}\left(\frac{dr}{dt}\right)_2^2 + \frac{1}{2}\left(\frac{dr}{dt}\right)_3^2 - \frac{1}{2}\left(\frac{dr}{dt}\right)_4^2\end{array}\right\}.$$

But this is the equation which we should have obtained had we set out from the following assumptions: that the ponderomotive force between two current-elements is the resultant of the force between the positive charge in *ds* and the positive charge in *ds'*, of the force between the positive charge in *ds* and the negative charge in *ds'*, etc.; and that any two electrified particles of charges *e* and *e'*, whose distance apart is *r*, repel each other with a force of magnitude

$$\frac{ee'}{r^2}\left\{r\frac{d^2r}{dt^2} - \frac{1}{2}\left(\frac{dr}{dt}\right)^2\right\}.$$

Two such charges would, of course, also exert on each other an electrostatic repulsion, whose magnitude in these units would be $ee'c^2/r^2$, where c denotes a constant* of the dimensions of a velocity, whose value is approximately 3×10^{10} cm./sec. So that on these assumptions the total repellent force would be

$$\frac{ee'c^2}{r^2}\left(1 + \frac{r\ddot{r}}{c^2} - \frac{\dot{r}^2}{2c^2}\right).$$

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* The units which have been adopted in the above investigation depend on the electrodynamic actions of currents; i.e. they are such that two unit currents flowing in parallel circular circuits at a certain distance apart exert unit ponderomotive force on each other. The quantity of electricity conveyed in unit time by such a unit current is adopted as the unit charge. This unit charge is not identical with the electrostatic unit charge, which is defined to be such that two unit charges at unit distance apart repel each other with unit ponderomotive force. Hence the necessity for introducing the factor *c*.

This expression for the force between two electric charges was taken by Weber as the basis of his theory. Weber's is the first of the *electron-theories*—a name given to any theory which attributes the phenomena of electrodynamics to the agency of moving electric charges, the forces on which depend not only on the position of the charges (as in electrostatics), but also on their velocity.

The latter feature of Weber's theory led its earliest critics to deny that his law of force could be reconciled with the principle of conservation of energy. They were, however, mistaken on this point, as may be seen from the following considerations. The above expression for the force between two charges may be written in the form

$$-\frac{\partial U}{\partial r} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{r}}\right),$$

where U denotes the expression

$$\frac{ee'c^2}{r}\left(1 + \frac{\dot{r}^2}{2c^2}\right).$$

Consider now two material particles at distance r apart, whose mechanical kinetic energy is T, and whose mechanical potential energy is V, and which carry charges e and e'. The equations of motion of these particles will be exactly the same as the equations of motion of a dynamical system for which the kinetic energy is

$$T - \frac{ee'\dot{r}^2}{2r},$$

and the potential energy is

$$V + \frac{ee'c^2}{r}.$$

To such a system the principle of conservation of energy may be applied: the equation of energy is, in fact,

$$T + V - \frac{1}{2r}ee'\dot{r}^2 + \frac{ee'c^2}{r} = \text{constant}.$$

The first objection made to Weber's theory is thus disposed of; but another and more serious one now presents itself. The occurrence of the negative sign with the term $-ee'\dot{r}^2/2r$ implies that a charge behaves somewhat as if its mass were negative, so that in certain circumstances its velocity might increase indefinitely under the action of a force opposed to the motion. This is one of the vulnerable points of Weber's theory, and has been the object of much criticism. In fact,* suppose that one charged particle of mass μ is free to move, and that the other charges are spread uniformly over the surface of a hollow spherical insulator in which the particle is enclosed. The equation of conservation of energy is

$$\tfrac{1}{2}(\mu - ep)v^2 + V = \text{constant},$$

where e denotes the charge of the particle, v its velocity, V its potential energy with respect to the mechanical forces which act on it, and p denotes the quantity

$$\iint \frac{\sigma}{r} \cos^2(v \overset{\wedge}{.} r)\, dS,$$

where the integration is taken over the sphere, and where σ denotes the surface-density; p is independent of the position of the particle μ within the sphere. If now the electric charge on the sphere is so great that ep is greater than μ, then v^2 and V must increase and diminish together, which is evidently absurd.

Leaving this objection unanswered, we proceed to show how Weber's law of force between electrons leads to the formulae for the induction of currents.

The mutual energy of two moving charges is

$$\frac{ee'c^2}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right),$$

or

$$\frac{ee'c^2}{r}\left[1 - \frac{\{(\mathbf{r}\,.\,\mathbf{v}') - (\mathbf{r}\,.\,\mathbf{v})\}^2}{2c^2r^2}\right],$$

where $\mathbf{v}$ and $\mathbf{v}'$ denote the velocities of the charges; so that the mutual energy of two current-elements containing charges e, e' respectively of each kind of electricity, is

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* This example was given by Helmholtz, Journal für Math. lxxv (1873), p. 35; Phil. Mag. xliv (1872), p. 530.

$$\frac{ee'}{2r^3}\left[-\{(\mathbf{r}.\mathbf{v}')-(\mathbf{r}.\mathbf{v})\}^2+\{(\mathbf{r}.\mathbf{v}')+(\mathbf{r}.\mathbf{v})\}^2+\{-(\mathbf{r}.\mathbf{v}')-(\mathbf{r}.\mathbf{v})\}^2-\{(-\mathbf{r}.\mathbf{v}')+(\mathbf{r}.\mathbf{v})\}^2\right],$$

or
$$\frac{4ee'(\mathbf{r}.\mathbf{v}')(\mathbf{r}.\mathbf{v})}{r^3}.$$

If ds, ds' denote the lengths of the elements, and i, i' the currents in them, we have

$$i\mathbf{ds} = 2e\mathbf{v}, \quad i'\mathbf{ds}' = 2e'\mathbf{v}';$$

so the mutual energy of two current-elements is

$$\frac{ii'}{r^3}(\mathbf{r}.\mathbf{ds}').(\mathbf{r}.\mathbf{ds}).$$

The mutual energy of $i\mathbf{ds}$ with all the other currents is therefore

$$i(\mathbf{ds}.\mathbf{a}),$$

where $\mathbf{a}$ denotes a vector-potential

$$\int_{s'} i'\frac{(\mathbf{r}.\mathbf{ds}').\mathbf{r}}{r^3}.$$

By reasoning similar to Neumann's, it may be shown that the electromotive force induced in $\mathbf{ds}$ by any alteration in the rest of the field is

$$-(\mathbf{ds}.\dot{\mathbf{a}});$$

and thus a complete theory of induced currents may be constructed.

The necessity for induced currents may be inferred by general reasoning from the first principles of Weber's theory. When a circuit s moves in the field due to currents, the velocity of the vitreous charges in s is, owing to the motion of s, not equal and opposite to that of the resinous charges : this gives rise to a difference in the forces acting on the vitreous and resinous charges in s; and hence the charges of opposite sign separate from each other and move in opposite directions.

The assumption that positive and negative charges move with equal and opposite velocities relative to the matter of the conductor is one to which, for various reasons which will appear later, objection may be taken; but it is an integral part of Weber's theory, and cannot be excised from it. In fact, if this condition were not satisfied, and if the law of force were Weber's, electric currents would exert forces on electrostatic charges at rest*; as may be seen by the following example. Let a current flow in a closed circuit formed by arcs of two concentric circles and the portions of the radii connecting their extremities; then, if Weber's law were true, and if only one kind of electricity were in motion, the current would evidently exert an electrostatic force on a charge placed at the centre of the circles. It has been shown,† indeed, that the assumption of opposite electricities moving with equal and opposite velocities in a circuit is almost inevitable in any theory of the type of Weber's, so long as the mutual action of two charges is assumed to depend only on their relative (as opposed to their absolute) motion.

The law of Weber is not the only one of its kind; an alternative to it was suggested by Bernhard Riemann (*b.* 1826, *d.* 1866), in a course of lectures which were delivered‡ at Göttingen in 1861, and which were published after his death by K. Hattendorff. Riemann proposed as the electrokinetic energy of two electrons $e(x, y, z)$ and $e'(x', y', z')$ the expression

$$-\frac{1}{2}\frac{ee'}{r}\{(\dot{x}-\dot{x}')^2+(\dot{y}-\dot{y}')^2+(\dot{z}-\dot{z}')^2\};$$

this differs from the corresponding expression given by Weber only in that the relative velocity of the two electrons is substituted in place of the component of this velocity along the radius vector. Eventually, as will be seen later, the laws of Riemann and Weber were both abandoned in favour of a third alternative.

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* This remark was first made by Clausius, Journal für Math. lxxxii (1877), p. 86: the simple proof given above is due to Grassmann, Journal für Math. lxxxiii (1877), p. 57.

† H. Lorberg, Journal für Math. lxxxiv (1878), p. 305.

‡ *Schwere, Elektricität und Magnetismus*, nach den Vorlesungen von B. Riemann: Hannover, 1875, p. 326. Another alternative to Weber's law had been discovered by Gauss so far back as 1835, but was not published until after his death: cf. Gauss' *Werke*, v, p. 616.

At the time, however, Weber's discovery was felt to be a great advance; and indeed it had, perhaps, the greatest share in awakening mathematical physicists to a sense of the possibilities latent in the theory of electricity. Beyond this, its influence was felt in general dynamics; for Weber's electrokinetic energy, which resembled kinetic energy in some respects and potential energy in others, could not be precisely classified under either head; and its introduction, by helping to break down the distinction which had hitherto subsisted between the two parts of the kinetic potential, prepared the way for the modern transformation-theory of dynamics.*

Another subject whose development was stimulated by the work of Weber was the theory of gravitation. That gravitation is propagated by the action of a medium, and consequently is a process requiring time for its accomplishment, had been an article of faith with many generations of physicists. Indeed, the dependence of the force on the distance between the attracting bodies seemed to suggest this idea; for a propagation which is truly instantaneous would, perhaps, be more naturally conceived to be effected by some kind of rigid connexion between the bodies, which would be more likely to give a force independent of the mutual distance.

It is obvious that, if the simple law of Newton is abandoned, there is a wide field of rival hypotheses from which to choose its successor. The first notable attempt to discuss the question was made by Laplace.† Laplace supposed gravity to be produced by the impulsion on the attracted body of a "gravific fluid," which flows with a definite velocity toward the centre of attraction—say, the sun. If the attracted body or planet is in motion, the velocity of the fluid relative to it will be compounded of the absolute velocity of the fluid and the reversed velocity of the planet, and the force of gravity will act in the direction thus determined, its magnitude being unaltered by the planet's motion. This amounts to supposing that gravity is subject to an aberrational effect similar to that observed in the case of light. It is easily seen that the modification thus introduced into Newton's law may be represented by an additional perturbing force, directed along the tangent to the orbit in the opposite sense to the motion, and proportional to the planet's velocity and to the inverse square of the distance from the sun. By considering the influence of this force on the secular equation of the moon's motion, Laplace found that the velocity of the gravific fluid must be at least a hundred million times greater than that of light.

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* Cf. Whittaker, *Analytical Dynamics*, chapters ii, iii, xi.

† *Mécanique Céleste*, Livre x, chap. vii, § 22.

The assumptions made by Laplace are evidently in the highest degree questionable; but the generation immediately succeeding, overawed by his fame, seems to have found no way of improving on them. Under the influence of Weber's ideas, however, astronomers began to think of modifying Newton's law by adding a term involving the velocities of the bodies. Tisserand* in 1872 discussed the motion of the planets round the sun on the supposition that the law of gravitation is the same as Weber's law of electrodynamic action, so that the force is

$$F = \frac{fm\mu}{r^2}\left\{1 - \frac{1}{h^2}\left(\frac{dr}{dt}\right)^2 + \frac{2}{h^2}r\frac{d^2r}{dt^2}\right\},$$

where f denotes the constant of gravitation, m the mass of the planet, μ the mass of the sun, r the distance of the planet from the sun, and h the velocity of propagation of gravitation. The equations of motion may be rigorously integrated by the aid of elliptic functions†; but the simplest procedure is to write

$$F = \frac{fm\mu}{r^2} + F_1,$$

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* Comptes Rendus, lxxv (1872), p. 760. Cf. also Comptes Rendus, cx (1890), p. 313, and Holzmüller, Zeitschrift für Math. u. Phys., 1870, p. 69.

† This had been done in an inaugural dissertation by Seegers, Göttingen, 1864.

and, regarding F_1 as a perturbing function, to find the variation of the constants of elliptic motion. Tisserand showed that the perturbations of all the elements are zero or periodic, and quite insensible, except that of the longitude of perihelion, which has a secular part. If h be assumed equal to the velocity of light, the effect would be to rotate the major axis of the orbit of Mercury in the direct sense 14″ in a century.

Now, as it happened, a discordance between theory and observation was known to exist in regard to the motion of Mercury's perihelion; for Le Verrier had found that the attraction of the planets might be expected to turn the perihelion 527″ in the direct sense in a century, whereas the motion actually observed was greater than this by 38″. It is evident, however, that only $\frac{3}{8}$ of the excess is explained by Tisserand's adoption of Weber's law; and it seemed therefore that this suggestion would prove as unprofitable as Le Verrier's own hypothesis of an intra-mercurial planet. But it was found later* that $\frac{3}{4}$ of the excess could be explained by substituting Riemann's electrodynamic law for Weber's, and that a combination of the laws of Riemann and Weber would give exactly the amount desired.†

After the publication of his memoir on the law of force between electrons, Weber turned his attention to the question of diamagnetism, and developed Faraday's idea regarding the explanation of diamagnetic phenomena by the effects of electric currents induced in the diamagnetic bodies.‡ Weber remarked that if, with Ampère, we assume the existence of molecular circuits in which there is no ohmic resistance, so that currents can flow without dissipation of energy, it is quite natural to suppose that currents would be induced in these molecular circuits if they were situated in a varying magnetic field; and he pointed out that such induced molecular currents would confer upon the substance the properties characteristic of diamagnetism.

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* By Maurice Lévy, Comptes Rendus, cx (1890), p. 545.

† The consequences of adopting the electrodynamic law of Clausius (for which see later) were discussed by Oppenheim, *Zur Frage nach der Fortpflanzungsgeschwindigkeit der Gravitation*, Wien, 1895.

‡ Leipzig Berichte, i (1847), p. 346; Ann. d. Phys. lxxiii (1848), p. 241; translated Taylor's *Scientific Memoirs*, v, p. 477; Abhandl. der K. Sächs. Ges. i (1852), p. 483; Ann. d. Phys. lxxxvii (1852), p. 145; trans. Tyndall and Francis' *Scientific Memoirs*, p. 163.

The difficulty with this hypothesis is to avoid explaining too much; for, if it be accepted, the inference seems to be that all bodies, without exception, should be diamagnetic. Weber escaped from this conclusion by supposing that in iron and other magnetic substances there exist permanent molecular currents, which do not owe their origin to induction, and which, under the influence of the impressed magnetic force, set themselves in definite orientations. Since a magnetic field tends to give such a direction to a pre-existing current that its course becomes opposed to that of the current which would be induced by the increase of the magnetic force, it follows that a substance stored with such pre-existing currents would display the phenomena of paramagnetism. The bodies ordinarily called paramagnetic are, according to this hypothesis, those bodies in which the paramagnetism is strong enough to mask the diamagnetism.

The radical distinction which Weber postulated between the natures of paramagnetism and diamagnetism accords with many facts which have been discovered subsequently. Thus in 1895 P. Curie showed* that the magnetic susceptibility per gramme-molecule is connected with the temperature by laws which are different for paramagnetic and diamagnetic bodies. For the former it varies in inverse proportion to the absolute temperature, whereas for diamagnetic bodies it is independent of the temperature.

The conclusions which followed from the work of Faraday and Weber were adverse to the hypothesis of magnetic fluids; for according to that hypothesis the induced polarity would be in the same direction whether due to a change of orientation of pre-existing molecular magnets, or to a fresh separation of magnetic fluids in the molecules. "Through the discovery of diamagnetism," wrote Weber* in 1852, "the hypothesis of electric molecular currents in the interior of bodies is corroborated, and the hypothesis of magnetic fluids in the interior of bodies is refuted." The latter hypothesis is, moreover, unable to account for the phenomena shown by bodies which are strongly magnetic, like iron: for it is found that when the magnetizing force is gradually increased to a very large value, the magnetization induced in such bodies does not increase in proportion, but tends to a saturation value This effect cannot be explained on the assumptions of Poisson, but is easily deducible from those of Weber; for, according to Weber's theory, the magnetizing force merely orients existing magnets; and when it has attained such a value that all of them are oriented in the same direction, there is nothing further to be done.

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* Annales de Chimie (7) v (1845), p. 289.

Weber's theory in its original form is, however, open to some objection. If the elementary magnets are supposed to be free to orient themselves without encountering any resistance, it is evident that a very small magnetizing force would suffice to turn them all parallel to each other, and thus would produce immediately the greatest possible intensity of induced magnetism. To overcome this difficulty, Weber assumed that every displacement of a molecular circuit is resisted by a couple, which tends to restore the circuit to its original orientation. This assumption fails, however, to account for the fact that iron which has been placed in a strong magnetic field does not return to its original condition when it is removed from the field, but retains a certain amount of residual magnetization.

Another alternative was to assume a frictional resistance to the rotation of the magnetic molecules; but if such a resistance existed, it could be overcome only by a finite magnetizing force; and this inference is inconsistent with the observation that some degree of magnetization is induced by every force, however feeble.

The hypothesis which has ultimately gained acceptance is that the orientation is resisted by couples which arise from the mutual action of the molecular magnets themselves. In the unmagnetized condition the molecules "arrange themselves so as to satisfy their mutual attraction by the shortest path, and thus form a complete closed circuit of attraction," as D. E. Hughes wrote* in 1883; when an external magnetizing force is applied, these small circuits are broken up; and at any stage of the process a molecular magnet is in equilibrium under the joint influence of the external force and the forces due to the other molecules.

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* Ann. d. Phys. lxxxvii (1852), p. 145; Tyndall and Francis' *Sci. Mem.*, p. 163.

This hypothesis was suggested by Maxwell,† and has been since developed by J. A. Ewing;‡ its consequences may be illustrated by the following simple example§:—

Consider two magnetic molecules, each of magnetic moment m, whose centres are fixed at a distance c apart. When undisturbed, they dispose themselves in the position of stable equilibrium, in which they point in the same direction along the line c. Now let an increasing magnetic force H be made to act on them in a direction at right angles to the line c. The magnets turn towards the direction of H; and when H attains the value $3m/c^3$, they become perpendicular to the line c, after which they remain in this position, when H is increased further. Thus they display the phenomena of induction initially proportional to the magnetizing force, and of saturation. If the magnetizing force H be supposed to act parallel to the line c, in the direction in which the axes originally pointed, the magnets will remain at rest. But if H acts in the opposite direction, the equilibrium will be stable only so long as H is less than m/c^3; when H increases beyond this limit, the equilibrium becomes unstable, and the magnets turn over so as to point in the direction of H; when H is gradually decreased to zero, they remain in their new positions, thus illustrating the phenomenon of residual magnetism.

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* Proc. Roy. Soc. xxxv (1883), p. 178.

† *Treatise on Elect. & Mag.*, § 443.

‡ Phil. Mag. xxx (1890), p. 205; *Magnetic Induction in Iron and other Metals*, 1891.

§ E. G. Gallop, Messenger of Math. xxvii (1897), p. 6.

By taking a large number of such pairs of magnetic molecules, originally oriented in all directions, and at such distances that the pairs do not sensibly influence each other, we may construct a model whose behaviour under the influence of an external magnetic field will closely resemble the actual behaviour of ferromagnetic bodies.

In order that the magnets in the model may come to rest in their new positions after reversal, it will be necessary to suppose that they experience some kind of dissipative force which damps the oscillations; to this would correspond in actual magnetic substances the electric currents which would be set up in the neighbouring mass when the molecular magnets are suddenly reversed; in either case, the sudden reversals are attended by a transformation of magnetic energy into heat.

The transformation of energy from one form to another is a subject which was first treated in a general fashion shortly before the middle of the nineteenth century. It had long been known that the energy of motion and the energy of position of a dynamical system are convertible into each other, and that the amount of their sum remains invariable when the system is self-contained. This principle of conservation of dynamical energy had been extended to optics by Fresnel, who had assumed* that the energy brought to an interface by incident light is equal to the energy carried away from the interface by the reflected and refracted beams. A similar conception was involved in Roget's and Faraday's defence† of the chemical theory of the voltaic cell; they argued that the work done by the current in the outer circuit must be provided at the expense of the chemical energy stored in the cell, and showed that the quantity of electricity sent round the circuit is proportional to the quantity of chemicals consumed, while its tension is proportional to the strength of the chemical affinities concerned in the reaction. This theory was extended and completed by James Prescott Joule, of Manchester, in 1841. Joule, who believed[*] that heat is producible from mechanical work and convertible into it, measured[†] the amount of heat evolved in unit time in a metallic wire, through which a current of known strength was passed; he found the amount to be proportional to the resistance of the wire multiplied by the square of the current-strength; or (as follows from Ohm's law) to the current-strength multiplied by the difference of electric tensions at the extremities of the wire.

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* Cf. p. 133.

† Cf. p. 203.

The quantity of energy yielded up as heat in the outer circuit being thus known, it became possible to consider the transference of energy in the circuit as a whole. "When," wrote Joule, "any voltaic arrangement, whether simple or compound, passes a current of electricity through any substance, whether an electrolyte or not, the total voltaic heat which is generated in any time is proportional to the number of atoms which are electrolyzed in each cell of the circuit, multiplied by the virtual intensity of the battery: if a decomposing cell be in the circuit, the virtual intensity of the battery is reduced in proportion to its resistance to electrolyzation." In the same year he[‡] enhanced the significance of this by showing that the quantities of heat which are evolved by the combustion of the equivalents of bodies are proportional to the intensities of their affinities for oxygen, as measured by the electromotive force of a battery required to decompose the oxide electrolytically.

The theory of Roget and Faraday, thus perfected by Joule, enables us to trace quantitatively the transformations of energy in the voltaic cell and circuit. The primary source of energy is the chemical reaction: in a Daniell cell, $Zn|Zn\,SO_4|Cu\,SO_4|Cu$, for instance, it is the substitution of zinc for copper as the partner of the sulphion. The strength of the chemical affinities concerned is in this case measured by the difference of the heats of formation of zinc sulphate and copper sulphate; and it is this which determines the electromotive force of the cell.* The amount of energy which is changed from the chemical to the electrical form in a given interval of time is measured by the product of the strength of the chemical affinity into the quantity of chemicals decomposed in that time, or (what is the same thing) by the product of the electromotive force of the cell into the quantity of electricity which is circulated. This energy may be either dissipated as heat in conformity to Joule's law, or otherwise utilized in the outer circuit.

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[*] Cf. p. 33.

[†] Phil. Mag. xix (1841), p. 260; Joule's *Scientific Papers* i, p. 60.

[‡] Phil. Mag. xx (1841), p. 98: cf. also Phil. Mag. xxii (1843), p. 204.

The importance of these principles was emphasized by Hermann von Helmholtz (*b.* 1821, *d.* 1894), in a memoir which was published in 1847, and which will be more fully noticed presently, and by W. Thomson (Lord Kelvin) in 1851†; the equations have subsequently received only one important modification, which is due to Helmholtz.‡ Helmholtz pointed out that the electrical energy furnished by a voltaic cell need not be derived exclusively from the energy of the chemical reactions: for the cell may also operate by abstracting heat-energy from neighbouring bodies, and converting this into electrical energy. The extent to which this takes place is determined by a law which was discovered in 1855 by Thomson.§ Thomson showed that if E denotes the "available energy," i.e., possible output of mechanical work, of a system maintained at the absolute temperature T, then a fraction

$$\frac{T}{E}\frac{dE}{dT}$$

of this work is obtained, not at the expense of the thermal or chemical energy of the system itself, but at the expense of the thermal energy of neighbouring bodies. Now in the case of the voltaic cell, the principle of Roget, Faraday, and Joule is expressed by the equation

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* The heat of formation of a gramme-molecule of $ZnSO_4$ is greater than the heat of formation of a gramme-molecule of $CuSO_4$ by about 50,000 calories; and with divalent metals, 46,000 calories per gramme-molecule corresponds to an e.m.f. of one volt; so the e.m.f. of a Daniell cell should be 50/46 volts, which is nearly the case.

† Kelvin's *Math. and Phys. Papers*, i, pp. 472, 490.

‡ Berlin Sitzungsber., 1882, pp. 22, 825; 1883, p. 647.

§ Quart. Journ. Math., April, 1855; Kelvin's *Math. and Phys. Papers*, i, p. 297, eqn. (7).

$$E = \lambda,$$

where E denotes the available or electrical energy, which is measured by the electromotive force of the cell, and where λ denotes the heat of the chemical reaction which supplies this energy. In accordance with Thomson's principle, we must replace this equation by

$$E = \lambda + T\frac{dE}{dT},$$

which is the correct relation between the electromotive force of a cell and the energy of the chemical reactions which occur in it. In general the term λ is much larger than the term $T\,dE/dT$; but in certain classes of cells—e.g., concentration-cells—λ is zero; in which case the whole of the electrical energy is procured at the expense of the thermal energy of the cells' surroundings.

Helmholtz's memoir of 1847, to which reference has already been made, bore the title, "On the Conservation of Force." It was originally read to the Physical Society of Berlin*; but though the younger physicists of the Society received it with enthusiasm, the prejudices of the older generation prevented its acceptance for the Annalen der Physik; and it was eventually published as a separate treatise.†

In this memoir it was asserted‡ that the conservation of energy is a universal principle of nature: that the kinetic and potential energy of dynamical systems may be converted into heat according to definite quantitative laws, as taught by Rumford, Joule, and Robert Mayer*; and that any of these forms of energy may be converted into the chemical, electrostatic, voltaic, and magnetic forms. The latter Helmholtz examined systematically.

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* On July 23rd, 1847.

† Berlin, G. A. Reimer. English Translation in Tyndall & Francis' *Scientific Memoirs*, p. 114. The publisher, to Helmholtz's "great surprise," gave him an honorarium. Cf. *Hermann von Helmholtz*, by Leo Koenigsberger; English translation by F. A. Welby.

‡ Helmholtz had been partly anticipated by W. R. Grove, in his lectures on the *Correlation of Physical Forces*, which were delivered in 1843 and published in 1846. Grove, after asserting that heat is "purely dynamical" in its nature, and that the various "physical forces" may be transformed into each other, remarked: "The great problem which remains to be solved, in regard to the correlation of physical forces, is the establishment of their equivalent of power, or their measurable relation to a given standard."

Consider first the energy of an electrostatic field. It will be convenient to suppose that the system has been formed by continually bringing from a very great distance infinitesimal quantities of electricity, proportional to the quantities already present at the various points of the system; so that the charge is always distributed proportionally to the final distribution. Let e typify the final charge at any point of space, and V the final potential at this point. Then at any stage of the process the charge and potential at this point will have the values λe and λV, where λ denotes a proper fraction. At this stage let charges $ed\lambda$ be brought from a great distance and added to the charges λe. The work required for this is

$$\Sigma\, e\, d\lambda \,.\, \lambda V,$$

so the total work required in order to bring the system from infinite dispersion to its final state is

$$\Sigma e V \,.\int_0^1 \lambda d\lambda, \quad \text{or} \quad \tfrac{1}{2}\Sigma e V.$$

By reasoning similar to that used in the case of electrostatic distributions, it may be shown that the energy of a magnetic field, which is due to permanent magnets and which also contains bodies susceptible to magnetic induction, is

$$\frac{1}{2}\iiint \rho_0\, \phi\, dx\, dy\, dz,$$

where ρ_0 denotes the density of Poisson's equivalent magnetization, for the permanent magnets only, and ϕ denotes the magnetic potential.*

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* Julius Robert Mayer (*b.* 1814, *d.* 1878), who was a medical man in Heilbronn, asserted the equivalence of heat and work in 1842, Annal. d. Chemie, xlii, p. 233; his memoir, like that of Helmholtz, was first declined by the editors of the Annalen der Physik. An English translation of one of Mayer's memoirs was printed in Phil. Mag. xxv (1863), p. 493.

Helmholtz, moreover, applied the principle of energy to systems containing electric currents. For instance, when a magnet is moved in the vicinity of a current, the energy taken from the battery may be equated to the sum of that expended as Joulian heat, and that communicated to the magnet by the electromagnetic force: and this equation shows that the current is not proportional to the electromotive force of the battery, i.e. it reveals the existence of Faraday's magneto-electric induction. As, however, Helmholtz was at the time unacquainted with the conception of the electrokinetic energy stored in connexion with a current, his equations were for the most part defective. But in the case of the mutual action of a current and a permanent magnet, he obtained the correct result that the time-integral of the induced electromotive force in the circuit is equal to the increase which takes place in the potential of the magnet towards a current of a certain strength in the circuit.

The correct theory of the energy of magnetic and electromagnetic fields is due mainly to W. Thomson (Lord Kelvin). Thomson's researches on this subject commenced with one or two short investigations regarding the ponderomotive forces which act on temporary magnets. In 1847 he discussed† the case of a small iron sphere placed in a magnetic field, showing that it is acted on by a ponderomotive force represented by $-\operatorname{grad} cR^2$, where c denotes a constant, and R denotes the magnetic force of the field; such a sphere must evidently tend to move towards the places where R^2 is greatest. The same analysis may be applied to explain why diamagnetic bodies tend to move, as in Faraday's experiments, from the stronger to the weaker parts of the field.

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* We suppose all transitions to be continuous, so as to avoid the necessity for writing surface-integrals separately.

† Camb. and Dub. Math. Journal, ii (1847), p. 230; W. Thomson's *Papers on Electrostatics and Magnetism*, p. 499; cf. also Phil. Mag. xxxvii (1850), p. 241.

Two years later Thomson presented to the Royal Society a memoir* in which the results of Poisson's theory of magnetism were derived from experimental data, without making use of the hypothesis of magnetic fluids; and this was followed in 1850 by a second memoir,† in which Thomson drew attention to the fact previously noticed by Poisson,‡ that the magnetic intensity at a point within a magnetized body depends on the shape of the small cavity in which the exploring magnet is placed. Thomson distinguished two vectors;§ one of these, by later writers generally denoted by **B**, represents the magnetic intensity at a point situated in a small crevice in the magnetized body, when the faces of the crevice are at right angles to the direction of magnetization; the vector **B** is always circuital. The other vector, generally denoted by **H**, represents the magnetic intensity in a narrow tubular cavity tangential to the direction of magnetization; it is an irrotational vector. The magnetic potential tends at any point to a limit which is independent of the shape of the cavity in which the point is situated; and the space-gradient of this limit is identical with **H**. Thomson called **B** the "magnetic force according to the electro-magnetic definition," and **H** the "magnetic force according to the polar definition"; but the names *magnetic induction* and *magnetic force,* proposed by Maxwell, have been generally used by later writers.

It may be remarked that the vector to which Faraday applied the term "magnetic force," and which he represented by lines of force, is not **H**, but **B**; for the number of unit lines of force passing through any gap must depend only on the gap, and not on the particular diaphragm filling up the gap, across which the flux is estimated; and this can be the case only if the vector which is represented by the lines of force is a circuital vector.

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* Phil. Trans., 1851, p. 243; Thomson's *Papers on Elect. and Mag.*, p. 345.

† Phil. Trans., 1851, p. 269; *Papers on Elect. and Mag.*, p. 382.

‡ Cf. p. 64.

§ Loc. cit., § 78 of the original paper, and § 517 of the reprint.

Thomson introduced a number of new terms into magnetic science—as indeed he did into every science in which he was interested. The ratio of the measure of the induced magnetization $\mathbf{I}_i$, in a temporary magnet, to the magnetizing force $\mathbf{H}$, he named the *susceptibility*; it is positive for paramagnetic and negative for diamagnetic bodies, and is connected with Poisson's constant k_p* by the relation

$$\kappa = \frac{3}{4\pi}\frac{k_p}{1 - k_p},$$

where κ denotes the susceptibility. By an easy extension of Poisson's analysis it is seen that the magnetic induction and magnetic force are connected by the equation

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{I},$$

where $\mathbf{I}$ denotes the total intensity of magnetization: so if $\mathbf{I}_0$ denote the permanent magnetization, we have

$$\begin{aligned}\mathbf{B} &= \mathbf{H} + 4\pi\mathbf{I}_i + 4\pi\mathbf{I}_0,\\ &= \mu\mathbf{H} + 4\pi\mathbf{I}_0,\end{aligned}$$

where μ denotes $(1 + 4\pi\kappa)$: μ was called by Thomson the *permeability*.

In 1851 Thomson extended his magnetic theory so as to include magnecrystallic phenomena. The mathematical foundations of the theory of magnecrystallic action had been laid by anticipation, long before the experimental discovery of the phenomenon, in a memoir read by Poisson to the Academy in February, 1824. Poisson, as will be remembered, had supposed temporary magnetism to be due to "magnetic fluids," movable within the infinitely small "magnetic elements" of which he assumed magnetizable matter to be constituted. He had not overlooked the possibility that in crystals these magnetic elements might be non-spherical (e.g. ellipsoidal), and symmetrically arranged; and had remarked that a portion of such a crystal, when placed in a magnetic field, would act in a manner depending on its orientation. The relations connecting the induced magnetization **I** with the magnetizing force **H** he had given in a form equivalent to

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* Cf. p. 65.

$$\begin{cases} I_x = aH_x + b'H_y + c''H_z, \\ I_y = a''H_x + bH_y + c'H_z, \\ I_z = a'H_x + b''H_y + cH_z. \end{cases}$$

Thomson now* showed that the nine coefficients $a, b', c'' \ldots$, introduced by Poisson, are not independent of each other. For a sphere composed of the magnecrystalline substance, if placed in a uniform field of force, would be acted on by a couple: and the work done by this couple when the sphere, supposed of unit volume, performs a complete revolution round the axis of x may be easily shown to be $\pi H(1 - H_x^2/H^2)(-b'' + c')$. But this work must be zero, since the system is restored to its primitive condition; and hence b'' and c' must be equal. Similarly $c'' = a'$, and $a'' = b'$. By change of axes three more coefficients may be removed, so that the equations may be brought to the form

$$I_x = \kappa_1 H_x, \quad I_y = \kappa_2 H_y, \quad I_z = \kappa_3 H_z,$$

where κ_1, κ_2, κ_3 may be called the *principal magnetic susceptibilities*.

In the same year (1851) Thomson investigated the energy which, as was evident from Faraday's work on self-induction, must be stored in connexion with every electric current. He showed that, in his own words,† "the value of a current in a closed conductor, left without electromotive force, is the quantity of work that would be got by letting all the infinitely small currents into which it may be divided along the lines of motion of the electricity come together from an infinite distance, and make it up. Each of these 'infinitely small currents' is of course in a circuit which is generally of finite length; it is the section of each partial conductor and the strength of the current in it that must be infinitely small."

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* Phil. Mag. (4) i (1851), p. 177: *Papers on Electrostatics and Magnetism*, p. 471.

† *Papers on Electrostatics and Magnetism*, p. 446.

Discussing next the mutual energy due to the approach of a permanent magnet and a circuit carrying a current, he arrived at the remarkable conclusion that in this case there is no electrokinetic energy which depends on the mutual action; the energy is simply the sum of that due to the permanent magnets and that due to the currents. If a permanent magnet is caused to approach a circuit carrying a current, the electromotive force acting in the circuit is thereby temporarily increased; the amount of energy dissipated as Joulian heat, and the speed of the chemical reactions in the cells, are temporarily increased also. But the increase in the Joulian heat is exactly equal to the increase in the energy derived from consumption of chemicals, together with the mechanical work done on the magnet by the operator who moves it; so that the balance of energy is perfect, and none needs to be added to or taken from the electrokinetic form. It will now be evident why it was that Helmholtz escaped in this case the errors into which he was led in other cases by his neglect of electrokinetic energy; for in this case there was no electrokinetic energy to neglect.

Two years later, in 1853, Thomson* gave a new form to the expression for the energy of a system of permanent and temporary magnets.

We have seen that the energy of such a system is represented by

$$\frac{1}{2}\iiint \rho_0 \phi \, dx \, dy \, dz,$$

where ρ_0 denotes the density of Poisson's equivalent magnetization for the permanent magnets, and ϕ denotes the magnetic potential, and where the integration may be extended over the whole of space. Substituting for ρ_0 its value $-\operatorname{div} \mathbf{I}_0$,† the expression may be written in the form

$$-\frac{1}{2}\iiint \phi \operatorname{div} \mathbf{I}_0 \, dx \, dy \, dz;$$

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* Proc. Glasgow Phil. Soc. iii (1853), p. 281; Kelvin's *Math. and Phys. Papers*, i, p. 521.

† Cf. p. 64.

or, integrating by parts,

$$-\tfrac{1}{2}\iiint (\mathbf{I}_0 \,.\, \text{grad}\, \phi)\, dx\, dy\, dz, \quad \text{or} \quad -\tfrac{1}{2}\iiint (\mathbf{H} \,.\, \mathbf{I}_0)\, dx\, dy\, dz.$$

Since $\mathbf{B} = \mu\mathbf{H} + 4\pi\mathbf{I}_0$, this expression may be written in the form

$$-\frac{1}{8\pi}\iiint (\mathbf{H} \,.\, \mathbf{B})\, dx\, dy\, dz + \frac{1}{8\pi}\iiint \mu\mathbf{H}^2\, dx\, dy\, dz;$$

but the former of these integrals is equivalent to

$$\iiint (\mathbf{B} \,.\, \text{grad}\, \phi)\, dx\, dy\, dz, \quad \text{or} \quad -\iiint \phi\, \text{div}\, \mathbf{B}\, dx\, dy\, dz,$$

which vanishes, since **B** is a circuital vector. The energy of the field, therefore, reduces to

$$\frac{1}{8\pi}\iiint \mu\mathbf{H}^2\, dx\, dy\, dz,$$

integrated over all space; which is equivalent to Thomson's form.*

In the same memoir Thomson returned to the question of the energy which is possessed by a circuit in virtue of an electric current circulating in it. As he remarked, the energy may be determined by calculating the amount of work which must be done in and on the circuit in order to double the circuit on itself while the current is sustained in it with constant strength; for Faraday's experiments show that a circuit doubled on itself has no stored energy. Thomson found that the amount of work required may be expressed in the form $\frac{1}{2}Li^2$, where i denotes the current strength, and L, which is called the *coefficient of self-induction*, depends only on the form of the circuit.

It may be noticed that in the doubling process the inherent

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* The form actually given by Thomson was

$$\frac{1}{8\pi}\iiint \left(\mathbf{H}^2 + \frac{4\pi}{\kappa}\mathbf{I}^2\right)\, dx\, dy\, dz,$$

which reduces to the above when we neglect that part of $\mathbf{I}^2$ which is due to the permanent magnetism, over which we have no control.

electrodynamic energy is being given up, and yet the operator is doing positive work. The explanation of this apparent paradox is that the energy derived from both these sources is being used to save the energy which would otherwise be furnished by the battery, and which is expended in Joulian heat.

Thomson next proceeded* to show that the energy which is stored in connexion with a circuit in which a current is flowing may be expressed as a volume-integral extended over the whole of space, similar to the integral by which he had already represented the energy of a system of permanent and temporary magnets. The theorem, as originally stated by its author, applied only to the case of a single circuit; but it may be established for a system formed by any number of circuits in the following way:—

If N_s denote the number of unit tubes of magnetic induction which are linked with the s^{th} circuit, in which a current i_s is flowing, the electrokinetic energy of the system is $\frac{1}{2}\sum\limits_s N_s i_s$; which may be written $\frac{1}{2}\sum\limits_r I_r$, where I_r denotes the total current flowing through the gap formed by the r^{th} unit tube of magnetic induction. But if $\mathbf{H}$ denote the (vector) magnetic force, and H its numerical magnitude, it is known that $(1/4\pi)\int Hds$, integrated along a closed line of magnetic induction, measures the total current flowing through the gap formed by the line. The energy is therefore $(1/8\pi)\Sigma\int Hds$, the summation being extended over all the unit tubes of magnetic induction, and the integration being taken along them. But if dS denote the cross-section of one of these tubes, we have $BdS = 1$, where B denotes the numerical magnitude of the magnetic induction $\mathbf{B}$: so the energy is $(1/8\pi)\,\Sigma BdS\int Hds$; and as the tubes fill all space, we may replace $\Sigma dS\int ds$ by $\iiint dx\,dy\,dz$. Thus the energy takes the form $(1/8\pi)\iiint BH dx\,dy\,dz$, where the integration is extended over the whole of space; and since in the present case $B = \mu H$, the energy may also be represented by $(1/8\pi)\iiint \mu H^2 dx\,dy\,dz$.

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* *Nichols' Cyclopaedia*, 2nd ed., 1860, article "Magnetism, dynamical relations of;" reprinted in Thomson's *Papers on Elect. and Mag.*, p. 447, and his *Math. and Phys. Papers*, p. 532.

But this is identical with the form which was obtained for a field due to permanent and temporary magnets. It thus appears that in all cases the stored energy of a system of electric currents and permanent and temporary magnets is

$$\frac{1}{8\pi}\iiint \mu H^2\, dx\, dy\, dz,$$

where the integration is extended over all space.

It must, however, be remembered that this represents only what in thermodynamics is called the "available energy"; and it must further be remembered that part even of this available energy may not be convertible into mechanical work within the limitations of the system: e.g., the electrokinetic energy of a current flowing in a single closed perfectly conducting circuit cannot be converted into any other form so long as the circuit is absolutely rigid. All that we can say is that the changes in this stored electrokinetic energy correspond to the work furnished by the system in any change.

The above form suggests that the energy may not be localized in the substance of the circuits and magnets, but may be distributed over the whole of space, an amount $(\mu H^2/8\pi)$ of energy being contained in each unit volume. This conception was afterwards adopted by Maxwell, in whose theory it is of fundamental importance.

While Thomson was investigating the energy stored in connexion with electric currents, the equations of flow of the currents were being generalized by Gustav Kirchhoff (*b.* 1824, *d.* 1887). In 1848 Kirchhoff* extended Ohm's theory of linear conduction to the case of conduction in three dimensions; this could be done without much difficulty by making use of the analogy with the flow of heat, which had proved so useful to Ohm. In Kirchhoff's memoir a system is supposed to be formed of three-dimensional conductors, through which steady currents are flowing. At any point let V denote the "tension" or "electroscopic force"—a quantity the significance of which

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*Ann. d. Phys. lxxv (1848), p. 189; Kirchhoff's *Ges. Abhandl.*, p. 33.

in electrostatics was not yet correctly known. Then, within the substance of any homogeneous conductor, the function V must satisfy Laplace's equation $\nabla^2 V = 0$; while at the air-surface of each conductor, the derivate of V taken along the normal must vanish. At the interface between two conductors formed of different materials, the function V has a discontinuity, which is measured by the value of Volta's contact force for the two conductors; and, moreover, the condition that the current shall be continuous across such an interface requires that $k\partial V/\partial N$ shall be continuous, where k denotes the ohmic specific conductivity of the conductor, and $\partial/\partial N$ denotes differentiation along the normal to the interface. The equations which have now been mentioned suffice to determine the flow of electricity in the system.

Kirchhoff also showed that the currents distribute themselves in the conductors in such a way as to generate the least possible amount of Joulian heat; as is easily seen, since the quantity of Joulian heat generated in unit time is

$$\iiint k \left\{ \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 \right\} dx\, dy\, dz,$$

where k, as before, denotes the specific conductivity; and this integral has a stationary value when V satisfies the equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial V}{\partial z}\right) = 0.$$

Kirchhoff next applied himself to establish harmony between electrostatical conceptions and the theory of Ohm. That theory had now been before the world for twenty years, and had been verified by numerous experimental researches; in particular, a careful investigation was made at this time (1848) by Rudolph Kohlrausch (*b.* 1809, *d.* 1858), who showed* that the difference of the electric "tensions" at the extremities of a voltaic cell, measured electrostatically with the circuit open, was for different cells proportional to the electromotive force measured by the electrodynamic effects of the cell with the circuit closed; and, further,* that when the circuit was closed, the difference of the tensions, measured electrostatically, at any two points of the outer circuit was proportional to the ohmic resistance existing between them. But in spite of all that had been done, it was still uncertain how "tension," or "electroscopic force," or "electromotive force" should be interpreted in the language of theoretical electrostatics; it will be remembered that Ohm himself, perpetuating a confusion which had originated with Volta, had identified electroscopic force with density of electric charge, and had assumed that the electricity in a conductor is at rest when it is distributed uniformly throughout the substance of the conductor.

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* Ann. d. Phys. lxxv (1848), p. 220.

The uncertainty was finally removed in 1849 by Kirchhoff,† who identified Ohm's electroscopic force with the electrostatic potential. That this identification is correct may be seen by comparing the different expressions which have been obtained for electric energy; Helmholtz's expression‡ shows that the energy of a unit charge at any place is proportional to the value of the electrostatic potential at that place; while Joule's result§ shows that the energy liberated by a unit charge in passing from one place in a circuit to another is proportional to the difference of the electric tensions at the two places. It follows that tension and potential are the same thing.

The work of Kirchhoff was followed by several other investigations which belong to the borderland between electrostatics and electrodynamics. One of the first of these was the study of the Leyden jar discharge.

Early in the century Wollaston, in the course of his experiments on the decomposition of water, had observed that when the decomposition is effected by a discharge of static electricity, the hydrogen and oxygen do not appear at separate electrodes; but that at each electrode there is evolved a mixture of the gases, as if the current had passed through the water in both directions. After this F. Savary* had noticed that the discharge of a Leyden jar magnetizes needles in alternating layers, and had conjectured that "the electric motion during the discharge consists of a series of oscillations." A similar remark was made in connexion with a similar observation by Joseph Henry (*b.* 1799, *d.* 1878), of Washington, in 1842.† "The phenomena," he wrote, "require us to admit the existence of a principal discharge in one direction, and then several reflex actions backward and forward, each more feeble than the preceding, until equilibrium is restored." Helmholtz had repeated the same suggestion in his essay on the conservation of energy: and in 1853 W. Thomson‡ verified it, by investigating the mathematical theory of the discharge, as follows:—

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* Ann. d. Phys, lxxviii (1849), p. 1.

† *Ib.* lxxviii (1849), p. 506; Kirchhoff's *Ges. Abhandl.*, p. 49; Phil. Mag. (3), xxxvii (1850), p. 463.

‡ Cf. p. 242.

§ Cf. p. 239.

Let C denote the capacity of the jar, i.e., the measure of the charge when there is unit difference of potential between the coatings; let R denote the ohmic resistance of the discharging circuit, and L its coefficient of self-induction. Then if at any instant t the charge of the condenser be Q, and the current in the wire be i, we have $i = dQ/dt$; while Ohm's law, modified by taking self-induction into account, gives the equation

$$Ri + L\frac{di}{dt} = -\frac{Q}{C}.$$

Eliminating i, we have

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0,$$

an equation which shows that when $R^2C < 4L$, the subsidence of Q to zero is effected by oscillations of period

$$2\pi\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{-\frac{1}{2}}.$$

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* Annales de Chimie, xxxiv (1827), p. 5.

† Proc. Am. Phil. Soc. ii (1842), p. 193.

‡ Phil. Mag. (4) v (1853), p. 400; Kelvin's *Math. and Phys. Papers* i, p. 540.

This simple result may be regarded as the beginning of the theory of electric oscillations.

Thomson was at this time much engaged in the problems of submarine telegraphy; and thus he was led to examine the vexed question of the "velocity of electricity" over long insulated wires and cables. Various workers had made experiments on this subject at different times, but with hopelessly discordant results. Their attempts had generally taken the form of measuring the interval of time between the appearance of sparks at two spark-gaps in the same circuit, between which a great length of wire intervened, but which were brought near each other in order that the discharges might be seen together. In one series of experiments, performed by Watson at Shooter's Hill in 1747-8,* the circuit was four miles in length, two miles through wire and two miles through the ground; but the discharges appeared to be perfectly simultaneous; whence Watson concluded that the velocity of propagation of electric effects is too great to be measurable.

In 1834 Charles Wheatstone,† Professor of Experimental Philosophy in King's College, London, by examining in a revolving mirror sparks formed at the extremities of a circuit, found the velocity of electricity in a copper wire to be about one and a half times the velocity of light. In 1850 H. Fizeau and E. Gounelle,‡ experimenting with the telegraph lines from Paris to Rouen and to Amiens, obtained a velocity about one-third that of light for the propagation of electricity in an iron wire, and nearly two-thirds that of light for the propagation in a copper wire.

The first step towards explaining these discrepancies was made by Faraday, who§ early in 1854 showed experimentally that a submarine cable, formed of copper wire covered with gutta-percha, "may be assimilated exactly to an immense Leyden battery; the glass of the jars represents the gutta-percha; the internal coating is the surface of the copper wire," while the outer coating corresponds to the sea-water. It follows that in all calculations relating to the propagation of electric disturbances along submarine cables, the electrostatic capacity of the cable must be taken into account.

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* Phil. Trans. xlv (1748), pp. 49, 491.

† Phil. Trans., 1834, p. 583.

‡ Comptes Rendus, xxx (1850), p. 437.

§ Proc. Roy. Inst., Jan. 20, 1854: Phil. Mag., June, 1854: *Exp. Res.* iii, pp. 508, 521.

The theory of signalling by cable originated in a correspondence between Stokes and Thomson in 1854. In the case of long submarine lines, the speed of signalling is so much limited by the electrostatic factor that electro-magnetic induction has no sensible effect; and it was accordingly neglected in the investigation. In view of other applications of the analysis, however, we shall suppose that the cable has a self-induction L per unit length, and that R denotes the ohmic resistance, and C the capacity per unit length, V the electric potential at a distance x from one terminal, and i the current at this place. Ohm's law, as modified for inductance, is expressed by the equation

$$-\frac{\partial V}{\partial x} = L\frac{\partial i}{\partial t} + Ri;$$

moreover, since the rate of accumulation of charge in unit length at x is $-\partial i/\partial x$, and since this increases the potential at the rate $-(1/C)\partial i/\partial x$, we have

$$C\frac{\partial V}{\partial t} = -\frac{\partial i}{\partial x}.$$

Eliminating i between these two equations, we have

$$\frac{1}{C}\frac{\partial^2 V}{\partial x^2} = L\frac{\partial^2 V}{\partial t^2} + R\frac{\partial V}{\partial t},$$

which is known as the *equation of telegraphy.*[*]

Thomson, in one of his letters[†] to Stokes in 1854, obtained this equation in the form which applies to Atlantic cables, i.e., with the term in L neglected. In this form it is the same as Fourier's equation for the linear propagation of heat: so that the known solutions of Fourier's theory may be used in a new interpretation. If we substitute

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[*] We have neglected leakage, which is beside our present purpose.

[†] Proc. Roy. Soc., May, 1855: Kelvin's *Math. and Phys. Papers*, ii, p. 61.

$$V = e^{2nt\sqrt{-1} + \lambda x},$$

we obtain

$$\lambda = \pm (1 + \sqrt{-1})(nCR)^{\frac{1}{2}};$$

and therefore a typical elementary solution of the equation is

$$V = e^{-(nCR)^{\frac{1}{2}}x} \sin \{2nt - (nCR)^{\frac{1}{2}} x\}.$$

The form of this solution shows that if a regular harmonic variation of potential is applied at one end of a cable, the phase is propagated with a velocity which is proportional to the square root of the frequency of the oscillations: since therefore the different harmonics are propagated with different velocities, it is evident that no definite "velocity of transmission" is to be expected for ordinary signals. If a potential is suddenly applied at one end of the cable, a certain time elapses before the current at the other end attains a definite percentage of its maximum value; but it may easily be shown* that this retardation is proportional to the square of the length of the cable, so that the apparent velocity of propagation would be less, the greater the length of cable used.

The case of a telegraph line insulated in the air on poles is different from that of a cable; for here the capacity is small, and it is necessary to take into account the inductance. If in the general equation of telegraphy we write

$$V = e^{nx\sqrt{-1} + \mu t},$$

we obtain the equation

$$\mu = -\frac{R}{2L} \pm \left(\frac{R^2}{4L^2} - \frac{n^2}{CL}\right)^{\frac{1}{2}};$$

as the capacity is small, we may replace the quantity under the radical by its second term: and thus we see that a typical elementary solution of the equation is

$$V = e^{-\frac{Rt}{2L}} \sin n\{x - (CL)^{-\frac{1}{2}} t\};$$

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* This result, indeed, follows at once from the theory of dimensions.

this shows that any harmonic disturbance, and therefore any disturbance whatever, is propagated along the wire with velocity $(CL)^{-\frac{1}{2}}$. The difference between propagation in an aerial wire and propagation in an oceanic cable is, as Thomson remarked, similar to the difference between the propagation of an impulsive pressure through a long column of fluid in a tube when the tube is rigid (case of the aerial wire) and when it is elastic, so as to be capable of local distension (case of the cable, the distension corresponding to the effect of capacity): in the former case, as is well known, the impulse is propagated with a definite velocity, namely, the velocity of sound in the fluid.

The work of Thomson on signalling along cables was followed in 1857 by a celebrated investigation* of Kirchhoff's, on the propagation of electric disturbance along an aerial wire of circular cross-section.

Kirchhoff assumed that the electric charge is practically all resident on the surface of the wire, and that the current is uniformly distributed over its cross-section; his idea of the current was the same as that of Fechner and Weber, namely, that it consists of equal streams of vitreous and resinous electricity flowing in opposite directions. Denoting the electric potential by V, the charge per unit length of wire by e, the length of the wire by l, and the radius of its cross-section by a, he showed that V is determined approximately by the equation†

$$V = 2e \log (l/a).$$

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* Ann. d. Phys. c (1857), pp. 193, 251: Kirchhoff's *Ges. Abhandl.*, p. 131; Phil. Mag. xiii (1857), p. 393.

† His method of obtaining this equation was to calculate separately the effects of (1) the portion of the wire within a distance ϵ on either side of the point considered, where ϵ denotes a length small compared with l, but large compared with a, and (2) the rest of the wire. He thus obtained the equation

$$V = 2e \log \frac{2\epsilon}{a} + \int \frac{e'ds'}{r},$$

where the integration is to be taken over all the length of the wire except the portion 2ϵ: the equation given in the text was then derived by an approximation, which, however, is open to some objection.

The next factor to be considered is the mutual induction of the current-elements in different parts of the wire. Assuming with Weber that the electromotive force induced in an element **ds** due to another element **ds**′ carrying a current i' is derivable from a vector-potential

$$i' \frac{(\mathbf{r} \cdot \mathbf{ds}') \cdot \mathbf{r}}{r^3},$$

Kirchhoff found for the vector-potential due to the entire wire the approximate value

$$w = 2i \log (l/a),$$

where i denotes the strength of the current;* the vector-potential being directed parallel to the wire. Ohm's law then gives the equation

$$i = -\pi k a^2 \left(\frac{\partial V}{\partial x} + \frac{1}{c^2} \frac{\partial w}{\partial t} \right),$$

where k denotes the specific conductivity of the material of which the wire is composed; and finally the principle of conservation of electricity gives the equation

$$\frac{\partial i}{\partial x} = -\frac{\partial e}{\partial t}.$$

Denoting log (l/a) by γ, and eliminating e, i, w from these four equations, we have

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} + \frac{1}{2\gamma \kappa \pi a^2} \frac{\partial V}{\partial t},$$

which is, as might have been expected, the equation of telegraphy. When the term in $\partial V / \partial t$ is ignored, as we have seen is in certain cases permissible, the equation becomes

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2},$$

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* This expression was derived in a similar way to that for V, by an intermediate formula

$$w = 2i \log \frac{2\epsilon}{a} + \int \frac{i' ds'}{r} \cos \theta \cos \theta',$$

where θ and θ' denote respectively the angles made with **r** by **ds** and **ds**′.

which shows that *the electric disturbance is propagated along the wire with the velocity c.*[*] Kirchhoff's procedure has, in fact, involved the calculation of the capacity and self-induction of the wire, and is thus able to supply the definite values of the quantities which were left undetermined in the general equation of telegraphy.

The velocity c, whose importance was thus demonstrated, has already been noticed in connexion with Weber's law of force; it is a factor of proportionality, which must be introduced when electrodynamic phenomena are described in terms of units which have been defined electrostatically,[†] or conversely when units which have been defined electrodynamically[‡] are used in the description of electrostatic phenomena. That the factor which is introduced on such occasions must be of the dimensions (length/time), may be easily seen: for the electrostatic repulsion between electric charges is a quantity of the same kind as the electrodynamic repulsion between two definite *lengths* of wire, carrying currents which may be specified by the amount of charge which travels past any point in unit *time*.

Shortly before the publication of Kirchhoff's memoir, the value of c had been determined by Weber and Kohlrausch[§]; their determination rested on a comparison of the measures of the charge of a Leyden jar, as obtained by a method depending on electrostatic attraction, and by a method depending on the magnetic effects of the current produced by discharging the jar. The resulting value was nearly

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[*] In referring to the original memoirs of Weber and Kirchhoff, it must be remembered that the quantity which in the present work is denoted by c, and which represents the velocity of light in free aether, was by these writers denoted by $c/\sqrt{2}$. Weber, in fact, denoted by c the relative velocity with which two charges must approach each other in order that the force between them, as calculated by his formula, should vanish.

It must also be remembered that those writers who accepted the hypothesis that currents consist of equal and opposite streams of vitreous and resinous electricity, were accustomed to write $2i$ to denote the current-strength.

[†] i.e., defining unit electric charge as that which exerts unit ponderomotive force on a conductor at unit distance which carries an equal charge; and then defining unit current as that which conveys unit charge in unit time.

[‡] i.e., defining unit current by means of the ponderomotive force which it exerts on an equal current, when the two currents flow in circuits of specified form at a specified distance apart.

[§] Ann. d. Phys. xcix (1856), p. 10.

$$c = 3{\cdot}1 \times 10^{10} \text{ cm./sec.};$$

which was the same, within the limits of the errors of measurement, as the speed with which light travels in interplanetary space. The coincidence was noticed by Kirchhoff, who was thus the first to discover the important fact that the velocity with which an electric disturbance is propagated along a perfectly-conducting aerial wire is equal to the velocity of light.

In a second memoir published in the same year, Kirchhoff* extended the equations of propagation of electric disturbance to the case of three-dimensional conductors.

As in his earlier investigation, he divided the electromotive force at any point into two parts, of which one is the gradient of the electrostatic potential ϕ, and the other is the derivate with respect to the time (with sign reversed) of a vector-potential $\mathbf{a}$; so that if $\mathbf{i}$ denote the current and k the specific conductivity, Ohm's law is expressed by the equation

$$\mathbf{i} = k\,(c^2 \operatorname{grad} \phi - \dot{\mathbf{a}}).$$

Kirchhoff calculated the value of $\mathbf{a}$ by aid of Weber's formula for the inductive action of one current element on another; the result is

$$\mathbf{a} = \iiint \frac{dx'\,dy'\,dz'}{r^3}\,(\mathbf{r}\,.\,\mathbf{i}')\,\mathbf{r},$$

where $\mathbf{r}$ denotes the vector from the point (x, y, z), at which $\mathbf{a}$ is measured, to any other point (x', y', z') of the conductor, at which the current is $\mathbf{i}'$; and the integration is extended over the whole volume of the conductor. The remaining general equations are the ordinary equation of the electrostatic potential

$$\nabla^2\phi + 4\pi\rho = 0$$

(where ρ denotes the density of electric charge), and the equation of conservation of electricity

$$\frac{\partial\rho}{\partial t} + \operatorname{div} \mathbf{i} = 0.$$

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* Ann. d. Phys. cii (1857), p. 529: *Ges. Abhandl.*, p. 154.

It will be seen that Kirchhoff's electrical researches were greatly influenced by those of Weber. The latter investigations, however, did not enjoy unquestioned authority; for there was still a question as to whether the expressions given by Weber for the mutual energy of two current elements, and for the mutual energy of two electrons, were to be preferred to the rival formulae of Neumann and Riemann. The matter was examined in 1870 by Helmholtz, in a series of memoirs* to which reference has already been made.† Helmholtz remarked that, for two elements **ds**, **ds**′, carrying currents i, i', the electrodynamic energy is

$$\frac{ii'(\mathbf{ds}\,.\,\mathbf{ds}')}{r},$$

according to Neumann, and

$$\frac{ii'}{r^3}(\mathbf{r}\,.\,\mathbf{ds})(\mathbf{r}\,.\,\mathbf{ds}'),$$

according to Weber; and that these expressions differ from each other only by the quantity

$$\frac{ii'dsds'}{r}\{-\cos(ds\,.\,ds') + \cos(r\,.\,ds)\cos(r\,.\,ds')\},$$

or

$$ii'dsds'\frac{d^2r}{ds\,ds'};$$

since this vanishes when integrated round either circuit, the two formulae give the same result when applied to entire currents. A general formula including both that of Neumann and that of Weber is evidently

$$\frac{ii'(\mathbf{ds}\,.\,\mathbf{ds}')}{r} + kii'\frac{d^2r}{ds\,ds'}\,ds\,ds',$$

where k denotes an arbitrary constant.‡

Helmholtz's result suggested to Clausius§ a new form for the law of force between electrons; namely, that which is obtained by supposing that two electrons of charges e, e', and velocities $\mathbf{v}$, $\mathbf{v}'$, possess electrokinetic energy of amount

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* Journal für Math., lxxii (1870), p. 57: lxxv (1873), p. 35: lxxviii (1874), p. 273.

† Cf. p. 229.

‡ Cf. H. Lamb, Proc. Lond. Math. Soc., xiv (1883), p. 301.

§ Journal für Math. lxxxii (1877), p. 85: Phil. Mag., x (1880), p. 255.

$$\frac{ee'\,(\mathbf{v}\,.\,\mathbf{v}')}{r} + kee'\,\frac{d^2 r}{ds\,ds'}\,vv'.$$

Subtracting from this the mutual electrostatic potential energy, which is $ee'c^2/r$, we may write the mutual kinetic potential of the two electrons in the form

$$\frac{ee'}{r}\,(\dot{x}\dot{x}' + \dot{y}\dot{y}' + \dot{z}\dot{z}' - c^2) + kee'\,\frac{d^2 r}{ds\,ds'}\,vv',$$

where (x, y, z) denote the coordinates of e, and (x', y', z') those of e'.

The unknown constant k has clearly no influence so long as closed circuits only are considered: if k be replaced by zero, the expression for the kinetic potential becomes

$$\frac{ee'}{r}\,(\dot{x}\dot{x}' + \dot{y}\dot{y}' + \dot{z}\dot{z}' - c^2),$$

which, as will appear later, closely resembles the corresponding expression in the modern theory of electrons.

Clausius' formula has the great advantage over Weber's, that it does not compel us to assume equal and opposite velocities for the vitreous and resinous charges in an electric current; on the other hand, Clausius' expression involves the absolute velocities of the electrons, while Weber's depends only on their relative motion; and therefore Clausius' theory requires the assumption of a fixed aether in space, to which the velocities $\mathbf{v}$ and $\mathbf{v}'$ may be referred.

When the behaviour of finite electrical systems is predicted from the formulae of Weber, Riemann, and Clausius, the three laws do not always lead to concordant results. For instance, if a circular current be rotated with constant angular velocity round its axis, according to Weber's law there would be a development of free electricity on a stationary conductor in the neighbourhood; whereas, according to Clausius' formula there would be no induction on a stationary body, but electrification would appear on a body turning with the circuit as if rigidly connected with it. Again,* let a magnet be suspended within a hollow metallic body, and let the hollow body be suddenly charged or discharged; then, according to Clausius' theory, the magnet is unaffected; but according to Weber's and Riemann's theories it experiences an impulsive couple. And again, if an electrified disk be rotated in its own plane, under certain circumstances a steady current will be induced in a neighbouring circuit according to Weber's law, but not according to the other formulae.

An interesting objection to Clausius' theory was brought forward in 1879 by Fröhlich†—namely, that when a charge of free electricity and a constant electric current are at rest relatively to each other, but partake together of the translatory motion of the earth in space, a force should act between them if Clausius' law were true. It was, however, shown by Budde‡ that the circuit itself acquires an electrostatic charge, partly as a result of the same action which causes the force on the external conductor, and partly as a result of electrostatic induction by the charge on the external conductor; and that the total force between the circuit and external conductor is thus reduced to zero.§

We have seen that the discrimination between the different laws of electrodynamic force is closely connected with the question whether in an electric current there are two kinds of electricity moving in opposite directions, or only one kind moving in one direction. On the unitary hypothesis, that the current consists in a transport of one kind of electricity with a definite velocity relative to the wire, it might be expected that a coil rotated rapidly about its own axis would generate a magnetic field different from that produced by the same coil at rest. Experiments to determine the matter were performed by A. Föppl* and by E. L. Nichols and W. S. Franklin,† but with negative results. The latter investigators found that the velocity of electricity must be such that the quantity conveyed past a specified point in a unit of time, when the direction of the current was that in which the coil was travelling, did not differ from that transferred when the current and coil were moving in opposite directions by as much as one part in ten million, even when the velocity of the wire was 9096 cm./sec. They considered that they would have been able to detect a change of deflexion due to the motion of the coil, even though the velocity of the current had been considerably greater than a thousand million metres per second.

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* The two following crucial experiments, with others, were suggested by E. Budde, Ann. d. Phys. xxx (1887), p. 100.

† Ann. d. Phys. ix (1880), p. 261.

‡ Ann. d. Phys. x (1880), p. 553.

§ This case of a charge and current moving side by side was afterwards examined by Fitz Gerald (Trans. Roy. Dub. Soc. i, 1882; *Scient. Writings of G. F. Fitz Gerald*, p. 111) without reference to Clausius' formula, from the standpoint of Maxwell's theory. The result obtained was the same—namely, that the electricity induced on the conductor carrying the current neutralizes the ponderomotive force between the current and the external charge.

During the decades in the middle of the century considerable progress was made in the science of thermo-electricity, whose beginnings we have already described.‡ In Faraday's laboratory note-book, under the date July 28th, 1836, we read§ :—"Surely the converse of thermo-electricity ought to be obtained experimentally. Pass current through a circuit of antimony and bismuth."

Unknown to Faraday, the experiment here indicated had already been made, although its author had arrived at it by a different train of ideas. In 1834 Jean Charles Peltier‖ (*b.* 1785, *d.* 1845) attempted the task, which was afterwards performed with success by Joule,¶ of measuring the heat evolved by the passage of an electric current through a conductor. He found that a current produces in a homogeneous conductor an elevation of temperature, which is the same in all parts of the conductor where the cross-section is the same; but he did not succeed in connecting the thermal phenomena quantitatively with the strength of the current—a failure which was due chiefly to the circumstance that his attention was fixed on the rise of temperature rather than on the amount of the heat evolved. But incidentally the investigation led to an important discovery—namely, that when a current was passed in succession through two conductors made of dissimilar metals, there was an evolution of heat at the junction; and that this depended on the direction of the current; for if the junction was heated when the current flowed in one sense, it was cooled when the current flowed in the opposite sense. This *Peltier effect*, as it is called, is quite distinct from the ordinary Joulian liberation of heat, in which the amount of energy set free in the thermal form is unaffected by a reversal of the current; the Joulian effect is, in fact, proportional to the square of the current-strength, while the Peltier effect is proportional to the current-strength directly. The Peltier heat which is absorbed from external sources when a current i flows for unit time through a junction from one metal B to another metal A may therefore be denoted by

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* Ann. d. Phys. xxvii (1886), p. 410.

† Amer. Jour. Sci., xxxvii (1889), p. 103.

‡ Cf. pp. 92, 93.

§ Bence Jones's *Life of Faraday*, ii, p. 76.

‖ Annales de Chimie, lvi (1834), p. 371.

¶ Cf. p. 239.

$$\Pi_B^A(T) \cdot i,$$

where T denotes the absolute temperature of the junction. The function $\Pi_B^A(T)$ is found to be expressible as the difference of two parts, of which one depends on the metal A only, and the other on the metal B only; thus we can write

$$\Pi_B^A(T) = \Pi_A(T) - \Pi_B(T).$$

In 1851 a general theory of thermo-electric phenomena was constructed on the foundation of Seebeck's* and Peltier's discoveries by W. Thomson.† Consider a circuit formed of two metals, A and B, and let one junction be maintained at a slightly higher temperature $(T + \delta T)$ than the temperature T of the other junction. As Seebeck had shown, a thermo-electric current will be set up in the circuit. Thomson saw that such a system might be regarded as a heat-engine, which absorbs a certain quantity of heat at the hot junction, and converts part of this into electrical energy, liberating the rest in the form of heat at the cold junction. If the Joulian evolution of heat be neglected, the process is reversible, and must obey the second law of thermodynamics; that is, the sum of the quantities of heat absorbed, each divided by the absolute temperature at which it is absorbed, must vanish. Thus we have

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* Cf. pp. 92, 93.

† Proc. R.S. Edinb. iii (1851), p. 91; Phil. Mag. iii (1852), p. 529: Kelvin's *Math. and Phys. Papers*, i, p. 316. Cf. also Trans. R. S. Edinb. xxi (1854), p. 123, reprinted in *Papers*, i, p. 232: and Phil. Trans., 1856, reprinted in *Papers*, ii, p. 189.

$$\frac{\Pi_B^A(T + \delta T)}{T + \delta T} - \frac{\Pi_B^A(T)}{T} = 0;$$

so the Peltier effect $\Pi_B^A(T)$ must be directly proportional to the absolute temperature T. This result, however, as Thomson well knew, was contradicted by the observations of Cumming, who had shown that when the temperature of the hot junction is gradually increased, the electromotive force rises to a maximum value and then decreases. The contradiction led Thomson to predict the existence of a hitherto unrecognized thermo-electric phenomenon—namely, a reversible absorption of heat at places in the circuit other than the junctions. Suppose that a current flows along a wire which is of the same metal throughout, but varies in temperature from point to point. Thomson showed that heat must be liberated at some points and absorbed at others, so as either to accentuate or to diminish the differences of temperature at the different points of the wire. Suppose that the heat absorbed from external sources when unit electric charge passes from the absolute temperature T to the temperature $(T + \delta T)$ in a metal A is denoted by $S_A(T) . \delta T$. The thermodynamical equation now takes the corrected form

$$\frac{\Pi_B^A(T + \delta T)}{T + \delta T} - \frac{\Pi_B^A(T)}{T} + \{S_B(T) - S_A(T)\}\frac{\delta T}{T} = 0.$$

Since the metals A and B are quite independent, this gives

$$\frac{\Pi_A(T+\delta T)}{T+\delta T}-\frac{\Pi_A(T)}{T}-S_A(T)\frac{\delta T}{T}=0,$$

or

$$S_A(T)=T\frac{d}{dT}\left\{\frac{\Pi_A(T)}{T}\right\}.$$

This equation connects Thomson's "specific heat of electricity" $S_A(T)$ with the Peltier effect.

In 1870 P. G. Tait* found experimentally that the specific heat of electricity in pure metals is proportional to the absolute temperature. We may therefore write $S_A(T)=\sigma_A T$, where σ_A denotes a constant characteristic of the metal A. The thermodynamical equation then becomes

$$\frac{d}{dT}\left\{\frac{\Pi_A(T)}{T}\right\}=\sigma_A,$$

or

$$\Pi_A(T)=\pi_A T+\sigma_A T^2,$$

where π_A denotes another constant characteristic of the metal. The chief part of the Peltier effect arises from the term $\pi_A T$.

By the investigations which have been described in the present chapter, the theory of electric currents was considerably advanced in several directions. In all these researches, however, attention was fixed on the conductor carrying the current as the seat of the phenomenon. In the following period, interest was centred not so much on the conductors which carry charges and currents, as on the processes which take place in the dielectric media around them.

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* Proc. R. S. Edinb. vii (1870), p. 308. Cf. also Batelli, Atti della R. Acc. di Torino, xxii (1886), p. 48, translated Phil. Mag. xxiv (1887), p. 295.

CHAPTER VIII.

MAXWELL.

SINCE the time of Descartes, natural philosophers have never ceased to speculate on the manner in which electric and magnetic influences are transmitted through space. About the middle of the nineteenth century, speculation assumed a definite form, and issued in a rational theory.

Among those who thought much on the matter was Karl Friedrich Gauss (*b.* 1777, *d.* 1855). In a letter* to Weber of date March 19, 1845, Gauss remarked that he had long ago proposed to himself to supplement the known forces which act between electric charges by other forces, such as would cause electric actions to be propagated between the charges with a finite velocity. But he expressed himself as determined not to publish his researches until he should have devised a mechanism by which the transmission could be conceived to be effected; and this he had not succeeded in doing.

More than one attempt to realize Gauss's aspiration was made by his pupil Riemann. In a fragmentary note,† which appears to have been written in 1853, but which was not published until after his death, Riemann proposed an aether whose elements should be endowed with the power of resisting compression, and also (like the elements of MacCullagh's aether) of resisting changes of orientation. The former property he conceived to be the cause of gravitational and electrostatic effects, and the latter to be the cause of optical and magnetic phenomena. The theory thus outlined was apparently not developed further by its author; but in a short investigation‡ which was published posthumously in 1867,§ he returned to the question of the process by which electric action is propagated through space. In this memoir he proposed to replace Poisson's equation for the electrostatic potential, namely,

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* Gauss' *Werke*, v, p. 629.

† Riemann's *Werke*, 2^e^ Aufl., p. 526.

‡ Ann. d. Phys. cxxxi (1867), p. 237; Riemann's *Werke*, 2^e^ Aufl., p. 288; Phil. Mag. xxxiv (1867), p. 368.

§ It had been presented to the Göttingen Academy in 1858, but afterwards withdrawn.

$$\nabla^2 V + 4\pi\rho = 0,$$

by the equation

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} + 4\pi\rho = 0,$$

according to which the changes of potential due to changing electrification would be propagated outwards from the charges with a velocity c. This, so far as it goes, is in agreement with the view which is now accepted as correct; but Riemann's hypothesis was too slight to serve as the basis of a complete theory. Success came only when the properties of the intervening medium were taken into account.

In that power to which Gauss attached so much importance, of devising dynamical models and analogies for obscure physical phenomena, perhaps no one has ever excelled W. Thomson*; and to him, jointly with Faraday, is due the credit of having initiated the theory of the electric medium. In one of his earliest papers, written at the age of seventeen,† Thomson compared the distribution of electrostatic force, in a region containing electrified conductors, with the distribution of the flow of heat in an infinite solid: the equipotential surfaces in the one case correspond to the isothermal surfaces in the other, and an electric charge corresponds to a source of heat.‡

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* As will appear from the present chapter, Maxwell had the same power in a very marked degree. It has always been cultivated by the "Cambridge school" of natural philosophers.

† Camb. Math. Journal, iii (Feb. 1842), p. 71; reprinted in Thomson's *Papers on Electrostatics and Magnetism*, p. 1. Also Camb. and Dub. Math. Journal, Nov., 1845; reprinted in *Papers*, p. 15.

‡ As regards this comparison, Thomson had been anticipated by Chasles, Journal de l'Éc. Polyt. xv (1837), p. 266, who had shown that attraction according to Newton's law gives rise to the same fields as the steady conduction of heat, both depending on Laplace's equation $\nabla^2 V = 0$.

It will be remembered that Ohm had used an analogy between thermal conduction and galvanic phenomena.

It may, perhaps, seem as if the value of such an analogy as this consisted merely in the prospect which it offered of comparing, and thereby extending, the mathematical theories of heat and electricity. But to the physicist its chief interest lay rather in the idea that formulae which relate to the electric field, and which had been deduced from laws of action at a distance, were shown to be identical with formulae relating to the theory of heat, which had been deduced from hypotheses of action between contiguous particles.

In 1846—the year after he had taken his degree as second wrangler at Cambridge—Thomson investigated* the analogies of electric phenomena with those of elasticity. For this purpose he examined the equations of equilibrium of an incompressible elastic solid which is in a state of strain; and showed that the distribution of the vector which represents the elastic displacement might be assimilated to the distribution of the electric force in an electrostatic system. This, however, as he went on to show, is not the only analogy which may be perceived with the equations of elasticity; for the elastic displacement may equally well be identified with a vector $\mathbf{a}$, defined in terms of the magnetic induction $\mathbf{B}$ by the relation

$$\operatorname{curl} \mathbf{a} = \mathbf{B}.$$

The vector $\mathbf{a}$ is equivalent to the vector-potential which had been used in the memoirs of Neumann, Weber, and Kirchhoff, on the induction of currents; but Thomson arrived at it independently by a different process, and without being at the time aware of the identification.

The results of Thomson's memoir seemed to suggest a picture of the propagation of electric or magnetic force: might it not take place in somewhat the same way as changes in the elastic displacement are transmitted through an elastic solid? These suggestions were not at the time pursued further by their author; but they helped to inspire another young

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* Camb. and Dub. Math. Journ. ii (1847), p. 61: Thomson's *Math. and Phys. Papers*, i, p. 76.

Cambridge man to take up the matter a few years later. James Clerk Maxwell, by whom the problem was eventually solved, was born in 1831, the son of a landed proprietor in Dumfriesshire. He was educated at Edinburgh, and at Trinity College, Cambridge, of which society he became in 1855 a Fellow; and not long after his election to Fellowship, he communicated to the Cambridge Philosophical Society the first of his endeavours* to form a mechanical conception of the electro-magnetic field.

Maxwell had been reading Faraday's *Experimental Researches*; and, gifted as he was with a physical imagination akin to Faraday's, he had been profoundly impressed by the theory of lines of force. At the same time, he was a trained mathematician; and the distinguishing feature of almost all his researches was the union of the imaginative and the analytical faculties to produce results partaking of both natures. This first memoir may be regarded as an attempt to connect the ideas of Faraday with the mathematical analogies which had been devised by Thomson.

Maxwell considered first the illustration of Faraday's lines of force which is afforded by the lines of flow of a liquid. The lines of force represent the direction of a vector; and the magnitude of this vector is everywhere inversely proportional to the cross-section of a narrow tube formed by such lines. This relation between magnitude and direction is possessed by any circuital vector; and in particular by the vector which represents the velocity at any point in a fluid, if the fluid be incompressible. It is therefore possible to represent the magnetic induction **B**, which is the vector represented by Faraday's lines of magnetic force, as the velocity of an incompressible fluid. Such an analogy had been indicated some years previously by Faraday himself,† who had suggested that along the lines of magnetic force there may be a "dynamic condition," analogous to that of the electric current, and that, in fact, "the physical lines of magnetic force are currents."

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* Trans. Camb. Phil. Soc. x, p. 27; Maxwell's *Scientific Papers*, i, p. 155.

† *Exp. Res.*, § 3269 (1852).

The comparison with the lines of flow of a liquid is applicable to electric as well as to magnetic lines of force. In this case the vector which corresponds to the velocity of the fluid is, in free aether, the electric force **E**. But when different dielectrics are present in the field, the electric force is not a circuital vector, and therefore cannot be represented by lines of force; in fact, the equation

$$\operatorname{div} \mathbf{E} = 0$$

is now replaced by the equation

$$\operatorname{div} (\epsilon \mathbf{E}) = 0,$$

where ε denotes the specific inductive capacity or dielectric constant at the place (x, y, z). It is, however, evident from this equation that the vector $\varepsilon\mathbf{E}$ is circuital; this vector, which will be denoted by **D**, bears to **E** a relation similar to that which the magnetic induction **B** bears to the magnetic force **H**. It is the vector **D** which is represented by Faraday's lines of electric force, and which in the hydrodynamical analogy corresponds to the velocity of the incompressible fluid.

In comparing fluid motion with electric fields it is necessary to introduce sources and sinks into the fluid to correspond to the electric charges; for **D** is not circuital at places where there is free charge. The magnetic analogy is therefore somewhat the simpler.

In the latter half of his memoir Maxwell discussed how Faraday's "electrotonic state" might be represented in mathematical symbols. This problem he solved by borrowing from Thomson's investigation of 1847 the vector **a**, which is defined in terms of the magnetic induction by the equation

$$\operatorname{curl} \mathbf{a} = \mathbf{B};$$

if, with Maxwell, we call **a** the *electrotonic intensity*, the equation is equivalent to the statement that "the entire electrotonic intensity round the boundary of any surface measures the number of lines of magnetic force which pass through that surface." The electromotive force of induction at the place (x, y, z) is $-\partial\mathbf{a}/\partial t$: as Maxwell said, "the electromotive force on any element of a conductor is measured by the instantaneous rate of change of the electrotonic intensity on that element." From this it is evident that **a** is no other than the vector-potential which had been employed by Neumann, Weber, and Kirchhoff, in the calculation of induced currents; and we may take* for the electrotonic intensity due to a current i' flowing in a circuit $\mathbf{s}'$ the value which results from Neumann's theory, namely,

$$\mathbf{a} = i' \int \frac{\mathbf{ds}'}{r}.$$

It may, however, be remarked that the equation

$$\operatorname{curl} \mathbf{a} = \mathbf{B},$$

taken alone, is insufficient to determine **a** uniquely; for we can choose **a** so as to satisfy this, and also to satisfy the equation

$$\operatorname{div} \mathbf{a} = \psi,$$

where ψ denotes any arbitrary scalar. There are, therefore, an infinite number of possible functions **a**. With the particular value of **a** which has been adopted, we have

$$\begin{aligned}\operatorname{div} \mathbf{a} &= \frac{\partial}{\partial x} i' \int_{s'} \frac{dx'}{r} + \frac{\partial}{\partial y} i' \int_{s'} \frac{dy'}{r} + \frac{\partial}{\partial z} i' \int_{s'} \frac{dz'}{r} \\ &= -i' \int_{s'} \left(dx' \cdot \frac{\partial}{\partial x'} + dy' \cdot \frac{\partial}{\partial y'} + dz' \cdot \frac{\partial}{\partial z'} \right) \left(\frac{1}{r} \right) \\ &= -i' \int_{s'} d \left(\frac{1}{r} \right) \\ &= 0;\end{aligned}$$

so the vector-potential **a** which we have chosen is circuital.

In this memoir the physical importance of the operators *curl* and *div* first became evident†; for, in addition to those applications which have been mentioned, Maxwell showed that he connexion between the strength $\imath$ of a current and the magnetic field **H**, to which it gives rise, may be represented by the equation

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* Cf. p. 224.

† These operators had, however, occurred frequently in the writings of Stokes, especially in his memoir of 1849 on the *Dynamical Theory of Diffraction*.

$$4\pi\imath = \text{curl}\,\mathbf{H};$$

this equation is equivalent to the statement that "the entire magnetic intensity round the boundary of any surface measures the quantity of electric current which passes through that surface."

In the same year (1856) in which Maxwell's investigation was published, Thomson* put forward an alternative interpretation of magnetism. He had now come to the conclusion, from a study of the magnetic rotation of the plane of polarization of light, that magnetism possesses a rotatory character; and suggested that the resultant angular momentum of the thermal motions of a body† might be taken as the measure of the magnetic moment. "The explanation," he wrote, "of all phenomena of electromagnetic attraction or repulsion, or of electromagnetic induction, is to be looked for simply in the inertia or pressure of the matter of which the motions constitute heat. Whether this matter is or is not electricity, whether it is a continuous fluid interpermeating the spaces between molecular nuclei, or is itself molecularly grouped: or whether all matter is continuous, and molecular heterogeneousness consists in finite vortical or other relative motions of contiguous parts of a body: it is impossible to decide, and, perhaps, in vain to speculate, in the present state of science."

The two interpretations of magnetism, in which the linear and rotatory characters respectively are attributed to it, occur frequently in the subsequent history of the subject. The former was amplified in 1858, when Helmholtz published his researches‡ on vortex motion; for Helmholtz showed that if a magnetic field produced by electric currents is compared to the flow of an incompressible fluid, so that the magnetic vector is represented by the fluid velocity, then the electric currents correspond to the vortex-filaments in the fluid. This analogy correlates many theorems in hydrodynamics and electricity; for instance, the theorem that a re-entrant vortex-filament is equivalent to a uniform distribution of doublets over any surface bounded by it, corresponds to Ampère's theorem of the equivalence of electric currents and magnetic shells.

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* Proc. Roy. Soc. viii (1856), p. 150; xi (1861), p. 327, foot-note: Phil. Mag. xiii (1857), p. 198; *Baltimore Lectures*, Appendix F.

† This was written shortly before the kinetic theory of gases was developed by Clausius and Maxwell.

‡ Journal für Math. lv (1858), p. 25; Helmholtz's *Wiss. Abh.* i, p. 101; translated Phil. Mag. xxxiii (1867), p. 485.

In his memoir of 1855, Maxwell had not attempted to construct a mechanical model of electrodynamic actions, but had expressed his intention of doing so. "By a careful study," he wrote,* "of the laws of elastic solids, and of the motions of viscous fluids, I hope to discover a method of forming a mechanical conception of this electrotonic state adapted to general reasoning"; and in a foot-note he referred to the effort which Thomson had already made in this direction. Six years elapsed, however, before anything further on the subject was published. In the meantime, Maxwell became Professor of Natural Philosophy in King's College, London—a position in which he had opportunities of personal contact with Faraday, whom he had long reverenced. Faraday had now concluded the *Experimental Researches*, and was living in retirement at Hampton Court; but his thoughts frequently recurred to the great problem which he had brought so near to solution. It appears from his note-book that in 1857† he was speculating whether the velocity of propagation of magnetic action is of the same order as that of light, and whether it is affected by the susceptibility to induction of the bodies through which the action is transmitted.

The answer to this question was furnished in 1861–2, when Maxwell fulfilled his promise of devising a mechanical conception of the electromagnetic field.‡

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* Maxwell's *Scientific Papers*, i, p. 188.

† Bence Jones's *Life of Faraday* ii, p. 379.

‡ Phil. Mag. xxi (1861), pp. 161, 281, 338; xxiii (1862), pp. 12, 85; Maxwell's *Scientific Papers*, i, p. 451.

In the interval since the publication of his previous memoir Maxwell had become convinced by Thomson's arguments that magnetism is in its nature rotatory. "The transference of electrolytes in fixed directions by the electric current, and the rotation of polarized light in fixed directions by magnetic force, are," he wrote, "the facts the consideration of which has induced me to regard magnetism as a phenomenon of rotation, and electric currents as phenomena of translation." This conception of magnetism he brought into connexion with Faraday's idea, that tubes of force tend to contract longitudinally and to expand laterally. Such a tendency may be attributed to centrifugal force, if it be assumed that each tube of force contains fluid which is in rotation about the axis of the tube. Accordingly Maxwell supposed that, in any magnetic field, the medium whose vibrations constitute light is in rotation about the lines of magnetic force; each unit tube of force may for the present be pictured as an isolated vortex.

The energy of the motion per unit volume is proportional to $\mu\mathbf{H}^2$, where μ denotes the density of the medium, and $\mathbf{H}$ denotes the linear velocity at the circumference of each vortex. But, as we have seen,* Thomson had already shown that the energy of any magnetic field, whether produced by magnets or by electric currents, is

$$\frac{1}{8\pi}\iiint \mu\mathbf{H}^2\,dx\,dy\,dz,$$

where the integration is taken over all space, and where μ denotes the magnetic permeability, and $\mathbf{H}$ the magnetic force. It was therefore natural to identify the density of the medium at any place with the magnetic permeability, and the circumferential velocity of the vortices with the magnetic force.

But an objection to the proposed analogy now presents itself. Since two neighbouring vortices rotate in the same direction, the particles in the circumference of one vortex must be moving in the opposite direction to the particles contiguous to them in the circumference of the adjacent vortex; and it seems, therefore, as if the motion would be discontinuous. Maxwell escaped from this difficulty by imitating a well-known mechanical arrangement. When it is desired that two wheels should revolve in the same sense, an "idle" wheel is inserted between them so as to be in gear with both. The model of the electromagnetic field to which Maxwell arrived by the introduction of this device greatly resembles that proposed by Bernoulli in 1736.* He supposed a layer of particles, acting as idle wheels, to be interposed between each vortex and the next, and to roll without sliding on the vortices; so that each vortex tends to make the neighbouring vortices revolve in the same direction as itself. The particles were supposed to be not otherwise constrained, so that the velocity of the centre of any particle would be the mean of the circumferential velocities of the vortices between which it is placed. This condition yields (in suitable units) the analytical equation

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* Cf. pp. 248, 250.

$$4\pi \mathbf{\imath} = \operatorname{curl} \mathbf{H},$$

where the vector $\mathbf{\imath}$ denotes the flux of the particles, so that its x-component i_x denotes the quantity of particles transferred in unit time across unit area perpendicular to the x-direction. On comparing this equation with that which represents Oersted's discovery, it is seen that the flux $\mathbf{\imath}$ of the movable particles interposed between neighbouring vortices is the analogue of the electric current.

It will be noticed that in Maxwell's model the relation between electric current and magnetic force is secured by a connexion which is not of a dynamical, but of a purely kinematical character. The above equation simply expresses the existence of certain non-holonomic constraints within the system.

If from any cause the rotatory velocity of some of the cellular vortices is altered, the disturbance will be propagated from that part of the model to all other parts, by the mutual action of the particles and vortices. This action is determined, as Maxwell showed, by the relation

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* Cf. p. 100.

$$\mu\dot{\mathbf{H}} = -\text{curl}\ \mathbf{E}$$

which connects **E**, the force exerted on a unit quantity of particles at any place in consequence of the tangential action of the vortices, with $\dot{\mathbf{H}}$, the rate of change of velocity of the neighbouring vortices. It will be observed that this equation is not kinematical but dynamical. On comparing it with the electromagnetic equations

$$\begin{cases} \text{curl}\ \mathbf{a} = \mu\mathbf{H}, \\ \text{Induced electromotive force} = -\dot{\mathbf{a}}, \end{cases}$$

it is seen that **E** must be interpreted electromagnetically as the induced electromotive force. Thus the motion of the particles constitutes an electric current, the tangential force with which they are pressed by the matter of the vortex-cells constitutes electromotive force, and the pressure of the particles on each other may be taken to correspond to the tension or potential of the electricity.

The mechanism must next be extended so as to take account of the phenomena of electrostatics. For this purpose Maxwell assumed that the particles, when they are displaced from their equilibrium position in any direction, exert a tangential action on the elastic substance of the cells; and that this gives rise to a distortion of the cells, which in turn calls into play a force arising from their elasticity, equal and opposite to the force which urges the particles away from the equilibrium position. When the exciting force is removed, the cells recover their form, and the electricity returns to its former position. The state of the medium, in which the electric particles are displaced in a definite direction, is assumed to represent an electrostatic field. Such a displacement does not itself constitute a current, because when it has attained a certain value it remains constant; but the variations of displacement are to be regarded as currents, in the positive or negative direction according as the displacement is increasing or diminishing.

The conception of the electrostatic state as a displacement of something from its equilibrium position was not altogether new, although it had not been previously presented in this form. Thomson, as we have seen, had compared electric force to the displacement in an elastic solid; and Faraday, who had likened the particles of a ponderable dielectric to small conductors embedded in an insulating medium,* had supposed that when the dielectric is subjected to an electrostatic field, there is a displacement of electric charge on each of the small conductors. The motion of these charges, when the field is varied, is equivalent to an electric current; and it was from this precedent that Maxwell derived the principle, which became of cardinal importance in his theory, that variations of displacement are to be counted as currents. But in adopting the idea, he altogether transformed it; for Faraday's conception of displacement was applicable only to ponderable dielectrics, and was in fact introduced solely in order to explain why the specific inductive capacity of such dielectrics is different from that of free aether; whereas according to Maxwell there is displacement wherever there is electric force, whether material bodies are present or not.

The difference between the conceptions of Faraday and Maxwell in this respect may be illustrated by an analogy drawn from the theory of magnetism. When a piece of iron is placed in a magnetic field, there is induced in it a magnetic distribution, say of intensity **I**; this induced magnetization exists only within the iron, being zero in the free aether outside. The vector **I** may be compared to the polarization or displacement, which according to Faraday is induced in dielectrics by an electric field; and the electric current constituted by the variation of this polarization is then analogous to $\partial \mathbf{I}/\partial t$. But the entity which was called by Maxwell the electric displacement in the dielectric is analogous not to **I**, but to the magnetic induction **B**: the Maxwellian displacement-current corresponds to $\partial\mathbf{B}/\partial t$, and may therefore have a value different from zero even in free aether.

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* Cf. p. 210.

It may be remarked in passing that the term *displacement*, which was thus introduced, and which has been retained in the later development of the theory, is perhaps not well chosen; what in the early models of the aether was represented as an actual displacement, has in later investigations been conceived of as a change of structure rather than of position in the elements of the aether.

Maxwell supposed the electromotive force acting on the electric particles to be connected with the displacement $\mathbf{D}$ which accompanies it, by an equation of the form

$$= \frac{1}{4\pi c_1^2}\,\mathbf{E},$$

where c_1 denotes a constant which depends on the elastic properties of the cells. The displacement-current $\dot{\mathbf{D}}$ must now be inserted in the relation which connects the current with the magnetic force; and thus we obtain the equation

$$\operatorname{curl} \mathbf{H} = 4\pi\mathbf{S},$$

where the vector $\mathbf{S}$, which is called the *total current*, is the sum of the convection-current $\mathbf{\imath}$ and the displacement-current $\dot{\mathbf{D}}$. By performing the operation *div* on both sides of this equation, it is seen that the total current is a circuital vector. In the model, the total current is represented by the total motion of the rolling particles; and this is conditioned by the rotations of the vortices in such a way as to impose the kinematic relation

$$\operatorname{div} \mathbf{S} = 0.$$

Having obtained the equations of motion of his system of vortices and particles, Maxwell proceeded to determine the rate of propagation of disturbances through it. He considered in particular the case in which the substance represented is a dielectric, so that the conduction-current is zero. If, moreover, the constant μ be supposed to have the value unity, the equations may be written

$$\begin{cases} \operatorname{div} \mathbf{H} = 0, \\ c_1^2 \operatorname{curl} \mathbf{H} = \dot{\mathbf{E}}, \\ -\operatorname{curl} \mathbf{E} = \dot{\mathbf{H}}. \end{cases}$$

Eliminating **E**, we see* that **H** satisfies the equations

$$\begin{cases} \operatorname{div} \mathbf{H} = 0, \\ \ddot{\mathbf{H}} = c_1^2 \nabla^2 \mathbf{H}. \end{cases}$$

But these are precisely the equations which the light-vector satisfies in a medium in which the velocity of propagation is c_1: it follows that disturbances are propagated through the model by waves which are similar to waves of light, the magnetic (and similarly the electric) vector being in the wave-front. For a plane-polarized wave propagated parallel to the axis of z, the equations reduce to

$$-c_1^2 \frac{\partial H_y}{\partial z} = \frac{\partial E_x}{\partial t}, \quad c_1^2 \frac{\partial H_x}{\partial z} = \frac{\partial E_y}{\partial t}, \quad \frac{\partial E_y}{\partial z} = \frac{\partial H_x}{\partial t}, \quad -\frac{\partial E_x}{\partial z} = \frac{\partial H_y}{\partial t},$$

whence we have

$$c_1 H_y = E_x, \qquad -c_1 H_x = E_y;$$

these equations show that the electric and magnetic vectors are at right angles to each other.

The question now arises as to the magnitude of the constant c_1.† This may be determined by comparing different expressions for the energy of an electrostatic field. The work done by an electromotive force **E** in producing a displacement **D** is

$$\int_0^{\mathbf{D}} \mathbf{E} \cdot d\mathbf{D} \quad \text{or} \quad \tfrac{1}{2}\mathbf{E}\mathbf{D}$$

per unit volume, since **E** is proportional to **D**. But if it be assumed that the energy of an electrostatic field is resident in the dielectric, the amount of energy per unit volume may be

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* For if **a** denote any vector, we have identically

$$\nabla^2 \mathbf{a} + \operatorname{grad} \operatorname{div} \mathbf{a} + \operatorname{curl} \operatorname{curl} \mathbf{a} = 0.$$

† For criticisms on the procedure by which Maxwell determined the velocity of propagation of disturbance, cf. P. Duhem, *Les Théories Électriques de J. Clerk Maxwell*, Paris, 1902.

calculated by considering the mechanical force required in order to increase the distance between the plates of a condenser, so as to enlarge the field comprised between them. The result is that the energy per unit volume of the dielectric is $\varepsilon \mathbf{E}'^2/8\pi$, where ε denotes the specific inductive capacity of the dielectric and $\mathbf{E}'$ denotes the electric force, measured in terms of the electrostatic unit: if $\mathbf{E}$ denotes the electric force expressed in terms of the electrodynamic units used in the present investigation, we have $\mathbf{E} = c\mathbf{E}'$, where c denotes the constant which* occurs in transformations of this kind. The energy is therefore $\varepsilon \mathbf{E}^2/8\pi c^2$ per unit volume. Comparing this with the expression for the energy in terms of $\mathbf{E}$ and $\mathbf{D}$, we have

$$\mathbf{D} = \epsilon \mathbf{E}/4\pi c^2,$$

and therefore the constant c_1 has the value $c\varepsilon^{-\frac{1}{2}}$. Thus the result is obtained that the velocity of propagation of disturbances in Maxwell's medium is $c\varepsilon^{-\frac{1}{2}}$, where ε denotes the specific inductive capacity and c denotes the velocity for which Kohlrausch and Weber had found† the value $3{\cdot}1 \times 10^{10}$ cm./sec.

Now by this time the velocity of light was known, not only from the astronomical observations of aberration and of Jupiter's satellites, but also by direct terrestrial experiments. In 1849 Hippolyte Louis Fizeau‡ had determined it by rotating a toothed wheel so rapidly that a beam of light transmitted through the gap between two teeth and reflected back from a mirror was eclipsed by one of the teeth on its return journey. The velocity of light was calculated from the dimensions and angular velocity of the wheel and the distance of the mirror; the result being $3{\cdot}15 \times 10^{10}$ cm./sec.§

* Cf. pp. 227, 259.

† Cf. p. 260.

‡ Comptes Rendus, xxix (1849), p. 90. A determination made by Cornu in 1874 was on this principle.

§ A different experimental method was employed in 1862 by Léon Foucault (Comptes Rendus, lv, pp. 501, 792); in this a ray from an origin O was reflected by a revolving mirror M to a fixed mirror, and so reflected back to M, and again to O. It is evident that the returning ray MO must be deviated by twice the angle through which M turns while the light passes from M to the fixed mirror and back. The value thus obtained by Foucault for the velocity of light was

Maxwell was impressed, as Kirchhoff had been before him, by the close agreement between the electric ratio c and the velocity of light*; and having demonstrated that the propagation of electric disturbance resembles that of light, he did not hesitate to assert the identity of the two phenomena. "We can scarcely avoid the inference," he said, "that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena." Thus was answered the question which Priestley had asked almost exactly a hundred years before:† "Is there any electric fluid *sui generis* at all, distinct from the aether?"

The presence of the dielectric constant ε in the expression $c\varepsilon^{-\frac{1}{2}}$, which Maxwell had obtained for the velocity of propagation of electromagnetic disturbances, suggested a further test of the identity of these disturbances with light: for the velocity of light in a medium is known to be inversely proportional to the refractive index of the medium, and therefore the refractive index should be, according to the theory, proportional to the square root of the specific inductive capacity. At the time, however, Maxwell did not examine whether this relation was confirmed by experiment.

In what has preceded, the magnetic permeability μ has been supposed to have the value unity. If this is not the case, the

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$2{\cdot}98 \times 10^{10}$ cm./sec. Subsequent determinations by Michelson in 1879 (Ast. Papers of the Amer. Ephemeris, i), and by Newcomb in 1882 (*ibid.*, ii) depended on the same principle.

As was shown afterwards by Lord Rayleigh (Nature, xxiv, p. 382, xxv, p. 52) and by Gibbs (Nature, xxxiii, p. 582), the value obtained for the velocity of light by the methods of Fizeau and Foucault represents the *group-velocity*, not the *wave-velocity*; the eclipses of Jupiter's satellites also give the group-velocity, while the value deduced from the coefficient of aberration is the wave-velocity. In a non-dispersive medium, the group-velocity coincides with the wave-velocity; and the agreement of the values of the velocity of light obtained by the two astronomical methods seems to negative the possibility of any appreciable dispersion in free aether.

The velocity of light in dispersive media was directly investigated by Michelson in 1883–4, with results in accordance with theory.

* He had "worked out the formulae in the country, before seeing Weber's result." Cf. Campbell and Garnett's *Life of Maxwell*, p. 244.

† Priestley's *History*, p. 488.

velocity of propagation of disturbance may be shown, by the same analysis, to be $c\epsilon^{-\frac{1}{2}}\mu^{-\frac{1}{2}}$; so that it is diminished when μ is greater than unity, i.e., in paramagnetic bodies. This inference had been anticipated by Faraday: "Nor is it likely," he wrote,* "that the paramagnetic body oxygen can exist in the air and not retard the transmission of the magnetism."

It was inevitable that a theory so novel and so capacious as that of Maxwell should involve conceptions which his contemporaries understood with difficulty and accepted with reluctance. Of these the most difficult and unacceptable was the principle that the total current is always a circuital vector; or, as it is generally expressed, that "all currents are closed." According to the older electricians, a current which is employed in charging a condenser is not closed, but terminates at the coatings of the condenser, where charges are accumulating. Maxwell, on the other hand, taught that the dielectric between the coatings is the seat of a process—the *displacement-current*—which is proportional to the rate of increase of the electric force in the dielectric; and that this process produces the same magnetic effects as a true current, and forms, so to speak, a continuation, through the dielectric, of the charging current, so that the latter may be regarded as flowing in a closed circuit.

Another characteristic feature of Maxwell's theory is the conception—for which, as we have seen, he was largely indebted to Faraday and Thomson—that magnetic energy is the kinetic energy of a medium occupying the whole of space, and that electric energy is the energy of strain of the same medium. By this conception electromagnetic theory was brought into such close parallelism with the elastic-solid theories of the aether, that it was bound to issue in an electromagnetic theory of light.

Maxwell's views were presented in a more developed form in a memoir entitled "A Dynamical Theory of the Electromagnetic Field," which was read to the Royal Society in 1864;†

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* Faraday's laboratory note-book for 1857: cf. Bence Jones's *Life of Faraday*, ii, p. 380.

† Phil. Trans. clv (1865), p. 459: Maxwell's *Scient. Papers*, i, p. 526

in this the architecture of his system was displayed, stripped of the scaffolding by aid of which it had been first erected.

As the equations employed were for the most part the same as had been set forth in the previous investigation, they need only be briefly recapitulated. The magnetic induction $\mu\mathbf{H}$, being a circuital vector, may be expressed in terms of a vector-potential $\mathbf{A}$ by the equation

$$\mu\mathbf{H} = \operatorname{curl} \mathbf{A}.$$

The electric displacement $\mathbf{D}$ is connected with the volume-density ρ of free electric charge by the electrostatic equation

$$\operatorname{div} \mathbf{D} = \rho.$$

The principle of conservation of electricity yields the equation

$$\operatorname{div} \imath = - \partial\rho/\partial t,$$

where $\imath$ denotes the conduction-current.

The law of induction of currents—namely, that the total electromotive force in any circuit is proportional to the rate of decrease of the number of lines of magnetic induction which pass through it—may be written

$$- \operatorname{curl} \mathbf{E} = \mu\dot{\mathbf{H}};$$

from which it follows that the electric force $\mathbf{E}$ must be expressible in the form

$$\mathbf{E} = - \dot{\mathbf{A}} + \operatorname{grad} \psi,$$

where ψ denotes some scalar function. The quantities $\mathbf{A}$ and ψ which occur in this equation are not as yet completely determinate; for the equation by which $\mathbf{A}$ is defined in terms of the magnetic induction specifies only the circuital part of $\mathbf{A}$; and as the irrotational part of $\mathbf{A}$ is thus indeterminate, it is evident that ψ also must be indeterminate. Maxwell decided the matter by assuming* $\mathbf{A}$ to be a circuital vector; thus

$$\operatorname{div} \mathbf{A} = 0,$$

and therefore

$$\operatorname{div} \mathbf{E} = - \nabla^2\psi,$$

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* This is the effect of the introduction of (F', G', H') in § 98 of the memoir; cf. also Maxwell's *Treatise on Electricity and Magnetism*, § 616.

from which equation it is evident that ψ represents the electrostatic potential.

The principle which is peculiar to Maxwell's theory must now be introduced. Currents of conduction are not the only kind of currents; even in the older theory of Faraday, Thomson, and Mossotti, it had been assumed that electric charges are set in motion in the particles of a dielectric when the dielectric is subjected to an electric field; and the predecessors of Maxwell would not have refused to admit that the motion of these charges is in some sense a current. Suppose, then, that $\mathbf{S}$ denotes the total current which is capable of generating a magnetic field: since the integral of the magnetic force round any curve is proportional to the electric current which flows through the gap enclosed by the curve, we have in suitable units

$$\operatorname{curl} \mathbf{H} = 4\pi\mathbf{S}.$$

In order to determine $\mathbf{S}$, we may consider the case of a condenser whose coatings are supplied with electricity by a conduction-current $\mathbf{i}$ per unit-area of coating. If $\pm\,\sigma$ denote the surface-density of electric charge on the coatings, we have

$$i = \partial\sigma/\partial t, \quad \text{and} \quad \sigma = D,$$

where D denotes the magnitude of the electric displacement $\mathbf{D}$ in the dielectric between the coatings; so $\mathbf{i} = \dot{\mathbf{D}}$. But since the total current is to be circuital, its value in the dielectric must be the same as the value $\mathbf{i}$ which it has in the rest of the circuit; that is, the current in the dielectric has the value $\dot{\mathbf{D}}$. We shall assume that the current in dielectrics always has this value, so that in the general equations the total current must be understood to be $\mathbf{i} + \dot{\mathbf{D}}$.

The above equations, together with those which express the proportionality of $\mathbf{E}$ to $\mathbf{D}$ in insulators, and to $\mathbf{i}$ in conductors, constituted Maxwell's system for a field formed by isotropic bodies which are not in motion. When the magnetic field is due entirely to currents (including both conduction-currents and displacement-currents), so that there is no magnetization, we have

$$\nabla^2 \mathbf{A} = - \operatorname{curl} \operatorname{curl} \mathbf{A} = - \operatorname{curl} \mathbf{H}$$
$$= - 4\pi \mathbf{S},$$

so that the vector-potential is connected with the total current by an equation of the same form as that which connects the scalar potential with the density of electric charge. To these potentials Maxwell inclined to attribute a physical significance; he supposed ψ to be analogous to a pressure subsisting in the mass of particles in his model, and **A** to be the measure of the electrotonic state. The two functions are, however, of merely analytical interest, and do not correspond to physical entities. For let two oppositely-charged conductors, placed close to each other, give rise to an electrostatic field throughout all space. In such a field the vector-potential **A** is everywhere zero, while the scalar potential ψ has a definite value at every point. Now let these conductors discharge each other; the electrostatic force at any point of space remains unchanged until the point in question is reached by a wave of disturbance, which is propagated outwards from the conductors with the velocity of light, and which annihilates the field as it passes over it. But this order of events is not reflected in the behaviour of Maxwell's functions ψ and **A**; for at the instant of discharge, ψ is everywhere annihilated, and **A** suddenly acquires a finite value throughout all space.

As the potentials do not possess any physical significance, it is desirable to remove them from the equations. This was afterwards done by Maxwell himself, who[*] in 1868 proposed to base the electromagnetic theory of light solely on the equations

$$\operatorname{curl} \mathbf{H} = 4\pi \mathbf{S},$$
$$- \operatorname{curl} \mathbf{E} = \dot{\mathbf{B}},$$

together with the equations which define **S** in terms of **E**, and **B** in terms of **H**.

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[*] Phil. Trans. clviii (1868), p. 643: Maxwell's *Scient. Papers*, ii, p. 125.

The memoir of 1864 contained an extension of the equations to the case of bodies in motion; the consideration of which naturally revives the question as to whether the aether is in any degree carried along with a body which moves through it. Maxwell did not formulate any express doctrine on this subject; but his custom was to treat matter as if it were merely a modification of the aether, distinguished only by altered values of such constants as the magnetic permeability and the specific inductive capacity; so that his theory may be said to involve the assumption that matter and aether move together. In deriving the equations which are applicable to moving bodies, he made use of Faraday's principle that the electromotive force induced in a body depends only on the relative motion of the body and the lines of magnetic force, whether one or the other is in motion absolutely. From this principle it may be inferred that the equation which determines the electric force* in terms of the potentials, in the case of a body which is moving with velocity **w**, is

$$\mathbf{E} = [\mathbf{w} \,.\, \mu\mathbf{H}] - \dot{\mathbf{A}} + \text{grad } \psi.$$

Maxwell thought that the scalar quantity ψ in this equation represented the electrostatic potential; but the researches of other investigators† have indicated that it represents the sum of the electrostatic potential and the quantity $(\mathbf{A} \,.\, \mathbf{w})$.

The electromagnetic theory of light was moreover extended in this memoir so as to account for the optical properties of crystals. For this purpose Maxwell assumed that in crystals the values of the coefficients of electric and magnetic induction depend on direction, so that the equation

$$\mu\mathbf{H} = \text{curl } \mathbf{A}$$

is replaced by

$$(\mu_1 H_x,\ \mu_2 H_y,\ \mu_3 H_z) = \text{curl } \mathbf{A};$$

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* It may be here remarked that later writers have distinguished between the electric force in a moving body and the electric force in the aether through which the body is moving, and that **E** in the present equation corresponds to the former of these vectors.

† Helmholtz, Journ. für Math., lxxviii (1874), p. 309; H. W. Watson, Phil. Mag. (5), xxv (1888), p. 271.

and similarly the equation

$$\mathbf{E} = 4\pi c^2 \mathbf{D}/\epsilon$$

is replaced by

$$\mathbf{E} = 4\pi\ (c_1^2 D_x,\ c_2^2 D_y,\ c_3^2 D_z).$$

The other equations are the same as in isotropic media; so that the propagation of disturbance is readily seen to depend on the equation

$$(\mu_1 \ddot{H}_x, \mu_2 \ddot{H}_y, \mu_3 \ddot{H}_z) = -\operatorname{curl}\{c_1^2 (\operatorname{curl} H)_x,\ c_2^2 (\operatorname{curl} H)_y,\ c_3^2 (\operatorname{curl} H)_z\}.$$

Now, if μ_1, μ_2, μ_3 are supposed equal to each other, this equation is the same as the equation of motion of MacCullagh's aether in crystalline media,* the magnetic force **H** corresponding to MacCullagh's elastic displacement; and we may therefore immediately infer that Maxwell's electromagnetic equations yield a satisfactory theory of the propagation of light in crystals, provided it is assumed that the magnetic permeability is (for optical purposes) the same in all directions, and provided the plane of polarization is identified with the plane which contains the magnetic vector. It is readily shown that the direction of the ray is at right angles to the magnetic vector and the electric force, and that the wave-front is the plane of the magnetic vector and the electric displacement.†

After this Maxwell proceeded to investigate the propagation of light in metals. The difference between metals and dielectrics, so far as electricity is concerned, is that the former are conductors; and it was therefore natural to seek the cause of the optical properties of metals in their ohmic conductivity. This idea at once suggested a physical reason for the opacity of metals—namely, that within a metal the energy of the light vibrations is converted into Joulian heat in the same way as the energy of ordinary electric currents.

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* Cf. pp. 154 et sqq.

† In the memoir of 1864 Maxwell left open the choice between the above theory and that which is obtained by assuming that in crystals the specific inductive capacity is (for optical purposes) the same in all directions, while the magnetic permeability is aeolotropic. In the latter case the plane of polarization must be identified with the plane which contains the electric displacement. Nine years later, in his *Treatise* (§ 794), Maxwell definitely adopted the former alternative.

The equations of the electromagnetic field in the metal may be written

$$\begin{cases} \text{curl}\, \mathbf{H} = 4\pi \mathbf{S}, \\ -\text{curl}\, \mathbf{E} = \dot{\mathbf{H}}, \\ \quad \mathbf{S} = \mathbf{i} + \dot{\mathbf{D}} = \kappa \mathbf{E} + \varepsilon \dot{\mathbf{E}}/4\pi c^2, \end{cases}$$

where κ denotes the ohmic conductivity; whence it is seen that the electric force satisfies the equation

$$\varepsilon \ddot{\mathbf{E}} + 4\pi\kappa c^2 \dot{\mathbf{E}} = c^2 \nabla^2 \mathbf{E}.$$

This is of the same form as the corresponding equation in the elastic-solid theory*; and, like it, furnishes a satisfactory general explanation of metallic reflexion. It is indeed correct in all details, so long as the period of the disturbance is not too short—i.e., so long as the light-waves considered belong to the extreme infra-red region of the spectrum; but if we attempt to apply the theory to the case of ordinary light, we are confronted by the difficulty which Lord Rayleigh indicated in the elastic-solid theory,† and which attends all attempts to explain the peculiar properties of metals by inserting a viscous term in the equation. The difficulty is that, in order to account for the properties of ideal silver, we must suppose the coefficient of $\ddot{\mathbf{E}}$ negative—that is, the dielectric constant of the metal must be negative, which would imply instability of electrical equilibrium in the metal. The problem, as we have already remarked,‡ was solved only when its relation to the theory of dispersion was rightly understood.

At this time important developments were in progress in the last-named subject. Since the time of Fresnel, theories of dispersion had proceeded§ from the assumption that the radii of action of the particles of luminiferous media are so large as to be comparable with the wave-length of light. It was generally supposed that the aether is loaded by the molecules of ponderable matter, and that the amount of dispersion depends on the ratio of the wave-length to the distance between adjacent molecules. This hypothesis was, however, seen to be inadequate, when, in 1862, F. P. Leroux* found that a prism filled with the vapour of iodine refracted the red rays to a greater degree than the blue rays; for in all theories which depend on the assumption of a coarse-grained luminiferous medium, the refractive index increases with the frequency of the light.

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* Cf. p. 180.

† Cf. p. 181. Cf. also Rayleigh, Phil. Mag. (5) xii (1881), p. 81, and H. A. Lorentz, *Over de Theorie de Terugkaatsing*, Arnhem, 1875.

‡ Cf. p. 181.

§ Cf. p. 182.

Leroux's phenomenon, to which the name *anomalous dispersion* was given, was shown by later investigators† to be generally associated with "surface-colour," i.e., the property of brilliantly reflecting incident light of some particular frequency. Such an association seemed to indicate that the dispersive property of a substance is intimately connected with a certain frequency of vibration which is peculiar to that substance, and which, when it happens to fall within the limits of the visible spectrum, is apparent in the surface-colour. This idea of a frequency of vibration peculiar to each kind of ponderable matter is found in the writings of Stokes as far back as the year 1852;‡ when, discussing fluorescence, he remarked:—"Nothing seems more natural than to suppose that the incident vibrations of the luminiferous aether produce vibratory movements among the ultimate molecules of sensitive substances, and that the molecules in turn, swinging on their own account, produce vibrations in the luminiferous aether, and thus cause the sensation of light. The periodic times of these vibrations depend on the periods in which the molecules are disposed to swing, not upon the periodic time of the incident vibrations."

The principle here introduced, of considering the molecules as dynamical systems which possess natural free periods, and which interact with the incident vibrations, lies at the basis of all modern theories of dispersion. The earliest of these was devised by Maxwell, who, in the Cambridge Mathematical Tripos for 1869,[*] published the results of the following investigation:—

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* Comptes Rendus, lv (1862), p. 126. In 1870 C. Christiansen (Ann. d. Phys. cxli, p. 479; cxliii, p. 250) observed a similar effect in a solution of fuchsin.

† Especially by Kundt, in a series of papers in the Annalen d. Phys., from vol. cxlii (1871) onwards.

‡ Phil. Trans., 1852, p. 463. Stokes's *Coll. Papers*, iii., p. 267.

A model of a dispersive medium may be constituted by embedding systems which represent the atoms of ponderable matter in a medium which represents the aether. We may picture each atom[†] as composed of a single massive particle supported symmetrically by springs from the interior face of a massless spherical shell: if the shell be fixed, the particle will be capable of executing vibrations about the centre of the sphere, the effect of the springs being equivalent to a force on the particle proportional to its distance from the centre. The atoms thus constituted may be supposed to occupy small spherical cavities in the aether, the outer shell of each atom being in contact with the aether at all points and partaking of its motion. An immense number of atoms is supposed to exist in each unit volume of the dispersive medium, so that the medium as a whole is fine-grained.

Suppose that the potential energy of strain of free aether per unit volume is

$$\tfrac{1}{2}E\left(\frac{\partial\eta}{\partial x}\right)^2,$$

where η denotes the displacement and E an elastic constant; so that the equation of wave-propagation in free aether is

$$\rho\frac{\partial^2\eta}{\partial t^2} = E\frac{\partial^2\eta}{\partial x^2},$$

where ρ denotes the aethereal density.

Then if σ denote the mass of the atomic particles in unit volume, $(\eta + \zeta)$ the total displacement of an atomic particle at the place x at time t, and $\sigma p^2\zeta$ the attractive force, it is evident that for the compound medium the kinetic energy per unit volume is

$$\tfrac{1}{2}\rho\left(\frac{\partial\eta}{\partial t}\right)^2 + \tfrac{1}{2}\sigma\left(\frac{\partial\eta}{\partial t} + \frac{\partial\zeta}{\partial t}\right)^2,$$

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[*] Cambridge Calendar, 1869; republished by Lord Rayleigh, Phil. Mag. xlviii (1899), p. 151.

[†] This illustration is due to W. Thomson.

and the potential energy per unit volume is

$$\tfrac{1}{2}E\,(\partial\eta/\partial x)^2 + \tfrac{1}{2}\sigma p^2\zeta^2.$$

The equations of motion, derived by the process usual in dynamics, are

$$\begin{cases} \rho\dfrac{\partial^2\eta}{\partial t^2} + \sigma\left(\dfrac{\partial^2\eta}{\partial t^2} + \dfrac{\partial^2\zeta}{\partial t^2}\right) - E\dfrac{\partial^2\eta}{\partial x^2} = 0, \\ \sigma\left(\dfrac{\partial^2\eta}{\partial t^2} + \dfrac{\partial^2\zeta}{\partial t^2}\right) + \sigma p^2\zeta = 0. \end{cases}$$

Consider the propagation, through the medium thus constituted, of vibrations whose frequency is n, and whose velocity of propagation in the medium is v; so that η and ζ are harmonic functions of $n(t - x/v)$. Substituting these values in the differential equations, we obtain

$$\frac{1}{v^2} = \frac{\rho}{E} + \frac{\sigma p^2}{E\,(p^2 - n^2)}\cdot$$

Now, ρ/E has the value $1/c^2$, where c denotes the velocity of light in free aether; and c/v is the refractive index μ of the medium for vibrations of frequency n. So the equation, which may be written

$$\mu^2 = 1 + \frac{\sigma p^2}{\rho\,(p^2 - n^2)},$$

determines the refractive index of the substance for vibrations of any frequency n. The same formula was independently obtained from similar considerations three years later by W. Sellmeier.*

If the oscillations are very slow, the incident light being in the extreme infra-red part of the spectrum, n is small, and the equation gives approximately $\mu^2 = (\rho + \sigma)/\rho$: for such oscillations, each atomic particle and its shell move together as a rigid body, so that the effect is the same as if the aether were simply loaded by the masses of the atomic particles, its rigidity remaining unaltered.

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* Ann. d. Phys. cxlv (1872), pp. 399, 520: cxlvii (1872), pp. 386, 525. Cf. also Helmholtz, Ann. d. Phys. cliv (1875), p. 582.

The dispersion of light within the limits of the visible spectrum is for most substances controlled by a natural frequency p which corresponds to a vibration beyond the violet end of the visible spectrum: so that, n being smaller than p, we may expand the fraction in the formula of dispersion, and obtain the equation

$$\mu^2 = 1 + \frac{\sigma}{\rho}\left(1 + \frac{n^2}{p^2} + \frac{n^4}{p^4} + \dots\right),$$

which resembles the formula of dispersion in Cauchy's theory*; indeed, we may say that Cauchy's formula is the expansion of Maxwell's formula in a series which, as it converges only when n has values within a limited range, fails to represent the phenomena outside that range.

The theory as given above is defective in that it becomes meaningless when the frequency n of the incident light is equal to the frequency p of the free vibrations of the atoms. This defect may be remedied by supposing that the motion of an atomic particle relative to the shell in which it is contained is opposed by a dissipative force varying as the relative velocity; such a force suffices to prevent the forced vibration from becoming indefinitely great as the period of the incident light approaches the period of free vibration of the atoms; its introduction is justified by the fact that vibrations in this part of the spectrum suffer absorption in passing through the medium. When the incident vibration is not in the same region of the spectrum as the free vibration, the absorption is not of much importance, and may be neglected.

It is shown by the spectroscope that the atomic systems which emit and absorb radiation in actual bodies possess more than one distinct free period. The theory already given may, however, readily be extended† to the case in which the atoms have several natural frequencies of vibration; we have only to suppose that the external massless rigid shell is connected by springs to an interior massive rigid shell, and that this again is connected by springs to another massive shell inside it, and so on. The corresponding extension of the equation for the refractive index is

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* Cf. p. 183.

† This subject was developed by Lord Kelvin in the *Baltimore Lectures.*

$$\mu^2 - 1 = \frac{c_1}{p_1^2 - n^2} + \frac{c_2}{p_2^2 - n^2} + \ldots,$$

where $p_1, p_2, \ldots$ denote the frequencies of the natural periods of vibration of the atom.

The validity of the Maxwell-Sellmeier formula of dispersion was strikingly confirmed by experimental researches in the closing years of the nineteenth century. In 1897 Rubens* showed that the formula represents closely the refractive indices of sylvin (potassium chloride) and rock-salt, with respect to light and radiant heat of wave-lengths between 4,240 A.U. and 223,000 A.U. The constants in the formula being known from this comparison, it was possible to predict the dispersion for radiations of still lower frequency; and it was found that the square of the refractive index should have a negative value (indicating complete reflexion) for wave-lengths 370,000 A.U. to 550,000 A.U. in the case of rock-salt, and for wave-lengths 450,000 to 670,000 A.U. in the case of sylvin. This inference was verified experimentally in the following year.†

It may seem strange that Maxwell, having successfully employed his electromagnetic theory to explain the propagation of light in isotropic media, in crystals, and in metals, should have omitted to apply it to the problem of reflexion and refraction. This is all the more surprising, as the study of the optics of crystals had already revealed a close analogy between the electromagnetic theory and MacCullagh's elastic-solid theory; and in order to explain reflexion and refraction electromagnetically, nothing more was necessary than to transcribe MacCullagh's investigation of the same problem, interpreting $\dot{\mathbf{e}}$ (the time-flux of the displacement of MacCullagh's aether) as the magnetic force, and curl $\mathbf{e}$ as the electric displacement. As in MacCullagh's theory the difference between the contiguous media is represented by a difference of their elastic constants, so in the electromagnetic theory it may be represented by a difference in their specific inductive capacities. From a letter which Maxwell wrote to Stokes in 1864, and which has been preserved,* it appears that the problem of reflexion and refraction was engaging Maxwell's attention at the time when he was preparing his Royal Society memoir on the electromagnetic field; but he was not able to satisfy himself regarding the conditions which should be satisfied at the interface between the media. He seems to have been in doubt which of the rival elastic-solid theories to take as a pattern; and it is not unlikely that he was led astray by relying too much on the analogy between the electric displacement and an elastic displacement.† For in the elastic-solid theory all three components of the displacement must be continuous across the interface between two contiguous media; but Maxwell found that it was impossible to explain reflexion and refraction if all three components of the electric displacement were supposed to be continuous across the interface; and, unwilling to give up the analogy which had hitherto guided him aright, yet unable to disprove‡ the Greenian conditions at bounding surfaces, he seems to have laid aside the problem until some new light should dawn upon it.

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* Ann. d. Phys. lx (1897), p. 454.

† Rubens and Aschkinass, Ann. d. Phys. lxiv (1898).

This was not the only difficulty which beset the electromagnetic theory. The theoretical conclusion, that the specific inductive capacity of a medium should be equal to the square of its refractive index with respect to waves of long period, was not as yet substantiated by experiment; and the theory of displacement-currents, on which everything else depended, was unfavourably received by the most distinguished of Maxwell's contemporaries. Helmholtz indeed ultimately accepted it, but only after many years; and W. Thomson (Kelvin) seems never to have thoroughly believed it to the end of his long life. In 1888 he referred to it as a "curious and ingenious, but not wholly tenable hypothesis,"* and proposed† to replace it by an extension of the older potential theories. In 1896 he had some inclination‡ to speculate that alterations of electrostatic force due to rapidly-changing electrification are propagated by condensational waves in the luminiferous aether. In 1904 he admitted§ that a bar-magnet rotating about an axis at right angles to its length is equivalent to a lamp emitting light of period equal to the period of the rotation, but gave his final judgment in the sentence‖:—"The so-called electromagnetic theory of light has not helped us hitherto."

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* Stokes's *Scientific Correspondence*, ii, pp. 25, 26.

† It must be remembered that Maxwell pictured the electric displacement as a real displacement of a medium. "My theory of electrical forces," he wrote, "is that they are called into play in insulating media by slight electric displacements, which put certain small portions of the medium into a state of distortion, which, being resisted by the elasticity of the medium, produces an electromotive force." Campbell and Garnett's *Life of Maxwell*, p. 244.

‡ The letter to Stokes already mentioned appears to indicate that Maxwell for a time doubted the correctness of Green's conditions.

Thomson appears to have based his ideas of the propagation of electric disturbance on the case which had first become familiar to him—that of the transmission of signals along a wire. He clung to the older view that in such a disturbance the wire is the actual medium of transmission; whereas in Maxwell's theory the function of the wire is merely to guide the disturbance, which is resident in the surrounding dielectric.

This opinion that conductors are the media of propagation of electric disturbance was entertained also by Ludwig Lorenz (*b.* 1829, *d.* 1891), of Copenhagen, who independently developed an electromagnetic theory of light¶ a few years after the publication of Maxwell's memoirs. The procedure which Lorenz followed was that which Riemann had suggested** in 1858—namely, to modify the accepted formulae of electrodynamics by introducing terms which, though too small to be appreciable in ordinary laboratory experiments, would be capable of accounting for the propagation of electrical effects through space with a finite velocity. We have seen that in Neumann's theory the electric force **E** was determined by the equation

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* Nature, xxxviii (1888) p. 571.

† Brit. Assoc. Report, 1888, p. 567.

‡ Cf. Bottomley, in Nature, liii (1896), p. 268; Kelvin, ib., p. 316; J. Willard Gibbs, ib., p. 509.

§ *Baltimore Lectures* (ed. 1904), p. 376.

‖ *Ibid.*, preface, p. 7.

¶ *Oversigt over det K. danske Vid. Selskaps Forhandlinger*, 1867, p. 26; Annal. der Phys. cxxxi (1867), p. 243; Phil. Mag., xxxiv (1867), p. 287.

** Cf. p. 268. Riemann's memoir was, however, published only in the same year (1867) as Lorenz's.

$$\mathbf{E} = c^2 \operatorname{grad} \phi - \dot{\mathbf{a}}, \qquad (1)$$

where ϕ denotes the electrostatic potential defined by the equation

$$\phi = \iiint (\rho'/r)\, dx'dy'dz',$$

ρ' being the density of electric charge at the point (x', y', z'), and where **a** denotes the vector-potential, defined by the equation

$$a = \iiint (\mathrm{\imath}'/r)\, dx'dy'dz',$$

$\mathrm{\imath}'$ being the conduction-current at (x', y', z'). We suppose the specific inductive capacity and the magnetic permeability to be everywhere unity.

Lorenz proposed to replace these by the equations

$$\phi = \iiint \{\rho'(t - r/c)/r\}\, dx'dy'dz',$$

$$\mathbf{a} = \iiint \{\mathrm{\imath}'(t - r/c)/r\}\, dx'dy'dz';$$

the change consists in replacing the values which ρ' and $\mathrm{\imath}'$ have at the instant t by those which they have at the instant $(t - r/c)$, which is the instant at which a disturbance travelling with velocity c must leave the place (x', y', z') in order to arrive at the place (x, y, z) at the instant t. Thus the values of the potentials at (x, y, z) at any instant t would, according to Lorenz's theory, depend on the electric state at the point (x', y', z') at the previous instant $(t - r/c)$: as if the potentials were propagated outwards from the charges and currents with velocity c. The functions ϕ and **a** formed in this way are generally known as the *retarded potentials.*

The equations by which ϕ and $\mathbf{a}$ have been defined are equivalent to the equations

$$\nabla^2\phi - \ddot{\phi}/c^2 = -4\pi\rho, \qquad (2)$$

$$\nabla^2\mathbf{a} - \ddot{\mathbf{a}}/c^2 = -4\pi\mathbf{\imath}, \qquad (3)$$

while the equation of conservation of electricity,

$$\operatorname{div}\mathbf{\imath} + \dot{\rho} = 0$$

gives

$$\operatorname{div}\mathbf{a} + \dot{\phi} = 0. \qquad (4)$$

From equations (1), (2), (4), we may readily derive the equation

$$\operatorname{div}\mathbf{E} = 4\pi c^2\rho; \qquad (\mathrm{I})$$

and from (1), (3), (4), we have

$$\operatorname{curl}\mathbf{H} = \dot{\mathbf{E}}/c^2 + 4\pi\mathbf{\imath}, \qquad (\mathrm{II})$$

where $\mathbf{H}$ or curl $\mathbf{a}$ denotes the magnetic force: while from (1) we have

$$\operatorname{curl}\mathbf{E} = -\dot{\mathbf{H}}. \qquad (\mathrm{III})$$

The equations (I), (II), (III) are, however, the fundamental equations of Maxwell's theory; and therefore the theory of L. Lorenz is practically equivalent to that of Maxwell, so far as concerns the propagation of electromagnetic disturbances through free aether. Lorenz himself, however, does not appear to have clearly perceived this; for in his memoir he postulated the presence of conducting matter throughout space, and was consequently led to equations resembling those which Maxwell had given for the propagation of light in metals. Observing that his equations represented periodic electric currents at right angles to the direction of propagation of the disturbance, he suggested that all luminous vibrations might be constituted by electric currents, and hence that there was "no longer any reason for maintaining the hypothesis of an aether, since we can admit that space contains sufficient ponderable matter to enable the disturbance to be propagated."

Lorenz was unable to derive from his equations any explanation of the existence of refractive indices, and his theory lacks the rich physical suggestiveness of Maxwell's; the value of his memoir lies chiefly in the introduction of the retarded potentials. It may be remarked in passing that Lorenz's retarded potentials are not identical with Maxwell's scalar and vector potentials; for Lorenz's $\mathbf{a}$ is not a circuital vector, and Lorenz's ϕ is not, like Maxwell's, the electrostatic potential, but depends on the positions occupied by the charges at certain previous instants.

For some years no progress was made either with Maxwell's theory or with Lorenz's. Meanwhile, Maxwell had in 1865 resigned his chair at King's College, and had retired to his estate in Dumfriesshire, where he occupied himself in writing a connected account of electrical theory. In 1871 he returned to Cambridge as Professor of Experimental Physics; and two years later published his *Treatise on Electricity and Magnetism.*

In this celebrated work is comprehended almost every branch of electric and magnetic theory; but the intention of the writer was to discuss the whole as far as possible from a single point of view, namely, that of Faraday; so that little or no account was given of the hypotheses which had been propounded in the two preceding decades by the great German electricians. So far as Maxwell's purpose was to disseminate the ideas of Faraday, it was undoubtedly fulfilled; but the *Treatise* was less successful when considered as the exposition of its author's own views. The doctrines peculiar to Maxwell —the existence of displacement-currents, and of electromagnetic vibrations identical with light—were not introduced in the first volume, or in the first half of the second volume; and the account which was given of them was scarcely more complete, and was perhaps less attractive, than that which had been furnished in the original memoirs.

Some matters were, however, discussed more fully in the *Treatise* than in Maxwell's previous writings; and among these was the question of stress in the electromagnetic field.

It will be remembered* that Faraday, when studying the curvature of lines of force in electrostatic fields, had noticed an apparent tendency of adjacent lines to repel each other, as if each tube of force were inherently disposed to distend laterally; and that in addition to this repellent or diverging force in the transverse direction, he supposed an attractive or contractile force to be exerted at right angles to it, that is to say, in the direction of the lines of force.

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* Cf. p. 209.

Of the existence of these pressures and tensions Maxwell was fully persuaded; and he determined analytical expressions suitable to represent them. The tension along the lines of force must be supposed to maintain the ponderomotive force which acts on the conductor on which the lines of force terminate; and it may therefore be measured by the force which is exerted on unit area of the conductor, i.e., $\varepsilon\mathbf{E}^2/8\pi c^2$ or $\frac{1}{2}\mathbf{DE}$. The pressure at right angles to the lines of force must then be determined so as to satisfy the condition that the aether is to be in equilibrium.

For this purpose, consider a thin shell of aether included between two equipotential surfaces. The equilibrium of the portion of this shell which is intercepted by a tube of force requires (as in the theory of the equilibrium of liquid films) that the resultant force per unit area due to the above-mentioned normal tensions on its two faces shall have the value $T(1/\rho_1 + 1/\rho_2)$, where ρ_1 and ρ_2 denote the principal radii of curvature of the shell at the place, and where T denotes the lateral stress across unit length of the surface of the shell, T being analogous to the surface-tension of a liquid film.

Now, if t denote the thickness of the shell, the area intercepted on the second face by the tube of force bears to the area intercepted on the first face the ratio $(\rho_1 + t)(\rho_2 + t)/\rho_1\rho_2$; and by the fundamental property of tubes of force, **D** and **E** vary inversely as the cross-section of the tube, so the total force on the second face will bear to that on the first face the ratio

$$\rho_1\rho_2/(\rho_1 + t)(\rho_2 + t),$$

or approximately

$$(1 - t/\rho_1 - t\rho_2);$$

the resultant force per unit area along the outward normal is therefore

$$-\tfrac{1}{2}\mathbf{DE}\,.\,t\,.\,(1/\rho_1 + 1/\rho_2),$$

and so we have

$$T = -\tfrac{1}{2}\mathbf{DE}\,.\,t\,;$$

or the pressure at right angles to the lines of force is $\tfrac{1}{2}\mathbf{DE}$ per unit area—that is, it is numerically equal to the tension along the lines of force.

The principal stresses in the medium being thus determined, it readily follows that the stress across any plane, to which the unit vector **N** is normal, is

$$(\mathbf{D}\,.\,\mathbf{N})\,\mathbf{E} - \tfrac{1}{2}\,(\mathbf{D}\,.\,\mathbf{E})\,\mathbf{N}.$$

Maxwell obtained* a similar formula for the case of magnetic fields; the ponderomotive forces on magnetized matter and on conductors carrying currents may be accounted for by assuming a stress in the medium, the stress across the plane **N** being represented by the vector

$$\frac{1}{4\pi}\,(\mathbf{B}\,.\,\mathbf{N})\,.\,\mathbf{H} - \frac{1}{8\pi}\,(\mathbf{B}\,.\,\mathbf{H})\,.\,\mathbf{N}.$$

This, like the corresponding electrostatic formula, represents a tension across planes perpendicular to the lines of force, and a pressure across planes parallel to them.

It may be remarked that Maxwell made no distinction between stress in the material dielectric and stress in the aether: indeed, so long as it was supposed that material bodies when displaced carry the contained aether along with them, no distinction was possible. In the modifications of Maxwell's theory which were developed many years afterwards by his followers, stresses corresponding to those introduced by Maxwell were assigned to the aether, as distinct from ponderable matter; and it was assumed that the only stresses set up in material bodies by the electromagnetic field are produced indirectly: they may be calculated by the methods of the theory of elasticity, from a knowledge of the ponderomotive forces exerted on the electric charges connected with the bodies.

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* Maxwell's *Treatise on Electricity and Magnetism*, § 643.

Another remark suggested by Maxwell's theory of stress he medium is that he considered the question from the ly statical point of view. He determined the stress so that ight produce the required forces on ponderable bodies, and elf-equilibrating in free aether. But* if the electric and ıetic phenomena are not really statical, but are kinetic in · nature, the stress or pressure need not be self-equilibrating. may be illustrated by reference to the hydrodynamical els of the aether shortly to be described, in which perforated s are immersed in a moving liquid: the ponderomotive s exerted on the solids by the liquid correspond to those h act on conductors carrying currents in a magnetic field, yet there is no stress in the medium beyond the pressure e liquid.

ımong the problems to which Maxwell applied his theory tress in the medium was one which had engaged the ıtion of many generations of his predecessors. The ad- ıts of the corpuscular theory of light in the eighteenth ıry believed that their hypothesis would be decisively con- d if it could be shown that rays of light possess momentum: termine the matter, several investigators directed powerful s of light on delicately-suspended bodies, and looked for nces of a pressure due to the impulse of the corpuscles. an experiment was performed in 1708 by Homberg,† who ined that he actually obtained the effect in question; but an and Du Fay in the middle of the century, having ted his operations, failed to confirm his conclusion.‡

'he subject was afterwards taken up by Michell, who "some ago," wrote Priestley§ in 1772, "endeavoured to ascertain ıomentum of light in a much more accurate manner than in which M. Homberg and M. Mairan had attempted it." xposed a very thin and delicately-suspended copper plate to the rays of the sun concentrated by a mirror, and observed a deflexion. He was not satisfied that the effect of the heating of the air had been altogether excluded, but " there seems to be no doubt," in Priestley's opinion, " but that the motion above mentioned is to be ascribed to the impulse of the rays of light." A similar experiment was made by A. Bennet,* who directed the light from the focus of a large lens on writing-paper delicately suspended in an exhausted receiver, but " could not perceive any motion distinguishable from the effects of heat." " Perhaps," he concluded, " sensible heat and light may not be caused by the influx or rectilineal projections of fine particles, but by the vibrations made in the universally diffused *caloric* or matter of heat, or fluid of light." Thus Bennet, and after him Young,† regarded the non-appearance of light-repulsion in this experiment as an argument in favour of the undulatory system of light. "For," wrote Young, "granting the utmost imaginable subtility of the corpuscles of light, their effects might naturally be expected to bear some proportion to the effects of the much less rapid motions of the electrical fluid, which are so very easily perceptible, even in their weakest states."

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* Cf. V. Bjerknes, Phil. Mag. ix (1905), p. 491.

† Histoire de l'Acad., 1708, p. 21.

‡ J. J. de Mairan, *Traité de l'Aurore boréale*, p. 370.

§ *History of Vision*, i, p. 387.

This attitude is all the more remarkable, because Euler many years before had expressed the opinion that light-pressure might be expected just as reasonably on the undulatory as on the corpuscular hypothesis. "Just as," he wrote,‡ "a vehement sound excites not only a vibratory motion in the particles of the air, but there is also observed a real movement of the small particles of dust which are suspended therein, it is not to be doubted but that the vibratory motion set up by the light causes a similar effect." Euler not only inferred the existence of light-pressure, but even (adopting a suggestion of Kepler's) accounted for the tails of comets by supposing that the solar rays, impinging on the atmosphere of a comet, drive off from it the more subtle of its particles.

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* Phil. Trans., 1792, p. 81.

† *Ibid.*, 1802, p. 46.

‡ *Histoire de l'Acad. de Berlin*, ii (1748), p. 117.

The question was examined by Maxwell[*] from the point of view of the electromagnetic theory of light; which readily furnishes reasons for the existence of light-pressure. For suppose that light falls on a metallic reflecting surface at perpendicular incidence. The light may be regarded as constituted of a rapidly-alternating magnetic field; and this must induce electric currents in the surface layers of the metal. But a metal carrying currents in a magnetic field is acted on by a ponderomotive force, which is at right angles to both the magnetic force and the direction of the current, and is therefore, in the present case, normal to the reflecting surface: this ponderomotive force is the light-pressure. Thus, according to Maxwell's theory, light-pressure is only an extended case of effects which may readily be produced in the laboratory.

The magnitude of the light-pressure was deduced by Maxwell from his theory of stresses in the medium. We have seen that the stress across a plane whose unit-normal is **N** is represented by the vector

$$(\mathbf{D}\,.\,\mathbf{N})\,.\,\mathbf{E} - \tfrac{1}{2}(\mathbf{D}\,.\,\mathbf{E})\,.\,\mathbf{N} + \frac{1}{4\pi}(\mathbf{B}\,.\,\mathbf{N})\,.\,\mathbf{H} - \frac{1}{8\pi}(\mathbf{B}\,.\,\mathbf{H})\,.\,\mathbf{N}.$$

Now, suppose that a plane wave is incident perpendicularly on a perfectly reflecting metallic sheet: this sheet must support the mechanical stress which exists at its boundary in the aether. Owing to the presence of the reflected wave, **D** is zero at the surface; and **B** is perpendicular to **N**, so (**B** . **N**) vanishes. Thus the stress is a pressure of magnitude $(1/8\pi)$ (**B** . **H**) normal to the surface: that is, the light-pressure is equal to the density of the aethereal energy in the region immediately outside the metal. This was Maxwell's result.

This conclusion has been reached on the assumption that the light is incident normally to the reflecting surface. If, on the other hand, the surface is placed in an enclosure completely surrounded by a radiating shell, so that radiation falls on it from all directions, it may be shown that the light-pressure is measured by one-third of the density of aethereal energy.

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[*] Maxwell's *Treatise on Electricity and Magnetism*, § 792.

A different way of inferring the necessity for light-pressure was indicated in 1876 by A. Bartoli,[*] who showed that, when radiant energy is transported from a cold body to a hot one by means of a moving mirror, the second law of thermodynamics would be violated unless a pressure were exerted on the mirror by the light.

The thermodynamical ideas introduced into the subject by Bartoli have proved very fruitful. If a hollow vessel be at a definite temperature, the aether within the vessel must be full of radiation crossing from one side to the other: and hence the aether, when in radiative equilibrium with matter at a given temperature, is the seat of a definite quantity of energy per unit volume.

If U denote this energy per unit volume, and P the light-pressure on unit area of a surface exposed to the radiation, we may apply[†] the equation of available energy[‡]

$$U = T\frac{dP}{dT} - P.$$

Since, as we have seen,

$$P = \tfrac{1}{3}U,$$

this equation gives

$$4U = T\frac{dU}{dT},$$

and therefore U must be proportional to T^4. From this it may be inferred that the intensity of emission of radiant energy by a body at temperature T is proportional to the fourth power of the absolute temperature—a law which was first discovered experimentally by Stefan[§] in 1879.

In the year in which Maxwell's treatise was published, Sir William Crookes[||] obtained experimental evidence of a pressure accompanying the incidence of light; but this was soon found to be due to thermal effects; and the existence of a true light-pressure was not confirmed experimentally* until 1899. Since then the subject has been considerably developed, especially in regard to the part played by the pressure of radiation in cosmical physics.

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[*] Bartoli, *Sopra i movimenti prodotti dalla luce e dal calore e sopra il radiometro di Crookes*. Firenze, 1876. Also Nuovo Cimento (3) xv (1884), p. 193; and Exner's Rep., xxi (1885), p. 198.

[†] Boltzmann, Ann. d. Phys. xxii (1884), p. 31. Cf. also B. Galitzine, Ann. d. Phys. xlvii (1892), p. 479.

[‡] Cf. p. 240.

[§] Wien. Ber. lxxix (1879), p. 391.

[||] Phil. Trans. clxiv (1874), p. 501. The radiometer was discovered in 1875.

Another matter which received attention in Maxwell's *Treatise* was the influence of a magnetic field on the propagation of light in material substances. We have already seen† that the theory of magnetic vortices had its origin in Thomson's speculations on this phenomenon; and Maxwell in his memoir of 1861–2 had attempted by the help of that theory to arrive at some explanation of it. The more complete investigation which is given in the *Treatise* is based on the same general assumptions, namely, that in a medium subjected to a magnetic field there exist concealed vortical motions, the axes of the vortices being in the direction of the lines of magnetic force; and that waves of light passing through the medium disturb the vortices, which thereupon react dynamically on the luminous motion, and so affect its velocity of propagation.

The manner of this dynamical interaction must now be more closely examined. Maxwell supposed that the magnetic vortices are affected by the light-waves in the same way as vortex-filaments in a liquid would be affected by any other coexisting motion in the liquid. The latter problem had been already discussed in Helmholtz's great memoir on vortex-motion; adopting Helmholtz's results, Maxwell assumed for the additional term introduced into the magnetic force by the displacement of the vortices the value $\partial \mathbf{e}/\partial\theta$, where $\mathbf{e}$ denotes the displacement of the medium (i.e. the light vector), and the operator $\partial/\partial\theta$ denotes $H_x\partial/\partial x + H_y\partial/\partial y + H_z\partial/\partial z$, $\mathbf{H}$ denoting the imposed magnetic field. Thus the luminous motion, by disturbing the vortices, gives rise to an electric current in the medium, proportional to curl $\partial\mathbf{e}/\partial\theta$.

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* P. Lebedew, Archives des Sciences Phys. et Nat. (4) viii (1899), p. 184. Ann. d. Phys. vi (1901), p. 433. E. F. Nichols and G. F. Hull, Phys. Rev. xiii (1901), p. 293; Astrophys. Jour., xvii (1903), p. 315.

† Cf. p. 274.

Maxwell further assumed that the current thus produced interacts dynamically with the luminous motion in such a manner that the kinetic energy of the medium contains a term proportional to the scalar product of $\dot{\mathbf{e}}$ and curl $\partial\mathbf{e}/\partial\theta$. The total kinetic energy of the medium may therefore be written

$$\tfrac{1}{2}\rho\dot{\mathbf{e}}^2 + \tfrac{1}{2}\sigma\,(\dot{\mathbf{e}}\,.\,\mathrm{curl}\;\partial\mathbf{e}/\partial\theta),$$

where ρ denotes the density of the medium, and σ denotes a constant which measures the capacity of the medium to rotate the plane of polarization of light in a magnetic field.

The equation of motion may now be derived as in the elastic-solid theories of light: it is

$$\rho\ddot{\mathbf{e}} = n\nabla^2\mathbf{e} - \sigma\frac{\partial^2}{\partial t\,\partial\theta}\,\mathrm{curl}\;\mathbf{e}.$$

When the light is transmitted in the direction of the lines of force, and the axis of x is taken parallel to this direction, the equation reduces to

$$\left\{\begin{aligned} \rho\,\frac{\partial^2 e_y}{\partial t^2} &= n\,\frac{\partial^2 e_y}{\partial x^2} + \sigma H\,\frac{\partial^3 e_z}{\partial t\partial x^2}, \\ \rho\,\frac{\partial^2 e_z}{\partial t^2} &= n\,\frac{\partial^2 e_z}{\partial x^2} - \sigma H\,\frac{\partial^3 e_y}{\partial t\partial x^2}; \end{aligned}\right.$$

and these equations, as we have seen,* furnish an explanation of Faraday's phenomenon.

It may be remarked that the term

$$\tfrac{1}{2}\sigma\,(\dot{\mathbf{e}}\,.\,\mathrm{curl}\;\partial\mathbf{e}/\partial\theta)$$

in the kinetic energy may by partial integration be transformed into a term

$$\tfrac{1}{2}\sigma\,(\mathrm{curl}\;\dot{\mathbf{e}}\,.\,\partial\mathbf{e}/\partial\theta),\dagger$$

together with surface-terms; or, again, into

$$-\tfrac{1}{2}\sigma\,(\mathrm{curl}\;\mathbf{e}\,.\,\partial\dot{\mathbf{e}}/\partial\theta),$$

together with surface-terms. These different forms all yield the same equation of motion for the medium; but, owing to the differences in the surface-terms, they yield different conditions at the boundary of the medium, and consequently give rise to different theories of reflexion.

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* Cf. p. 215.

† This form was suggested by Fitz Gerald six years later, Phil. Trans., 1880, p. 691: Fitz Gerald's *Scientific Writings*, p. 45.

The assumptions involved in Maxwell's treatment of the magnetic rotation of light were such as might scarcely be justified in themselves; but since the discussion as a whole proceeded from sound dynamical principles, and its conclusions were in harmony with experimental results, it was fitted to lead to the more perfect explanations which were afterwards devised by his successors. At the time of Maxwell's death, which happened in 1879, before he had completed his forty-ninth year, much yet remained to be done both in this and in the other investigations with which his name is associated; and the energies of the next generation were largely spent in extending and refining that conception of electrical and optical phenomena whose origin is correctly indicated in its name of *Maxwell's Theory*.

CHAPTER IX.

MODELS OF THE AETHER.

THE early attempts of Thomson and Maxwell to represent the electric medium by mechanical models opened up a new field of research, to which investigators were attracted as much by its intrinsic fascination as by the importance of the services which it promised to render to electric theory.

Of the models to which reference has already been made, some—such as those described in Thomson's memoir* of 1847 and Maxwell's memoir† of 1861–2—attribute a linear character to electric force and electric current, and a rotatory character to magnetism; others—such as that devised by Maxwell in 1855‡ and afterwards amplified by Helmholtz§—regard magnetic force as a linear and electric current as a rotatory phenomenon. This distinction furnishes a natural classification of models into two principal groups.

Even within the limits of the former group diversity has already become apparent; for in Maxwell's analogy of 1861–2, a continuous vortical motion is supposed to be in progress about the lines of magnetic induction; whereas in Thomson's analogy the vector-potential was likened to the displacement in an elastic solid, so that the magnetic induction at any point would be represented by the twist of an element of volume of the solid from its equilibrium position; or, in symbols,

$$\mathbf{a} = \mathbf{e}, \quad \mathbf{E} = -\dot{\mathbf{e}}, \quad \mathbf{B} = \text{curl}\, \mathbf{e},$$

where **a** denotes the vector-potential, **E** the electric force, **B** the magnetic induction, and **e** the elastic displacement.

Thomson's original memoir concluded with a notice of his intention to resume the discussion in another communication His purpose was fulfilled only in 1890, when‖ he showed tha in his model a linear current could be represented by a piece of endless cord, of the same quality as the solid and embedded in it, if a tangential force were applied to the cord uniformly all round the circuit. The forces so applied tangentially produce a tangential drag on the surrounding solid; and the rotatory displacement thus caused is everywhere proportional to the magnetic vector.

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* Cf. p. 270. † Cf. p. 276. ‡ Cf. p. 271. § Cf. p. 274.

‖ Kelvin's *Math. and Phys. Papers*, iii, p. 436.

In order to represent the effect of varying permeability, Thomson abandoned the ordinary type of elastic solid, and replaced it by an aether of MacCullagh's type; that is to say, an ideal incompressible substance, having no rigidity of the ordinary kind (i.e. elastic resistance to change of shape), but capable of resisting absolute rotation—a property to which the name *gyrostatic rigidity* was given. The rotation of the solid representing the magnetic induction, and the coefficient of gyrostatic rigidity being inversely proportional to the permeability, the normal component of magnetic induction will be continuous across an interface, as it should be.*

We have seen above that in models of this kind the electric force is represented by the translatory velocity of the medium. It might therefore be expected that a strong electric field would perceptibly affect the velocity of propagation of light; and that this does not appear to be the case,† is an argument against the validity of the scheme.

We now turn to the alternative conception, in which electric phenomena are regarded as rotatory, and magnetic force is represented by the linear velocity of the medium; in symbols,

$$\begin{cases} 4\pi\mathbf{D} = \operatorname{curl} \mathbf{e}, \\ \mathbf{H} = \dot{\mathbf{e}}, \end{cases}$$

where **D** denotes the electric displacement, **H** the magnetic force, and **e** the displacement of the medium. In Maxwell's memoir of 1855, and in most of the succeeding writings for many years, attention was directed chiefly to magnetic fields of a steady, or at any rate non-oscillatory, character; in such fields, the motion of the particles of the medium is continuously progressive; and it was consequently natural to suppose the medium to be fluid.

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* Thomson inclined to believe (*Papers*, iii, p. 465) that light might be correctly represented by the vibratory motion of such a solid.

† Wilberforce, Trans. Camb. Phil. Soc. xiv (1887), p. 170; Lodge, Phil. Trans. clxxxix (1897), p. 149.

Maxwell himself, as we have seen,* afterwards abandoned this conception in favour of that which represents magnetic phenomena as rotatory. "According to Ampère and all his followers," he wrote in 1870,† "electric currents are regarded as a species of translation, and magnetic force as depending on rotation. I am constrained to agree with this view, because the electric current is associated with electrolysis, and other undoubted instances of translation, while magnetism is associated with the rotation of the plane of polarization of light." But the other analogy was felt to be too valuable to be altogether discarded, especially when in 1858 Helmholtz extended it‡ by showing that if magnetic induction is compared to fluid velocity, then electric currents correspond to vortex-filaments in the fluid. Two years afterwards Kirchhoff§ developed it further. If the analogy has any dynamical (as distinguished from a merely kinematical) value, it is evident that the ponderomotive forces between metallic rings carrying electric currents should be similar to the ponderomotive forces between the same rings when they are immersed in an infinite incompressible fluid; the motion of the fluid being such that its circulation through the aperture of each ring is proportional to the strength of the electric current in the corresponding ring. In order to decide the question, Kirchhoff attempted, and solved, the hydrodynamical problem of the motion of two thin, rigid rings in an incompressible frictionless fluid, the fluid motion being irrotational; and found that the forces between the rings are *numerically* equal to those which the rings would exert on each other if they were traversed by electric currents proportional to the circulations.

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* Cf. p. 276.

† Proc. Lond. Math. Soc. iii (1870), p. 224; Maxwell's *Scient. Papers*, ii, p. 263.

‡ Cf. p. 274.

§ Journal für Math. lxxi (1869); Kirchhoff's *Gesamm. Abhandl.*, p. 404. Cf. also C. Neumann, Leipzig Berichte, xliv (1892), p. 86.

There is, however, an important difference between the two cases, which was subsequently discussed by W. Thomson, who pursued the analogy in several memoirs.[*] In order to represent the magnetic field by a conservative dynamical system, we shall suppose that it is produced by a number of rings of perfectly conducting material, in which electric currents are circulating; the surrounding medium being free aether. Now any perfectly conducting body acts as an impenetrable barrier to lines of magnetic force; for, as Maxwell showed,[†] when a perfect conductor is placed in a magnetic field, electric currents are induced on its surface in such a way as to make the total magnetic force zero throughout the interior of the conductor.[‡] Lines of force are thus deflected by the body in the same way as the lines of flow of an incompressible fluid would be deflected by an obstacle of the same form, or as the lines of flow of electric current in a uniform conducting mass would be deflected by the introduction of a body of this form and of infinite resistance. If, then, for simplicity we consider two perfectly conducting rings carrying currents, those lines of force which are initially linked with a ring cannot escape from their entanglement, and new lines cannot become involved in it. This implies that the total number of lines of magnetic force which pass through the aperture of each ring is invariable. If the coefficients of self and mutual induction of the rings are denoted by L_1, L_2, L_{12}, the electrokinetic energy of the system may be represented by

$$T = \tfrac{1}{2}\,(L_1 i_1^2 + 2L_{12} i_1 i_2 + L_2 i_2^2),$$

where i_1, i_2 denote the strengths of the currents; and the condition that the number of lines of force linked with each circuit is to be invariable gives the equations

$$L_1 i_1 + L_{12} i_2 = \text{constant},$$
$$L_{12} i_1 + L_2 i_2 = \text{constant}.$$

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[*] Thomson's *Reprint of Papers in Elect. and Mag.*, §§ 573, 733, 751 (1870–1872).

[†] Maxwell's *Treatise on Elect. and Mag.*, § 654.

[‡] For this reason W. Thomson called a perfect conductor an *ideal extreme diamagnetic*.

It is evident that, when the system is considered from the point of view of general dynamics, the electric currents must be regarded as generalized velocities, and the quantities

$$(L_1 i_1 + L_{12} i_2) \quad \text{and} \quad (L_{12} i_1 + L_2 i_2)$$

as momenta. The electromagnetic ponderomotive force on the rings tending to increase any coordinate x is $\partial T/\partial x$. In the analogous hydrodynamical system, the fluid velocity corresponds to the magnetic force: and therefore the circulation through each ring (which is defined to be the integral $\int v ds$, taken round a path linked once with the ring) corresponds kinematically to the electric current; and the flux of fluid through each ring corresponds to the number of lines of magnetic force which pass through the aperture of the ring. But in the hydrodynamical problem the circulations play the part of generalized momenta; while the fluxes of fluid through the rings play the part of generalized velocities. The kinetic energy may indeed be expressed in the form

$$K = \tfrac{1}{2} (N_1 \kappa_1^2 + 2N_{12} \kappa_1 \kappa_2 + N_2 \kappa_2^2),$$

where κ_1, κ_2, denote the circulations (so that κ_1 and κ_2 are proportional respectively to i_1 and i_2), and N_1, N_{12}, N_2, depend on the positions of the rings; but this is the Hamiltonian (as opposed to the Lagrangian) form of the energy-function,* and the ponderomotive force on the rings tending to increase any coordinate x is $-\partial K/\partial x$. Since $\partial K/\partial x$ is equal to $\partial T/\partial x$, we see that the ponderomotive forces on the rings in any position in the hydrodynamical system are equal, *but opposite*, to the ponderomotive forces on the rings in the electric system.

The reason for the difference between the two cases may readily be understood. The rings cannot cut through the lines of magnetic force in the one system, but they can cut through the stream-lines in the other: consequently the flux of fluid through the rings is not invariable when the rings are moved, the invariants in the hydrodynamical system being the circulations.

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* Cf. Whittaker, *Analytical Dynamics*, § 109.

If a thin ring, for which the circulation is zero, is introduced into the fluid, it will experience no ponderomotive forces; but if a ring initially carrying no current is introduced into a magnetic field, it will experience ponderomotive forces, owing to the electric currents induced in it by its motion.

Imperfect though the analogy is, it is not without interest. A bar-magnet, being equivalent to a current circulating in a wire wound round it, may be compared (as W. Thomson remarked) to a straight tube immersed in a perfect fluid, the fluid entering at one end and flowing out by the other, so that the particles of fluid follow the lines of magnetic force. If two such tubes are presented with like ends to each other, they attract; with unlike ends, they repel. The forces are thus diametrically opposite in direction to those of magnets; but in other respects the laws of mutual action between these tubes and between magnets are precisely the same.*

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* The mathematical analysis in this case is very simple. A narrow tube through which water is flowing may be regarded as equivalent to a source at one end of the tube and a sink at the other; and the problem may therefore be reduced to the consideration of sinks in an unlimited fluid. If there are two sinks in such a fluid, of strengths m and m', the velocity-potential is

$$m/r + m'/r',$$

where r and r' denote distance from the sinks. The kinetic energy per unit volume of the fluid is

$$\tfrac{1}{2}\rho\left[\left\{\frac{\partial}{\partial x}\left(\frac{m}{r}+\frac{m'}{r'}\right)\right\}^2+\left\{\frac{\partial}{\partial y}\left(\frac{m}{r}+\frac{m'}{r'}\right)\right\}^2+\left\{\frac{\partial}{\partial z}\left(\frac{m}{r}+\frac{m'}{r'}\right)\right\}^2\right],$$

where ρ denotes the density of the fluid; whence it is easily seen that the total energy of the fluid, when the two sinks are at a distance l apart, exceeds the total energy when they are at an infinite distance apart by an amount

$$\rho m m' \iiint \left\{\frac{\partial}{\partial x}\left(\frac{1}{r}\right)\frac{\partial}{\partial x}\left(\frac{1}{r'}\right)+\frac{\partial}{\partial y}\left(\frac{1}{r}\right)\frac{\partial}{\partial y}\left(\frac{1}{r'}\right)+\frac{\partial}{\partial z}\left(\frac{1}{r}\right)\frac{\partial}{\partial z}\left(\frac{1}{r'}\right)\right\} dx\, dy\, dz,$$

the integration being taken throughout the whole volume of the fluid, except two small spheres s, s', surrounding the sinks. By Green's theorem, this expression reduces at once to

$$\rho m m' \iint \frac{1}{r'}\frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS,$$

where the integration is taken over s and s', and n denotes the interior normal to s or s'. The integral taken over s' vanishes; evaluating the remaining integral, we have

$$-\frac{\rho m m'}{l}\iint \frac{\partial}{\partial r}\left(\frac{1}{r}\right) dS, \quad \text{or} \quad 4\pi\rho m m'/l.$$

The energy of the fluid is therefore greater when sinks of strengths m, m' are at a

Thomson, moreover, investigated* the ponderomotive forces which act between two solid bodies immersed in a fluid, when one of the bodies is constrained to perform small oscillations. If, for example, a small sphere immersed in an incompressible fluid is compelled to oscillate along the line which joins its centre to that of a much larger sphere, which is free, the free sphere will be attracted if it is denser than the fluid; while if it is less dense than the fluid, it will be repelled or attracted according as the ratio of its distance from the vibrator to its radius is greater or less than a certain quantity depending on the ratio of its density to the density of the fluid. Systems of this kind were afterwards extensively investigated by C. A. Bjerknes.† Bjerknes showed that two spheres which are immersed in an incompressible fluid, and which pulsate (i.e., change in volume) regularly, exert on each other (by the mediation of the fluid) an attraction, determined by the inverse square law, if the pulsations are concordant; and exert on each other a repulsion, determined likewise by the inverse square law, if the phases of the pulsations differ by half a period. It is necessary to suppose that the medium is incompressible, so that all pulsations are propagated instantaneously: otherwise attractions would change to repulsions and vice versa at distances greater than a quarter wave-length.‡ If the spheres, instead of pulsating, oscillate to and fro in straight lines about their mean positions, the forces between them are proportional in magnitude and the same in direction, but

mutual distance l than when sinks of the same strengths are at infinite distance apart by an amount $4\pi\rho mm'/l$. Since, in the case of the tubes, the quantities m correspond to the fluxes of fluid, this expression corresponds to the Lagrangian form of the kinetic energy; and therefore the force tending to increase the coordinate x of one of the sinks is $(\partial/\partial x)(4\pi\rho mm'/l)$. Whence it is seen that the *like* ends of two tubes *attract*, and the unlike ends *repel*, according to the inverse square law.

* Phil. Mag. xli (1870), p. 427.

† Repertorium d. Mathematik von Konisberger und Zeuner (1876), p. 268. Göttinger Nachrichten, 1876, p. 245. Comptes Rendus, lxxxiv (1877), p. 1377. Cf. Nature, xxiv (1881), p. 360.

‡ On the mathematical theory of the force between two pulsating spheres in a fluid, cf. W. M. Hicks, Proc. Camb. Phil. Soc. iii (1879), p. 276; iv (1880), p. 29.

opposite in sign, to those which act between two magnets oriented along the directions of oscillation.*

The results obtained by Bjerknes were extended by A. H. Leahy† to the case of two spheres pulsating in an elastic medium; the wave-length of the disturbance being supposed large in comparison with the distance between the spheres. For this system Bjerknes' results are reversed, the law being now that of attraction in the case of unlike phases, and of repulsion in the case of like phases: the intensity is as before proportional to the inverse square of the distance.

The same author afterwards discussed‡ the oscillations which may be produced in an elastic medium by the displacement, in the direction of the tangent to the cross-section, of the surfaces of tubes of small sectional area: the tubes either forming closed curves, or extending indefinitely in both directions. The direction and circumstances of the motion are in general analogous to ordinary vortex-motions in an incompressible fluid; and it was shown by Leahy that, if the period of the oscillation be such that the waves produced are long compared with ordinary finite distances, the displacement due to the tangential disturbances is proportional to the velocity due to vortex-rings of the same form as the tubular surfaces. One of these "oscillatory twists," as the tubular surfaces may be called, produces a displacement which is analogous to the magnetic force due to a current flowing in a curve coincident with the tube; the strength of the current being proportional to $b^2\omega \sin pt$, where b denotes the radius of the twist, and $\omega \sin pt$ its angular displacement. If the field of vibration is explored by a rectilineal twist of the same period as that of the vibration, the twist will experience a force at right angles to the plane containing the twist and the direction of the displacement which would exist if the twist were removed; if the displacement of the medium be represented by $F \sin pt$, and the angular displacement of the twist by $\omega \sin pt$, the magnitude of the force is proportional to the vector-product of F (in the direction of the displacement) and ω (in the direction of the axis of the twist).

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* A theory of gravitation has been based by Korn on the assumption that gravitating particles resemble slightly compressible spheres immersed in an incompressible perfect fluid: the spheres execute pulsations, whose intensity corresponds to the mass of the gravitating particles, and thus forces of the Newtonian kind are produced between them. Cf. Korn, *Eine Theorie der Gravitation und der elect. Erscheinungen*, Berlin, 1898.

† Trans. Camb. Phil. Soc. xiv (1884), p. 45.

‡ Trans. Camb. Phil. Soc. xiv (1885), p. 188.

A model of magnetic action may evidently be constructed on the basis of these results. A bar-magnet must be regarded as vibrating tangentially, the direction of vibration being parallel to the axis of the body. A cylindrical body carrying a current will have its surface also vibrating tangentially; but in this case the direction of vibration will be perpendicular to the axis of the cylinder. A statically electrified body, on the other hand, may, as follows from the same author's earlier work, be regarded as analogous to a body whose surface vibrates in the normal direction.

We have now discussed models in which the magnetic force is represented as the velocity in a liquid, and others in which it is represented as the displacement in an elastic solid. Some years before the date of Leahy's memoir, George Francis Fitz Gerald (*b.* 1851, *d.* 1901)* had instituted a comparison between magnetic force and the velocity in a quasi-elastic solid of the type first devised by MacCullagh.† An analogy is at once evident when it is noticed that the electromagnetic equation

$$4\pi\dot{\mathbf{D}} = \operatorname{curl} \mathbf{H}$$

is satisfied identically by the values

$$\begin{cases} 4\pi\mathbf{D} = \operatorname{curl} \mathbf{e}, \\ \mathbf{H} = \dot{\mathbf{e}}, \end{cases}$$

where $\mathbf{e}$ denotes any vector; and that, on substituting these values in the other electromagnetic equation,

$$-\operatorname{curl}(4\pi c^2\mathbf{D}/\epsilon) = \dot{\mathbf{H}},$$

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* Phil. Trans., 1880, p. 691 (presented October, 1878). Fitz Gerald's *Scientific Writings*, p. 45.

† Cf. p. 155.

we obtain the equation

$$\epsilon\ddot{\mathbf{e}} + c^2 \operatorname{curl} \operatorname{curl} \mathbf{e} = 0,$$

which is no other than the equation of motion of MacCullagh's aether,* the specific inductive capacity ε corresponding to the reciprocal of MacCullagh's constant of elasticity. In the analogy thus constituted, electric displacement corresponds to the twist of the elements of volume of the aether; and electric charge must evidently be represented as an intrinsic rotational strain. Mechanical models of the electromagnetic field, based on FitzGerald's analogy, were afterwards studied by A. Sommerfeld,† by R. Reiff,‡ and by Sir J. Larmor.§ The last-named author‖ supposed the electric charge to exist in the form of discrete electrons, for the creation of which he suggested the following ideal process¶ :—A filament of aether, terminating at two nuclei, is supposed to be removed, and circulatory motion is imparted to the walls of the channel so formed, at each point of its length, so as to produce throughout the medium a rotational strain. When this has been accomplished, the channel is to be filled up again with aether, which is to be made continuous with its walls. When the constraint is removed from the walls of the channel, the circulation imposed on them proceeds to undo itself, until this tendency is balanced by the elastic resistance of the aether with which the channel has been filled up; thus finally the system assumes a state of equilibrium in which the nuclei, which correspond to a positive and a negative electron, are surrounded by intrinsic rotational strain.

Models in which magnetic force is represented by the velocity of an aether are not, however, secure from objection. It is necessary to suppose that the aether is capable of flowing like a perfect fluid in irrotational motion (which would correspond to a steady magnetic field), and that it is at the same time endowed with the power (which is requisite for the explanation of electric phenomena) of resisting the rotation of any element of volume.* But when the aether moves irrotationally in the fashion which corresponds to a steady magnetic field, each element of volume acquires after a finite time a rotatory displacement from its original orientation, in consequence of the motion; and it might therefore be expected that the quasi-elastic power of resisting rotation would be called into play—i.e., that a steady magnetic field would develop electric phenomena.†

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* Cf. p. 155.

† Ann. d. Phys. xlvi (1892), p. 139.

‡ Reiff, *Elasticität und Elektricität*, Freiburg, 1893.

§ Phil. Trans. clxxxv (1893), p. 719.

‖ In a supplement, of date August, 1894, to his above-cited memoir of 1893.

¶ Phil. Trans. clxxxv (1894), p. 810; cxc (1897), p. 210; Larmor, *Aether and Matter* (1900), p. 326.

A further objection to all models in which magnetic force corresponds to velocity is that a strong magnetic field, being in such models represented by a steady drift of the aether, might be expected to influence the velocity of propagation of light. The existence of such an effect appears, however, to be disproved by the experiments of Sir Oliver Lodge;‡ at any rate, unless it is assumed that the aether has an inertia at least of the same order of magnitude as that of ponderable matter, in which case the motion might be too slow to be measurable.

Again, the evidence in favour of the rotatory as opposed to the linear character of magnetic phenomena has perhaps, on the whole, been strengthened since Thomson originally based his conclusion on the magnetic rotation of light. This brings us to the consideration of an experimental discovery.

In 1879 E. H. Hall,§ at that time a student at Baltimore, repeating an experiment which had been previously suggested by H. A. Rowland, obtained a new action of a magnetic field on electric currents. A strip of gold leaf mounted on glass, forming part of an electric circuit through which a current was passing, was placed between the poles of an electromagnet, the plane of the strip being perpendicular to the lines of magnetic force. The two poles of a sensitive galvanometer were then placed in connexion with different parts of the strip, until two points at the same potential were found. When the magnetic field was created or destroyed, a deflection of the galvanometer needle was observed, indicating a change in the relative potential of the two poles. It was thus shown that the magnetic field produces in the strip of gold leaf a new electromotive force, at right angles to the primary electromotive force and to the magnetic force, and proportional to the product of these forces.

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* Larmor (loc. cit.) suggested the analogy of a liquid filled with magnetic molecules under the action of an external magnetic field.

It has often been objected to the mathematical conception of a perfect fluid that it contains no safeguard against slipping between adjacent layers, so that there is no justification for the usual assumption that the motion of a perfect fluid is continuous. Larmor remarked that a rotational elasticity, such as is attributed to the medium above considered, furnishes precisely such a safeguard; and that without some property of this kind a continuous frictionless fluid cannot be imagined.

† Larmor proposed to avoid this by assuming that the rotation which is resisted by an element of volume of the aether is the vector sum of the series of differential rotations which it has experienced.

‡ Phil. Trans. clxxxix (1897), p. 149.

§ Am. Jour. Math. ii, p. 287; Am. J. Sci. xix, p. 200, and xx, p. 161; Phil. Mag. ix, p. 225, and x, p. 301.

From the physical point of view we may therefore regard Hall's effect as an additional electromotive force generated by the action of the magnetic field on the current; or alternatively we may regard it as a modification of the ohmic resistance of the metal, such as would be produced if the molecules of the metal assumed a helicoidal structure about the lines of magnetic force. From the latter point of view, all that is needed is to modify Ohm's law

$$\mathbf{S} = k\mathbf{E}$$

(where **S** denotes electric current, k specific conductivity, and **E** electric force) so that it takes the form

$$\mathbf{S} = k\mathbf{E} + h\,[\mathbf{E}\,.\,\mathbf{H}]$$

where **H** denotes the imposed magnetic force, and h denotes a constant on which the magnitude of Hall's phenomenon depends. It is a curious circumstance that the occurrence, in the case of magnetized bodies, of an additional term in Ohm's law, formed from a vector-product of **E**, had been expressly suggested in Maxwell's *Treatise**: although Maxwell had not indicated the possibility of realizing it by Hall's experiment.

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* *Elect. and Mag.*, § 303. Cf. Hopkinson, Phil. Mag. x (1880), p. 430.

An interesting application of Hall's discovery was made in the same year by Boltzmann,[*] who remarked that it offered a prospect of determining the absolute velocity of the electric charges which carry the current in the strip. For if it is supposed that only one kind (vitreous or resinous) of electricity is in motion, the force on one of the charges tending to drive it to one side of the strip will be proportional to the vector-product of its velocity and the magnetic intensity. Assuming that Hall's phenomenon is a consequence of this tendency of charges to move to one side of the strip, it is evident that the velocity in question must be proportional to the magnitude of the Hall electromotive force due to a unit magnetic field. On the basis of this reasoning, A. von Ettingshausen[†] found for the current sent by one or two Daniell's cells through a gold strip a velocity of the order of 0·1 cm. per second. It is clear, however, that, if the current consists of both vitreous and resinous charges in motion in opposite directions, Boltzmann's argument fails; for the two kinds of electricity would give opposite directions to the current in Hall's phenomenon.

In the year following his discovery, Hall[‡] extended his researches in another direction, by investigating whether a magnetic field disturbs the distribution of equipotential lines in a dielectric which is in an electric field; but no effect could be observed.[§] Such an effect, indeed,[||] was not to be expected on theoretical grounds; for when, in a material system, all the velocities are reversed, the motion is reversed, it being understood that, in the application of this theorem to electrical theory, an electrostatic state is to be regarded as one of rest, and a current as a phenomenon of motion; and if such a reversal be performed in the present system, the poles of the electromagnet are exchanged, while in the dielectric no change takes place.

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[*] Wien Anz., 1880, p. 12. Phil. Mag. ix (1880), p. 307.

[†] Ann. d. Phys. xi (1880), pp. 432, 1044.

[‡] Am. Jour. Sci. xx (1880), p. 164.

[§] In 1885–6 E. van Aubel, Bull. de l'Acad. Roy. de Belgique (3) x, p. 609; xii, p. 280, repeated the investigation in an improved form, and confirmed the result that a magnetic field has no influence on the electrostatic polarization of dielectrics.

[||] H. A. Lorentz, Arch. Neérl. xix (1884), p. 123.

We must now consider the bearing of Hall's effect on the question as to whether magnetism is a rotatory or a linear phenomenon.* If magnetism be linear, electric currents must be rotatory; and if Hall's phenomenon be supposed to take place in a horizontal strip of metal, the magnetic force being directed vertically upwards, and the primary current flowing horizontally from north to south, the only geometrical entities involved are the vertical direction and a rotation in the east-and-west vertical plane; and these are indifferent with respect to a rotation in the north-and-south vertical plane, so that there is nothing in the physical circumstances of the system to determine in which direction the secondary current shall flow. The hypothesis that magnetism is linear appears therefore to be inconsistent with the existence of Hall's effect.† There are, however, some considerations which may be urged on the other side. Hall's effect, like the magnetic rotation of light, takes place only in ponderable bodies, not in free aether; and its direction is sometimes in one sense, sometimes in the other, according to the nature of the substance. It may therefore be doubted whether these phenomena are not of a secondary character, and the argument based on them invalid. Moreover, as FitzGerald remarked,‡ the magnetic lines of force associated with a system of currents are circuital and have no open ends, making it difficult to imagine how alteration of rotation inside them could be produced.

Of the various attempts to represent electric and magnetic phenomena by the motions and strains of a continuous medium, none of those hitherto considered has been found free from objection.* Before proceeding to consider models which are not constituted by a continuous medium, mention must be made of a suggestion offered by Riemann in his lectures† of 1861. Riemann remarked that the scalar-potential ϕ and vector-potential **a**, corresponding to his own law of force between electrons, satisfy the equation

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* Cf. F. Koláček, Ann. d. Phys. lv (1895), p. 503.

† Further evidence in favour of the hypothesis that it is the electric phenomena which are linear is furnished by the fact that pyro-electric effects (the production of electric polarization by warming) occur in acentric crystals, and only in such. Cf. M. Abraham, *Encyklopädie der math. Wiss.* iv (2), p. 43.

‡ Cf. Larmor, Phil. Trans. clxxxv, p. 780.

$$\dot{\phi} + \operatorname{div} \mathbf{a} = 0;$$

an equation which, as we have seen, is satisfied also by the potentials of L. Lorenz.‡ This appeared to Riemann to indicate that ϕ might represent the density of an aether, of which **a** represents the velocity. It will be observed that on this hypothesis the electric and magnetic forces correspond to *second* derivates of the displacement—a circumstance which makes it somewhat difficult to assimilate the energy possessed by the electromagnetic field to the energy of the model.

We must now proceed to consider those models in which the aether is represented as composed of more than one kind of constituent: of these Maxwell's model of 1861–2, formed of vortices and rolling particles, may be taken as the type. Another device of the same class was described in 1885 by Fitz Gerald§; this was constituted of a number of wheels, free to rotate on axes fixed perpendicularly in a plane board; the axes were fixed at the intersections of two systems of perpendicular lines; and each wheel was geared to each of its four neighbours by an indiarubber band. Thus all the wheels could rotate without any straining of the system, provided they all had the same angular velocity; but if some of the wheels were revolving faster than others, the indiarubber bands would become strained. It is evident that the wheels in this model play the same part as the vortices in Maxwell's model of 1861–2: their rotation is the analogue of magnetic force; and a region in which the masses of the wheels are large corresponds to a region of high magnetic permeability. The indiarubber bands of Fitz Gerald's model correspond to the medium in which Maxwell's vortices were embedded; and a strain on the bands represents dielectric polarization, the line joining the tight and slack sides of any band being the direction of displacement. A body whose specific inductive capacity is large would be represented by a region in which the elasticity of the bands is feeble. Lastly, conduction may be represented by a slipping of the bands on the wheels.

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* Cf. H. Witte, *Ueber den gegenwärtigen Stand der Frage nach einer mechanischen Erklärung der elektrischen Erscheinungen*; Berlin, 1906.

† Edited after his death by K. Hattendorff, under the title *Schwere, Elektricität, und Magnetismus*, 1875, p. 330.

‡ Cf. p. 299.

§ Scient. Proc. Roy. Dublin Soc., 1885; Phil. Mag. June, 1885; Fitz Gerald's *Scient. Writings*, pp. 142, 157.

Such a model is capable of transmitting vibrations analogous to those of light. For if any group of wheels be suddenly set in rotation, those in the neighbourhood will be prevented by their inertia from immediately sharing in the motion; but presently the rotation will be communicated to the adjacent wheels, which will transmit it to their neighbours; and so a wave of motion will be propagated through the medium. The motion constituting the wave is readily seen to be directed in the plane of the wave, i.e. the vibration is transverse. The axes of rotation of the wheels are at right angles to the direction of propagation of the wave, and the direction of polarization of the bands is at right angles to both these directions.

The elastic bands may be replaced by lines of governor balls:* if this be done, the energy of the system is entirely of the kinetic type.†

Models of types different from the foregoing have been suggested by the researches of Helmholtz and W. Thomson on vortex-motion. The earliest attempts in this direction, however, were intended to illustrate the properties of ponderable matter rather than of the luminiferous medium. A vortex existing in a perfect fluid preserves its individuality throughout all changes, and cannot be destroyed; so that if, as Thomson* suggested in 1867, the atoms of matter are constituted of vortex-rings in a perfect fluid, the conservation of matter may be immediately explained. The mutual interactions of atoms may be illustrated by the behaviour of smoke-rings, which after approaching each other closely are observed to rebound: and the spectroscopic properties of matter may be referred to the possession by vortex-rings of free periods of vibration.†

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* Fitz Gerald's *Scient. Writings*, p. 271.

† It is of course possible to devise models of this class in which the rotation may be interpreted as having the electric instead of the magnetic character. Such a model was proposed by Boltzmann, *Vorlesungen über Maxwell's Theorie*, ii.

There are, however, objections to the hypothesis of vortex-atoms. It is not easy to understand how the large density of ponderable matter as compared with aether is to be explained; and further, the virtual inertia of a vortex-ring increases as its energy increases; whereas the inertia of a ponderable body is, so far as is known, unaffected by changes of temperature. It is, moreover, doubtful whether vortex-atoms would be stable. "It now seems to me certain," wrote W. Thomson‡ (Kelvin) in 1905, "that if any motion be given within a finite portion of an infinite incompressible liquid, originally at rest, its fate is necessarily dissipation to infinite distances with infinitely small velocities everywhere; while the total kinetic energy remains constant. After many years of failure to prove that the motion in the ordinary Helmholtz circular ring is stable, I came to the conclusion that it is essentially unstable, and that its fate must be to become dissipated as now described."

The vortex-atom hypothesis is not the only way in which the theory of vortex-motion has been applied to the construction of models of the aether. It was shown in 1880 by W. Thomson§ that in certain circumstances a mass of fluid can exist in a state in which portions in rotational and irrotational motion are finely mixed together, so that on a large scale the mass is homogeneous, having within any sensible volume an equal amount of vortex-motion in all directions. To a fluid having such a type of motion he gave the name *vortex-sponge.*

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* Phil. Mag. xxxiv (1867), p. 15; Proc. R.S. Edinb. vi, p. 94.

† An attempt was made in 1883 by J. J. Thomson, Phil. Mag. xv (1883), p. 427, to explain the phenomena of the electric discharge through gases in terms of the theory of vortex-atoms. The electric field was supposed to consist in a distribution of velocity in the medium whose vortex-motion constituted the atoms of the gas; and Thomson considered the effect of this field on the dissociation and recoupling of vortex-rings.

‡ Proc. Roy. Soc. Edinb., xxv (1905), p. 565.

§ Brit. Assoc. Rep., 1880, p. 473.

Five years later, FitzGerald* discussed the suitability of the vortex-sponge as a model of the aether. Since vorticity in a perfect fluid cannot be created or destroyed, the modification of the system which is to be analogous to an electric field must be a polarized state of the vortex motion, and light must be represented by a communication of this polarized motion from one part of the medium to another. Many distinct types of polarization may readily be imagined : for instance, if the turbulent motion were constituted of vortex-rings, these might be in motion parallel to definite lines or planes; or if it were constituted of long vortex filaments, the filaments might be bent spirally about axes parallel to a given direction. The energy of any polarized state of vortex-motion would be greater than that of the unpolarized state; so that if the motion of matter had the effect of reducing the polarization, there would be forces tending to produce that motion. Since the forces due to a small vortex vary inversely as a high power of the distance from it, it seems probable that in the case of two infinite planes, separated by a region of polarized vortex-motion, the forces due to the polarization between the planes would depend on the polarization, but not on the mutual distance of the planes—a property which is characteristic of plane distributions whose elements attract according to the Newtonian law.

It is possible to conceive polarized forms of vortex-motion which are steady so far as the interior of the medium is concerned, but which tend to yield up their energy in producing motion of its boundary—a property parallel to that of the aether, which, though itself in equilibrium, tends to move objects immersed in it.

In the same year Hicks† discussed the possibility of transmitting waves through a medium consisting of an incompressible fluid in which small vortex-rings are closely packed together. The wave-length of the disturbance was supposed large in comparison with the dimensions and mutual distances of the rings; and the translatory motion of the latter was supposed to be so slow that very many waves can pass over any one before it has much changed its position. Such a medium would probably act as a fluid for larger motions. The vibration in the wave-front might be either swinging oscillations of a ring about a diameter, or transverse vibrations of the ring, or apertural vibrations; vibrations normal to the plane of the ring appear to be impossible. Hicks determined in each case the velocity of translation, in terms of the radius of the rings, the distance of their planes, and their cyclic constant.

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* Scient. Proc. Roy. Dublin Soc., 1885; *Scientific Writings of FitzGerald,* p. 154.

† Brit. Assoc. Rep., 1885, p. 930.

The greatest advance in the vortex-sponge theory of the aether was made in 1887, when W. Thomson* showed that the equation of propagation of laminar disturbances in a vortex-sponge is the same as the equation of propagation of luminous vibrations in the aether. The demonstration, which in the circumstances can scarcely be expected to be either very simple or very rigorous, is as follows:—

Let (u, v, w) denote the components of velocity, and p the pressure, at the point (x, y, z) in an incompressible fluid. Let the initial motion be supposed to consist of a laminar motion $\{f(y), 0, 0\}$, superposed on a homogeneous, isotropic, and fine-grained distribution (u'_0, v_0, w_0): so that at the origin of time the velocity is $\{f(y) + u'_0, v_0, w_0\}$: it is desired to find a function $f(y, t)$ such that at any time t the velocity shall be $\{f(y, t) + u', v, w\}$, where $u', v, w,$ are quantities of which every average taken over a sufficiently large space is zero.

Substituting these values of the components of velocity in the equation of motion

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{\partial p}{\partial x},$$

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* Phil. Mag. xxiv (1887), p. 342: Kelvin's *Math. and Phys. Papers*, iv, p. 308.

there results

$$\frac{\partial f(y,t)}{\partial t}+\frac{\partial u'}{\partial t}=-f(y,t)\frac{\partial u'}{\partial x}-v\frac{\partial f(y,t)}{\partial y}-u'\frac{\partial u'}{\partial x}-v\frac{\partial u'}{\partial y}$$
$$-w\frac{\partial u'}{\partial z}-\frac{\partial p}{\partial x}.$$

Take now the xz-averages of both members. The quantities $\partial u'/\partial t$, $\partial u'/\partial x$, v, $\partial p/\partial x$ have zero averages; so the equation takes the form

$$\frac{\partial f(y,t)}{\partial t}=-A\,.\left(u'\frac{\partial u'}{\partial x}+v\frac{\partial u'}{\partial y}+w\frac{\partial u'}{\partial z}\right),$$

if the symbol A is used to indicate that the xz-average is to be taken of the quantity following. Moreover, the incompressibility of the fluid is expressed by the equation

$$\frac{\partial u'}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0,$$

whence

$$0=-A\,.\left(u'\frac{\partial u'}{\partial x}+u'\frac{\partial v}{\partial y}+u'\frac{\partial w}{\partial z}\right).$$

When this is added to the preceding equation, the first and third pairs of terms of the second member vanish, since the x-average of any derivate $\partial Q/\partial x$ vanishes if Q is finite for infinitely great values of x; and the equation thus becomes

$$\frac{\partial f(y,t)}{\partial t}=-A\,.\frac{\partial(u'v)}{\partial y}. \tag{1}$$

From this it is seen that if the turbulent motion were to remain continually isotropic as at the beginning, $f(y,t)$ would constantly retain its critical value $f(y)$. In order to examine the deviation from isotropy, we shall determine $A\partial(u'v)/\partial t$, which may be done in the following way:—Multiplying the u- and v-equations of motion by v, u' respectively, and adding, we have

$$v\frac{\partial f(y,t)}{\partial t}+\frac{\partial(u'v)}{\partial t}=-f(y,t)\frac{\partial(u'v)}{\partial x}-v^2\frac{\partial f(y,t)}{\partial y}-u'\frac{\partial(u'v)}{\partial x}$$
$$-v\frac{\partial(u'v)}{\partial y}-w\frac{\partial(u'v)}{\partial z}-v\frac{\partial p}{\partial x}-u'\frac{\partial p}{\partial y}.$$

Taking the xz-average of this, we observe that the first term of the first member disappears, since $A \,.\, v$ is zero, and the first term of the second member disappears, since $A \,.\, \partial(u'v)/\partial x$ is zero. Denoting by $\frac{1}{3}R^2$ the average value of u^2, v^2, or w^2, so that R may be called the *average velocity* of the turbulent motion, the equation becomes

$$\frac{\partial}{\partial t}\{A \,.\, (u'v)\} = -\tfrac{1}{3}R^2 \frac{\partial f(y, t)}{\partial y} - Q,$$

where

$$Q = A \,.\left\{u' \frac{\partial(u'v)}{\partial x} + v \frac{\partial(u'v)}{\partial y} + w \frac{\partial(u'v)}{\partial z} + v \frac{\partial p}{\partial x} + u' \frac{\partial p}{\partial y}\right\}.$$

Let p be written $(p' + \varpi)$, where p' denotes the value which p would have if f were zero. The equations of motion immediately give

$$-\nabla^2 p = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 + 2\frac{\partial v}{\partial z}\frac{\partial w}{\partial y} + 2\frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x};$$

and on subtracting the forms which this equation takes in the two cases, we have

$$-\nabla^2\varpi = 2\frac{\partial f(y, t)}{\partial y}\frac{\partial v}{\partial x},$$

which, when the turbulent motion is fine-grained, so that $f(y, t)$ is sensibly constant over ranges within which u', v, w pass through all their values, may be written

$$\varpi = -2\frac{\partial f(y, t)}{\partial y}\nabla^{-2}\frac{\partial v}{\partial x}.$$

Moreover, we have

$$0 = A\left\{u' \frac{\partial(u'v)}{\partial x} + v \frac{\partial(u'v)}{\partial y} + w \frac{\partial(u'v)}{\partial z} + v \frac{\partial p'}{\partial x} + u' \frac{\partial p'}{\partial y}\right\};$$

for positive and negative values of u', v, w are equally probable; and therefore the value of the second member of this equation is doubled by adding to itself what it becomes when for u', v, w we substitute $-u'$, $-v$, $-w$; which (as may be seen by inspection of the above equation in $\nabla^2 p$) does not change the value of p'.

Comparing this equation with that which determines the value of Q, we have

$$Q = A\,.\left(v\frac{\partial\varpi}{\partial x} + u'\frac{\partial\varpi}{\partial y}\right),$$

or substituting for ϖ,

$$Q = -\,2\,\frac{\partial f(y, t)}{\partial y}\,.\,A\,.\left(v\frac{\partial}{\partial x} + u'\frac{\partial}{\partial y}\right).\,\nabla^{-2}\frac{\partial v}{\partial x}\,.$$

The isotropy with respect to x and z gives the equation

$$2A\,.\left(v_0\frac{\partial}{\partial x} + u_0'\frac{\partial}{\partial y}\right).\,\nabla^{-2}\frac{\partial v_0}{\partial x} = A\,.\left\{v_0\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\right.$$
$$\left. + \left(u_0'\frac{\partial}{\partial x} + w_0\frac{\partial}{\partial z}\right)\frac{\partial}{\partial y}\right\}\nabla^{-2}v_0.$$

But by integration by parts we obtain the equation

$$A\,.\left(u_0'\frac{\partial}{\partial x} + w_0\frac{\partial}{\partial z}\right)\frac{\partial}{\partial y}\,.\,\nabla^{-2}v_0 = -\,A\left(\frac{\partial u_0'}{\partial x} + \frac{\partial w_0}{\partial z}\right)\frac{\partial}{\partial y}\nabla^{-2}v_0;$$

and by the condition of incompressibility the second member may be written

$$A\,.(\partial v_0/\partial y)\,.\,(\partial/\partial y)\,.\,\nabla^{-2}v_0, \quad \text{or} \quad -\,A\,.\,v_0\,.(\partial/^2\partial y^2)\,.\,\nabla^{-2}v_0\,;$$

so we have

$$Q_0 = -\,\frac{\partial f(y, t)}{\partial y}\,A\,.\left\{v_0\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial y^2}\right)\right\}.\,\nabla^{-2}v_0.$$

On account of the isotropy, we may write $\frac{1}{3}$ for

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\nabla^{-2};$$

so $$Q_0 = -\,\tfrac{1}{9}R^2\,\frac{\partial f(y, t)}{\partial y}\,;$$

and, therefore,

$$\frac{\partial}{\partial t}_{(t=0)}\{A\,.(u'v)\} = -\,\tfrac{2}{9}R^2\left\{\frac{\partial f(y, t)}{\partial y}\right\}_{t=0}.$$

The deviation from isotropy shown by this equation is very small, because of the smallness of $\partial f(y, t)/\partial y$. The equation is therefore not restricted to the initial values of the two members, for we may neglect an infinitesimal deviation from $(2/9)R^2$ in the first factor of the second member, in consideration of the smallness of the second factor. Hence for all values of t we have the equation

$$\frac{\partial}{\partial t} A . (u'v) = - \tfrac{2}{9} R^2 \frac{\partial f(y, t)}{\partial y},$$

which, in combination with (1), yields the result

$$\frac{\partial^2}{\partial t^2} f(y, t) = \tfrac{2}{9} R^2 \frac{\partial^2}{\partial y^2} f(y, t);$$

the form of this equation shows that *laminar disturbances are propagated through the vortex-sponge in the same manner as waves of distortion in a homogeneous elastic solid.*

The question of the stability of the turbulent motion remained undecided; and at the time Thomson seems to have thought it likely that the motion would suffer diffusion. But two years later* he showed that stability was ensured at any rate when space is filled with a set of approximately straight hollow vortex filaments. Fitz Gerald† subsequently determined the energy per unit-volume in a turbulent liquid which is transmitting laminar waves. Writing for brevity

$$(2/9) R^2 = V^2, \quad f(y, t) = P, \quad \text{and} \quad A(u'v) = \gamma,$$

the equations are

$$\frac{\partial P}{\partial t} = - \frac{\partial \gamma}{\partial y}, \quad \text{and} \quad \frac{\partial \gamma}{\partial t} = - V^2 \frac{\partial P}{\partial y}$$

If the quantity

$$P^2 + \gamma^2 / V^2 = 2\Sigma$$

is integrated throughout space, and the variations of the integral with respect to time are determined, it is found that

$$\frac{\partial}{\partial t} \iiint \Sigma \, dx \, dy \, dz = \iiint \left(P \frac{\partial P}{\partial t} + \frac{\gamma}{V^2} \frac{\partial \gamma}{\partial t} \right) dx \, dy \, dz$$
$$= \iiint \left(P \frac{\partial P}{\partial t} - \gamma \frac{\partial P}{\partial y} \right) dx \, dy \, dz.$$

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* Proc. Roy. Irish Acad. (3) i (1889), p. 340; Kelvin's *Math. and Phys. Papers*, iv, p. 202.

† Brit. Assoc. Rep., 1899. Fitz Gerald's *Scientific Writings*, p. 484.

Integrating the second term under the integral by parts, and omitting the superficial terms (which may be at infinity, or wherever energy enters the space under consideration), we have

$$\frac{\partial}{\partial t}\iiint \Sigma \, dx\, dy\, dz = \iiint P\left(\frac{\partial P}{\partial t} + \frac{\partial \gamma}{\partial y}\right) dx\, dy\, dz = 0.$$

Hence it appears that the quantity Σ, which is of the dimensions of energy, must be proportional to the energy per unit-volume of the medium—a result which shows that there is a pronounced similarity between the dynamics of a vortex-sponge and of Maxwell's elastic aether.

A definite vortex-sponge model of the aether was described by Hicks in his Presidential Address to the mathematical section of the British Association in 1895.* In this the small motions whose function is to confer the quasi-rigidity were not completely chaotic, but were disposed systematically. The medium was supposed to be constituted of cubical elements of fluid, each containing a rotational circulation complete in itself: in any element, the motion close to the central vertical diameter of the element is vertically upwards: the fluid which is thus carried to the upper part of the element flows outwards over the top, down the sides, and up the centre again. In each of the six adjoining elements the motion is similar to this, but in the reverse direction. The rotational motion in the elements confers on them the power of resisting distortion, so that waves may be propagated through the medium as through an elastic solid; but the rotations are without effect on irrotational motions of the fluid, provided the velocities in the irrotational motion are slow compared with the velocity of propagation of distortional vibrations.

A different model was described four years later by Fitz Gerald.† Since the distribution of velocity of a fluid in the neighbourhood of a vortex filament is the same as the distribution of magnetic force around a wire of identical form carrying an electric current, it is evident that the fluid has more energy when the filament has the form of a helix than when it is straight; so if space were filled with vortices, whose axes were all parallel to a given direction, there would be an increase in the energy per unit volume when the vortices were bent into a spiral form; and this could be measured by the square of a vector—say, **E**—which may be supposed parallel to this direction.

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* Brit. Assoc. Rep., 1895, p. 595.

† Proc. Roy. Dublin Soc., December 12, 1899; Fitz Gerald's *Scientific Writings*, p. 472.

If now a single spiral vortex is surrounded by parallel straight ones, the latter will not remain straight, but will be bent by the action of their spiral neighbour. The transference of spirality may be specified by a vector **H**, which will be distributed in circles round the spiral vortex; its magnitude will depend on the rate at which spirality is being lost by the original spiral, and can be taken such that its square is equal to the mean energy of this new motion. The vectors **E** and **H** will then represent the electric and magnetic vectors; the vortex spirals representing tubes of electric force.

Fitz Gerald's spirality is essentially similar to the laminar motion investigated by Lord Kelvin, since it involves a flow in the direction of the axis of the spiral, and such a flow cannot take place along the direction of a vortex filament without a spiral deformation of a filament.

Other vortex analogues have been devised for electrostatical systems. One such, which was described in 1888 by W. M. Hicks,* depends on the circumstance that if two bodies in contact in an infinite fluid are separated from each other, and if there be a vortex filament which terminates on the bodies, there will be formed at the point where they separate a hollow vortex filament† stretching from one to the other, with rotation equal and opposite to that of the original filament. As the bodies are moved apart, the hollow vortex may, through failure of stability, dissociate into a number of smaller ones; and if the resulting number be very large, they will ultimately take up a position of stable equilibrium. The two sets of filaments —the original filaments and their hollow companions—will be intermingled, and each will distribute itself according to the same law as the lines of force between the two bodies which are equally and oppositely electrified.

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* Brit. Assoc. Rep., 1888, p. 577.

† A hollow vortex is a cyclic motion existing in a fluid without the presence of any actual rotational filaments. On the general theory cf. Hicks, Phil. Trans. clxxv (1883), p. 161; clxxvi (1885), p. 725; cxcii (1898), p. 33.

Since the pressure inside a hollow vortex is zero, the portion of the surface on which it abuts experiences a diminution of pressure; the two bodies are therefore attracted. Moreover, as the two bodies separate further, the distribution of the filaments being the same as that of lines of electric force, the diminution of pressure for each line is the same at all distances, and therefore the force between the two bodies follows the same law as the force between two bodies equally and oppositely electrified. It may be shown that the effect of the original filaments is similar, the diminution of pressure being half as large again as for the hollow vortices.

If another surface were brought into the presence of the others, those of the filaments which encounter it would break off and rearrange themselves so that each part of a broken filament terminates on the new body. This analogy thus gives a complete account of electrostatic actions both quantitatively and qualitatively: the electric charge on a body corresponds to the number of ends of filaments abutting on it, the sign being determined by the direction of rotation of the filament as viewed from the body.

A magnetic field may be supposed to be produced by the motion of the vortex filaments through the stationary aether, the magnetic force being at right angles to the filament and to its direction of motion. Electrostatic and magnetic fields thus correspond to states of motion in the medium, in which, however, there is no bodily flow; for the two kinds of filament produce circulation in opposite directions.

It is possible that hollow vortices are better adapted than ordinary vortex-filaments for the construction of models of the aether. Such, at any rate, was the opinion of Thomson (Kelvin) in his later years.* The analytical difficulties of the subject are formidable, and progress is consequently slow; but among the many mechanical schemes which have been devised to represent electrical and optical phenomena, none possesses greater interest than that which pictures the aether as a vortex-sponge.

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* Proc. Roy. Irish Acad., November 30, 1889; Kelvin's *Math. and Phys. Papers*, iv, p. 202. "Rotational vortex-cores," he wrote, "must be absolutely discarded; and we must have nothing but irrotational revolution and vacuous cores."

CHAPTER X.

THE FOLLOWERS OF MAXWELL.

THE most notable imperfection in the electromagnetic theory of light, as presented in Maxwell's original memoirs, was the absence of any explanation of reflexion and refraction. Before the publication of Maxwell's *Treatise*, however, a method of supplying the omission was indicated by Helmholtz.[*] The principles on which the explanation depends are that the normal component of the electric displacement **D**, the tangential components of the electric force **E**, and the magnetic vector **B** or **H**, are to be continuous across the interface at which the reflexion takes place; the optical difference between the contiguous bodies being represented by a difference in their dielectric constants, and the electric vector being assumed to be at right angles to the plane of polarization.[†] The analysis required is a mere transcription of MacCullagh's theory of reflexion,[‡] if the derivate of MacCullagh's displacement **e** with respect to the time be interpreted as the magnetic force, μ curl **e** as the electric force, and curl **e** as the electric displacement. The mathematical details of the solution were not given by Helmholtz himself, but were supplied a few years later in the inaugural dissertation of H. A. Lorentz.[§]

In the years immediately following the publication of Maxwell's *Treatise*, a certain amount of evidence in favour of his theory was furnished by experiment. That an electric field is closely concerned with the propagation of light was demonstrated in 1875, when John Kerr* showed that dielectrics subjected to powerful electrostatic force acquire the property of double refraction, their optical behaviour being similar to that of uniaxal crystals whose axes are directed along the lines of force.

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[*] Journal für Math. lxxii (1870), p. 68, note.

[†] Helmholtz (loc. cit.) pointed out that if the optical difference between the media were assumed to be due to a difference in their magnetic permeabilities, it would be necessary to suppose the magnetic vector at right angles to the plane of polarization in order to obtain Fresnel's sine and tangent formulae of reflexion.

[‡] Cf. pp. 148, 149, 154–156.

[§] Zeitschrift für Math. u. Phys. xxii (1877), pp. 1, 205: *Over de theorie der terugkaatsing en breking van het licht*, Arnhem, 1875. Lorentz's work was based on Helmholtz's equations, but remains substantially unchanged when Maxwell's formulae are substituted.

Other researches undertaken at this time had a more direct bearing on the questions at issue between the hypothesis of Maxwell and the older potential theories. In 1875–6 Helmholtz† and his pupil Schiller‡ attempted to discriminate between the various doctrines and formulae relative to unclosed circuits by performing a crucial experiment.

It was agreed in all theories that a ring-shaped magnet, which returns into itself so as to have no poles, can exert no ponderomotive force on other magnets or on closed electric currents. Helmholtz§ had, however, shown in 1873 that according to the potential-theories such a magnet would exert a ponderomotive force on an unclosed current. The matter was tested by suspending a magnetized steel ring by a long fibre in a closed metallic case, near which was placed a terminal of a Holtz machine. No ponderomotive force could be observed when the machine was put in action so as to produce a brush discharge from the terminal: from which it was inferred that the potential-theories do not correctly represent the phenomena, at least when displacement-currents and convection-currents (such as that of the electricity carried by the electrically repelled air from the terminal) are not taken into account.

The researches of Helmholtz and Schiller brought into prominence the question as to the effects produced by the translatory motion of electric charges. That the convection of electricity is equivalent to a current had been suggested long before by Faraday.* "If," he wrote in 1838, "a ball be electrified positively in the middle of a room and be then moved in any direction, effects will be produced as if a current in the same direction had existed." To decide the matter a new experiment inspired by Helmholtz was performed by H. A. Rowland† in 1876. The electrified body in Rowland's disposition was a disk of ebonite, coated with gold leaf and capable of turning rapidly round a vertical axis between two fixed plates of glass, each gilt on one side. The gilt faces of the plates could be earthed, while the ebonite disk received electricity from a point placed near its edge; each coating of the disk thus formed a condenser with the plate nearest to it. An astatic needle was placed above the upper condenser-plate, nearly over the edge of the disk; and when the disk was rotated a magnetic field was found to be produced. This experiment, which has since been repeated under improved conditions by Rowland and Hutchinson,‡ H. Pender§, and Eichenwald,‖ shows that the "convection-current" produced by the rotation of a charged disk, when the other ends of the lines of force are on an earthed stationary plate parallel to it, produces the same magnetic field as an ordinary conduction-current flowing in a circuit which coincides with the path of the convection-current. When two disks forming a condenser are rotated together, the magnetic action is the sum of the magnetic actions of each of the disks separately. It appears, therefore, that electric charges cling to the matter of a conductor and move with it, so far as Rowland's phenomenon is concerned.

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* Phil. Mag. (4) l (1875), pp. 337, 446; (5) viii (1879), pp. 85, 229; xiii (1882), pp. 153, 248.

† Monatsberichte d. Acad. d. Berlin, 1875, p. 400. Ann. d. Phys., clviii (1876), p. 87.

‡ Ann. d. Phys. clix (1876), pp. 456, 537; clx (1877), p. 333.

§ The valuable memoirs by Helmholtz in Journal für Math. lxxii (1870), p. 57; lxxv (1873), p. 35; lxxviii (1874), p. 273, to which reference has already been made, contain a full discussion of the various possibilities of the potential-theories.

The first examination of the matter from the point of view of Maxwell's theory was undertaken by J. J. Thomson,¶ in 1881. If an electrostatically charged body is in motion, the change in the location of the charge must produce a continuous alteration of the electric field at any point in the surrounding medium; or, in the language of Maxwell's theory, there must be displacement-currents in the medium. It was to these displacement-currents that Thomson, in his original investigation, attributed the magnetic effects of moving charges. The particular system which he considered was that formed by a charged spherical conductor, moving uniformly in a straight line. It was assumed that the distribution of electricity remains uniform over the surface during the motion, and that the electric field in any position of the sphere is the same as if the sphere were at rest; these assumptions are true so long as quantities of order $(v/c)^2$ are neglected, where v denotes the velocity of the sphere and c the velocity of light.

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* *Exper. Res.*, § 1644.

† Monatsberichte d. Akad. d. Berlin, 1876, p. 211: Ann. d. Phys. clviii (1876), p. 487: Annales de Chim. et de Phys. xii (1877) p. 119.

‡ Phil. Mag. xxvii (1889), p. 445.

§ *Ibid.* ii (1901), p. 179: v (1903), p. 34.

‖ Ann. d. Phys. xi (1901), p. 1.

¶ Phil. Mag. xi (1881), p. 229.

Thomson's method was to determine the displacement-currents in the space outside the sphere from the known values of the electric field, and then to calculate the vector-potential due to these displacement-currents by means of the formula

$$\mathbf{A} = \iiint (\mathbf{S}'/r)\, dx'\, dy'\, dz',$$

where $\mathbf{S}'$ denotes the displacement-current at $(x'y'z')$. The magnetic field was then determined by the equation

$$\mathbf{H} = \text{curl } \mathbf{A}.$$

A defect in this investigation was pointed out by Fitz Gerald, who, in a short but most valuable note,* published a few months afterwards, observed that the displacement-currents of Thomson do not satisfy the circuital condition. This is most simply seen by considering the case in which the system consists of two parallel plates forming a condenser; if one of the plates is fixed, and the other plate is moved towards it, the electric field is annihilated in the space over which the moving plate travels: this destruction of electric displacement constitutes a displacement-current, which, considered alone, is evidently not a closed current. The defect, as Fitz Gerald showed, may be immediately removed by assuming that a moving charge itself is to be counted as a current-element: the total current, thus composed of the displacement-currents and the convection-current, is circuital. Making this correction, Fitz Gerald found that the magnetic force due to a sphere of charge e moving with velocity v along the axis of z is curl $(0, 0, ev/r)$—a formula which shows that the displacement-currents have no resultant magnetic effect, since the term ev/r would be obtained from the convection-current alone.

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* Proc. Roy. Dublin Soc., November, 1881; Fitz Gerald's *Scientific Writings*, p. 102.

The expressions obtained by Thomson and Fitz Gerald were correct only to the first order of the small quantity v/c. The effect of including terms of higher order was considered in 1889 by Oliver Heaviside,* whose solution may be derived in the following manner:—

Suppose that a charged system is in motion with uniform velocity v parallel to the axis of z; the total current consists of the displacement-current $\dot{\mathbf{E}}/4\pi c^2$ where $\mathbf{E}$ denotes the electric force, and the convection-current $\rho\mathbf{v}$ where ρ denotes the volume-density of electricity. So the equation which connects magnetic force with electric current may be written

$$\dot{\mathbf{E}}/c^2 = \text{curl } \mathbf{H} - 4\pi\rho\mathbf{v}.$$

Eliminating $\mathbf{E}$ between this and the equation

$$\text{curl } \mathbf{E} = -\dot{\mathbf{H}},$$

and remembering that $\mathbf{H}$ is here circuital, we have

$$\ddot{\mathbf{H}}/c^2 - \nabla^2\mathbf{H} = 4\pi \text{ curl } \rho\mathbf{v}.$$

If, therefore, a vector-potential $\mathbf{a}$ be defined by the equation

$$\ddot{\mathbf{a}}/c^2 - \nabla^2\mathbf{a} = 4\pi\rho\mathbf{v},$$

the magnetic force will be the curl of $\mathbf{a}$; and from the equation for $\mathbf{a}$ it is evident that the components a_x and a_y are zero, and that a_z is to be determined from the equation

$$\ddot{a}_z/c^2 - \nabla^2 a_z = 4\pi\rho v.$$

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* Phil. Mag. xxvii (1889), p. 324.

Now, let (x, y, ζ) denote coordinates relative to axes which are parallel to the axes (x, y, z), and which move with the charged bodies; then a_z is a function of (x, y, ζ) only; so we have

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta}, \quad \text{and} \quad \frac{\partial}{\partial t} = -v\frac{\partial}{\partial \zeta};$$

and the preceding equation is readily seen to be equivalent to

$$\frac{\partial^2 a_z}{\partial x^2} + \frac{\partial^2 a_z}{\partial y^2} + \frac{\partial^2 a_z}{\partial \zeta_1^2} = -4\pi\rho v,$$

where ζ_1 denotes $(1 - v^2/c^2)^{-\frac{1}{2}}\zeta$. But this is simply Poisson's equation, with ζ_1 substituted for z; so the solution may be transcribed from the known solution of Poisson's equation: it is

$$a_z = \iiint \frac{\rho' v' \, dx' \, dy' \, d\zeta_1'}{\{(\zeta_1 - \zeta_1')^2 + (x - x')^2 + (y - y')^2\}^{\frac{1}{2}}},$$

the integrations being taken over all the space in which there are moving charges; or

$$a_z = \iiint \frac{\rho' v' \, dx' \, dy' \, d\zeta'}{\{(\zeta - \zeta')^2 + (1 - v^2/c^2)(x - x')^2 + (1 - v^2/c^2)(y - y')^2\}^{\frac{1}{2}}}.$$

If the moving system consists of a single charge e at the point $\zeta = 0$, this gives

$$a_z = \frac{cv}{r(1 - v^2 \sin^2 \theta/c^2)^{\frac{1}{2}}},$$

where $\sin^2 \theta = (x^2 + y^2)/r^2$.

It is readily seen that the lines of magnetic force due to the moving point-charge are circles whose centres are on the line of motion, the magnitude of the magnetic force being

$$\frac{ev(1 - v^2/c^2)\sin\theta}{r^2(1 - v^2\sin^2\theta/c^2)^{\frac{3}{2}}}.$$

The electric force is radial, its magnitude being

$$\frac{c^2 e(1 - v^2/c^2)}{r^2(1 - v^2\sin^2\theta/c^2)^{\frac{3}{2}}}.$$

The fact that the electric vector due to a moving point-charge is everywhere radial led Heaviside to conclude that the same solution is applicable when the charge is distributed over a perfectly conducting sphere whose centre is at the point, the only change being that **E** and **H** would now vanish inside the sphere. This inference was subsequently found* to be incorrect: a distribution of electric charge on a moving sphere could in fact not be in equilibrium if the electric force were radial, since there would then be nothing to balance the mechanical force exerted on the moving charge (which is equivalent to a current) by the magnetic field. The moving system which gives rise to the same field as a moving point-charge is not a sphere, but an oblate spheroid whose polar axis (which is in the direction of motion) bears to its equatorial axis the ratio $(1 - v^2/c^2)^{\frac{1}{2}} : 1$.†

The energy of the field surrounding a charged sphere is greater when the sphere is in motion than when it is at rest. To determine the additional energy quantitatively (retaining only the lowest significant powers of v/c), we have only to integrate, throughout the space outside the sphere, the expression $\mathbf{H}^2/8\pi$, which represents the electrokinetic energy per unit volume: the result is $e^2v^2/3a$, where e denotes the charge, v the velocity, and a the radius of the sphere.

It is evident from this result that the work required to be done in order to communicate a given velocity to the sphere is greater when the sphere is charged than when it is uncharged; that is to say, the virtual mass of the sphere is increased by an amount $2e^2/3a$, owing to the presence of the charge. This may be regarded as arising from the self-induction of the convection-current which is formed when the charge is set in motion. It was suggested by J. Larmor‡ and by W. Wien§ that the inertia of ordinary ponderable matter may ultimately prove to be of this nature, the atoms being constituted of systems of electrons.||

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* By G. F. C. Searle.

† Cf. Searle, Phil. Trans. clxxxvii (1896), p. 675, and Phil. Mag. xliv (1897), p. 329. On the theory of the moving electrified sphere, cf. also J. J. Thomson, *Recent Researches in Elect. and Mag.*, p. 16; O. Heaviside, *Electrical Papers*, ii, p. 514; *Electromag. Theory*, i, p. 269; W. B. Morton, Phil. Mag. xli (1896), p. 488; A. Schuster, Phil. Mag. xliii (1897), p. 1.

‡ Phil. Trans. clxxxvi (1895), p. 697.

§ Arch. Néerl (3) v (1900), p. 96.

|| Experimental evidence that the inertia of electrons is purely electromagnetic was afterwards furnished by W. Kaufmann, Gött. Nach., 1901, p. 143; 1902, p 291.

It may, however, be remarked that this view of the origin of mass is not altogether consistent with the principle that the electron is an indivisible entity. For the so-called self-induction of the spherical electron is really the *mutual* induction of the convection-currents produced by the elements of electric charge which are distributed over its surface; and the calculation of this quantity presupposes the divisibility of the total charge into elements capable of acting severally in all respects as ordinary electric charges; a property which appears scarcely consistent with the supposed fundamental nature of the electron.

After the first attempt of J. J. Thomson to determine the field produced by a moving electrified sphere, the mathematical development of Maxwell's theory proceeded rapidly. The problems which admit of solution in terms of known functions are naturally those in which the conducting surfaces involved have simple geometrical forms—planes, spheres, and cylinders.*

A result which was obtained by Horace Lamb,† when investigating electrical motions in a spherical conductor, led to interesting consequences. Lamb found that if a spherical conductor is placed in a rapidly alternating field, the induced currents are almost entirely confined to a superficial layer; and his result was shortly afterwards generalized by Oliver Heaviside,‡ who showed that whatever be the form of a conductor rapidly alternating currents do not penetrate far into its substance.§ The reason for this may be readily understood: it is virtually an application of the principle|| that a perfect conductor is impenetrable to magnetic lines of force. No perfect conductor is known to exist; but¶ if the alternations of magnetic force to which a good conductor such as copper is exposed are very

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* Cf., e.g., C. Niven, Phil. Trans. clxxii (1881), p. 307; H. Lamb, Phil. Trans. clxxiv (1883), p. 519; J. J. Thomson, Proc. Lond. Math. Soc. xv (1884), p. 197; H. A. Rowland, Phil. Mag. xvii (1884), p. 413; J. J. Thomson, Proc. Lond. Math. Soc. xvii (1886), p. 310; xix (1888), p. 520; and many investigations of Oliver Heaviside, collected in his *Electrical Papers.*

† Loc. cit.

‡ Electrician, Jan. 1885.

§ The mathematical theory was given by Lord Rayleigh, Phil. Mag. xxi. (1886), p. 381. Cf. Maxwell's *Treatise*, § 689.

|| Cf. p. 313.

¶ As was first remarked by Lord Rayleigh, Phil. Mag. xiii (1882), p. 344.

rapid, the conductor has not time (so to speak) to display the imperfection of its conductivity, and the magnetic field is therefore unable to extend far below the surface.

The same conclusion may be reached by different reasoning.* When the alternations of the current are very rapid, the ohmic resistance ceases to play a dominant part, and the ordinary equations connecting electromotive force, induction, and current are equivalent to the conditions that the currents shall be so distributed as to make the electrokinetic or magnetic energy a minimum. Consider now the case of a single straight wire of circular cross-section. The magnetic energy in the space outside the wire is the same whatever be the distribution of current in the cross-section (so long as it is symmetrical about the centre), since it is the same as if the current were flowing along the central axis; so the condition is that the magnetic energy in the wire shall be a minimum; and this is obviously satisfied when the current is concentrated in the superficial layer, since then the magnetic force is zero in the substance of the wire.

In spite of the advances which were effected by Maxwell and his earliest followers in the theory of electric oscillations, the gulf between the classical electrodynamics and the theory of light was not yet completely bridged. For in all the cases considered in the former science, energy is merely exchanged between one body and another, remaining within the limits of a given system; while in optics the energy travels freely through space, unattached to any material body. The first discovery of a more complete connexion between the two theories was made by Fitz Gerald, who argued that if the unification which had been indicated by Maxwell is valid, it ought to be possible to generate radiant energy by purely electrical means; and in 1883† he described methods by which this could be done.

Fitz Gerald's system is what has since become known as the *magnetic oscillator*: it consists of a small circuit, in which the strength of the current is varied according to the simple periodic law. The circuit will be supposed to be a circle of small area S, whose centre is the origin and whose plane is the plane of xy; and the surrounding medium will be supposed to be free aether. The current may be taken to be of strength $A \cos (2\pi t/T)$, so that the moment of the equivalent magnet is $SA \cos (2\pi t/T)$. Now in the older electrodynamics, the vector-potential due to a magnetic molecule of (vector) moment $\mathbf{M}$ at the origin is $(1/4\pi)$ curl $(\mathbf{M}/r)$, where r denotes distance from the origin. The vector-potential due to Fitz Gerald's magnetic oscillator would therefore be $(1/4\pi)$ curl $\mathbf{K}$, where $\mathbf{K}$ denotes a vector parallel to the axis of z, and of magnitude $(1/r)\, SA \cos (2\pi t/T)$. The change which is involved in replacing the assumptions of the older electrodynamics by those of Maxwell's theory is in the present case equivalent* to retarding the potential; so that the vector-potential $\mathbf{a}$ due to the oscillator is $(1/4\pi)$ curl $\mathbf{K}$ where $\mathbf{K}$ is still directed parallel to the axis of z, and is of magnitude

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* Cf. J. Stefan, Wiener, Sitzungsber. xcix (1890), p. 319; Ann. d. Phys. xli (1890), p. 400.

† Trans. Roy. Dublin Soc. iii (1883); Fitz Gerald's *Scient. Writings*, p. 122.

$$K = \frac{SA}{r} \cos \frac{2\pi}{T}\left(t - \frac{r}{c}\right).$$

The electric force $\mathbf{E}$ at any point of space is $-\dot{\mathbf{a}}$, and the magnetic force $\mathbf{H}$ is curl $\mathbf{a}$: so that these quantities may be calculated without difficulty. The electric energy per unit volume is $\mathbf{E}^2/8\pi c^2$: performing the calculations, it is found that the value of this quantity averaged over a period of the oscillation and also averaged over the surface of a sphere of radius r is

$$\frac{\pi A^2 S^2}{6c^2 r^4 T^2}\left(1 + \frac{4\pi^2 r^2}{c^2 T^2}\right).$$

The part of this which is radiated is evidently that which is proportional to the inverse square of the distance,† so the average value of the radiant energy of electric type at distance r from the oscillator is $2\pi^3A^2S^2/3c^4r^2T^4$ per unit volume. The radiant energy of magnetic type may be calculated in a similar way, and is found to have the same value; so the total radiant energy at distance r is $4\pi^3A^2S^2/3c^4r^2T^4$ per unit volume; and therefore the energy radiated in unit time is $16\pi^4A^2S^2/3c^3T^4$. This is small, unless the frequency is very high; so that ordinary alternating currents would give no appreciable radiation. Fitz Gerald, however, in the same year* indicated a method by which the difficulty of obtaining currents of sufficiently high frequency might be overcome: this was, to employ the alternating currents which are produced when a condenser is discharged.

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* Cf. pp. 298, 299.

† The other term, which is neglected, is very small compared to the term retained, at great distances from the origin; it is what would be obtained if the effects of induction of the displacement-currents were neglected: i.e. it is the energy of the forced displacement-currents which are produced directly by the variation of the primary current, and which originate the radiating displacement-currents.

The Fitz Gerald radiator constructed on this principle is closely akin to the radiator afterwards developed with such success by Hertz: the only difference is that in Fitz Gerald's arrangement the condenser is used merely as the store of energy (its plates being so close together that the electrostatic field due to the charges is practically confined to the space between them), and the actual source of radiation is the alternating magnetic field due to the circular loop of wire: while in Hertz's arrangement the loop of wire is abolished, the condenser plates are at some distance apart, and the source of radiation is the alternating electrostatic field due to their charges.

In the study of electrical radiation, valuable help is afforded by a general theorem on the transfer of energy in the electromagnetic field, which was discovered in 1884 by John Henry Poynting.† We have seen that the older writers on electric currents recognized that an electric current is associated with the transport of energy from one place (e.g. the voltaic cell which maintains the current) to another (e.g. an electromotor which is worked by the current); but they supposed the energy to be conveyed by the current itself within the wire, in much the same way as dynamical energy is carried by water flowing in a pipe; whereas in Maxwell's theory, the storehouse and vehicle of energy is the dielectric medium surrounding the wire. What Poynting achieved was to show that the flux of energy at any place might be expressed by a simple formula in terms of the electric and magnetic forces at the place.

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* Brit. Assoc. Rep., 1883; FitzGerald's *Scientific Writings*, p. 129.

† Phil. Trans. clxxv (1884), p. 343.

Denoting as usual by **E** the electric force, by **D** the electric displacement, by **H** the magnetic force, and by **B** the magnetic induction, the energy stored in unit volume of the medium is[*]

$$\tfrac{1}{2}\,\mathbf{ED} + (1/8\pi)\,\mathbf{BH}\,;$$

so the increase of this in unit time is (since in isotropic media **D** is proportional to **E**, and **B** is proportional to **H**)

$$\mathbf{E}\dot{\mathbf{D}} + (1/4\pi)\,\mathbf{H}\dot{\mathbf{B}}$$

or

$$\mathbf{E}\,(\mathbf{S} - \mathbf{\imath}) + (1/4\pi)\,\mathbf{H}\dot{\mathbf{B}},$$

where **S** denotes the total current, and $\mathbf{\imath}$ the current of conduction; or (in virtue of the fundamental electromagnetic equations)

$$-\,(\mathbf{E}\,.\,\mathbf{\imath}) + (1/4\pi)\,(\mathbf{E}\,.\,\mathrm{curl}\,\mathbf{H}) - (1/4\pi)\,(\mathbf{H}\,.\,\mathrm{curl}\,\mathbf{E}),$$

or

$$-\,(\mathbf{E}\,.\,\mathbf{\imath}) - (1/4\pi)\,\mathrm{div}\,[\mathbf{E}\,.\,\mathbf{H}].$$

Now $(\mathbf{E}\,.\,\mathbf{\imath})$ is the amount of electric energy transformed into heat per unit volume per second; and therefore the quantity $-\,(1/4\pi)\,\mathrm{div}\,[\mathbf{E}\,.\,\mathbf{H}]$ must represent the deposit of energy in unit volume per second due to the streaming of energy; which shows that the flux of energy is represented by the vector $(1/4\pi)\,[\mathbf{E}\,.\,\mathbf{H}]$.[†] This is Poynting's theorem: *that the flux of energy at any place is represented by the vector-product of the electric and magnetic forces, divided by* 4π.[‡]

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[*] Cf. pp. 248, 250, 282.

[†] Of course any circuital vector may be added. H. M. Macdonald, *Electric Waves*, p. 72, propounded a form which differs from Poynting's by a non-circuital vector.

[‡] The analogue of Poynting's theorem in the theory of the vibrations of an isotropic elastic solid may be easily obtained; for from the equation of motion of an elastic solid,

$$\rho\ddot{\mathbf{e}} = -\,(k + 4n/3)\ \mathrm{grad\ div}\ \mathbf{e} - n\ \mathrm{curl\ curl}\ \mathbf{e},$$

it follows that

$$\frac{\partial}{\partial t}\{\tfrac{1}{2}\rho\dot{\mathbf{e}}^2 + \tfrac{1}{2}\,(k + \tfrac{4}{3}n)\,(\mathrm{div}\ \mathbf{e})^2 + \tfrac{1}{2}n\,(\mathrm{curl}\ \mathbf{e})^2\} = -\,\mathrm{div}\ \mathbf{W},$$

In the special case of the field which surrounds a straight wire carrying a continuous current, the lines of magnetic force are circles round the axis of the wire, while the lines of electric force are directed along the wire; hence energy must be flowing in the medium in a direction at right angles to the axis of the wire. A current in any conductor may therefore be regarded as consisting essentially of a convergence of electric and magnetic energy from the medium upon the conductor, and its transformation there into other forms.

This association of a current with motions at right angles to the wire in which it flows doubtless suggested to Poynting the conceptions of a memoir which he published* in the following year. When an electric current flowing in a straight wire is gradually increased in strength from zero, the surrounding space becomes filled with lines of magnetic force, which have the form of circles round the axis of the wire. Poynting, adopting Faraday's idea of the physical reality of lines of force, assumed that these lines of force arrive at their places by moving outwards from the wire; so that the magnetic field grows by a continual emission from the wire of lines of force, which enlarge and spread out like the circular ripples from the place where a stone is dropped into a pond. The electromotive force which is associated with a changing magnetic field was now attributed directly to the motion of the lines of force, so that wherever electromotive force is produced by change in the magnetic field, or by motion of matter through the field, the electric intensity is equal to the number of tubes of magnetic force intersected by unit length in unit time.

A similar conception was introduced in regard to lines of electric force. It was assumed that any change in the total

where **W** denotes the vector

$$-(k + 4n/3)\ \mathrm{div}\ \mathbf{e} \,.\, \mathbf{e} + n\,[\mathrm{curl}\ \mathbf{e} \,.\, \dot{\mathbf{e}}]\,;$$

and since the expression which is differentiated with respect to t represents the sum of the kinetic and potential energies per unit volume of the solid (save for terms which give only surface-integrals), it is seen that **W** is the analogue of the Poynting vector. Cf. L. Donati, Bologna Mem. (5) vii (1899), p. 633.

* Phil. Trans. clxxvi (1885), p. 277.

electric induction through a curve is caused by the passage of tubes of force in or out across the boundary; so that whenever magnetomotive force is produced by change in the electric field, or by motion of matter through the field, the magnetomotive force is proportional to the number of tubes of electric force intersected by unit length in unit time.

Poynting, moreover, assumed that when a steady current C flows in a straight wire, C tubes of electric force close in upon the wire in unit time, and are there dissolved, their energy appearing as heat. If E denote the magnitude of the electric force, the energy of each tube per unit length is $\frac{1}{2}E$, so the amount of energy brought to the wire is $\frac{1}{2}CE$ per unit length per unit time. This is, however, only half the energy actually transformed into heat in the wire: so Poynting further assumed that E tubes of magnetic force also move in per unit length per unit time, and finally disappear by contraction to infinitely small rings. This motion accounts for the existence of the electric field; and since each tube (which is a closed ring) contains energy of amount $\frac{1}{2}C$, the disappearance of the tubes accounts for the remaining $\frac{1}{2}CE$ units of energy dissipated in the wire.

The theory of moving tubes of force has been extensively developed by Sir Joseph Thomson.* Of the two kinds of tubes —magnetic and electric—which had been introduced by Faraday and used by Poynting, Thomson resolved to discard the former and employ only the latter. This was a distinct departure from Faraday's conceptions, in which, as we have seen, great significance was attached to the physical reality of the magnetic lines; but Thomson justified his choice by inferences drawn from the phenomena of electric conduction in liquids and gases. As will appear subsequently, these phenomena indicate that molecular structure is closely connected with tubes of electrostatic force—perhaps much more closely than with tubes of magnetic force; and Thomson therefore decided to regard magnetism as the secondary effect, and to ascribe magnetic fields, not to the presence of magnetic tubes, but to the motion of electric tubes. In order to account for the fact that magnetic fields may occur without any manifestation of electric force, he assumed that tubes exist in great numbers everywhere in space, either in the form of closed circuits or else terminating on atoms, and that electric force is only perceived when the tubes have a greater tendency to lie in one direction than in another. In a steady magnetic field the positive and negative tubes might be conceived to be moving in opposite directions with equal velocities.

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* Phil. Mag. xxxi (1891), p. 149; Thomson's *Recent Researches in Elect. and Mag.* (1893), chapter i.

A beam of light might, from this point of view, be regarded simply as a group of tubes of force which are moving with the velocity of light at right angles to their own length. Such a conception almost amounts to a return to the corpuscular theory; but since the tubes have definite directions perpendicular to the direction of propagation, there would now be no difficulty in explaining polarization.

The energy accompanying all electric and magnetic phenomena was supposed by Thomson to be ultimately kinetic energy of the aether; the electric part of it being represented by rotation of the aether inside and about the tubes, and the magnetic part being the energy of the additional disturbance set up in the aether by the movement of the tubes. The inertia of this latter motion he regarded as the cause of induced electromotive force.

There was, however, one phenomenon of the electromagnetic field as yet unexplained in terms of these conceptions—namely, the ponderomotive force which is exerted by the field on a conductor carrying an electric current. Now any ponderomotive force consists in a transfer of mechanical momentum from the agent which exerts the force to the body which experiences it; and it occurred to Thomson that the ponderomotive forces of the electromagnetic field might be explained if the moving tubes of force, which enter a conductor carrying a current and are there dissolved, were supposed to possess mechanical momentum, which could be yielded up to the conductor. It is readily seen that such momentum must be directed at right angles to the tube and to the magnetic induction—a result which suggests that the momentum stored in unit volume of the aether may be proportional to the vector-product of the electric and magnetic vectors.

For this conjecture reasons of a more definite kind may be given.[*] We have already seen[†] that the ponderomotive forces on material bodies in the electromagnetic field may be accounted for by Maxwell's supposition that across any plane in the aether whose unit normal is $\mathbf{N}$, there is a stress represented by

$$\mathbf{P_N} = (\mathbf{D}\,.\,\mathbf{N})\,\mathbf{E} - \tfrac{1}{2}\,(\mathbf{D}\,.\,\mathbf{E})\,\mathbf{N} + (1/4\pi)\,(\mathbf{B}\,.\,\mathbf{N})\,\mathbf{H} - (1/8\pi)\,(\mathbf{B}\,.\,\mathbf{H})\,\mathbf{N}.$$

So long as the field is steady (i.e. electrostatic or magnetostatic) the resultant of the stresses acting on any element of volume of the aether is zero, so that the element is in equilibrium. But when the field is variable, this is no longer the case. The resultant stress on the aether contained within a surface S is

$$\iint \mathbf{P_N}\,.\,dS$$

integrated over the surface: transforming this into a volume-integral, the term $(\mathbf{D}\,.\,\mathbf{N})\,\mathbf{E}$ gives a term $\operatorname{div}\mathbf{D}\,.\,\mathbf{E} + (\mathbf{D}\,.\,\nabla)\,\mathbf{E}$, where ∇ denotes the vector operator $(\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$; and the first of these terms vanishes, since $\mathbf{D}$ is a circuital vector; the term $-\tfrac{1}{2}\,(\mathbf{D}\,.\,\mathbf{E})\,\mathbf{N}$ gives in the volume-integral a term $\tfrac{1}{2}\operatorname{grad}(\mathbf{D}\,.\,\mathbf{E})$; and the magnetic terms give similar results. So the resultant force on unit-volume of the aether is

$$(\mathbf{D}\,.\,\nabla)\,\mathbf{E} + \tfrac{1}{2}\operatorname{grad}(\mathbf{D}\,.\,\mathbf{E}) + (1/4\pi)\,(\mathbf{B}\,.\,\nabla)\,\mathbf{E} + (1/8\pi)\operatorname{grad}(\mathbf{B}\,.\,\mathbf{H}),$$

which may be written

$$[\operatorname{curl}\mathbf{E}\,.\,\mathbf{D}] + (1/4\pi)\,[\operatorname{curl}\mathbf{H}\,.\,\mathbf{B}];$$

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[*] The hypothesis that the aether is a storehouse of mechanical momentum, which was first advanced by J. J. Thomson (*Recent Researches in Elect. and Mag.* (1893), p. 13), was afterwards developed by H. Poincaré, Archives Neérl. (2) v (1900), p. 252, and by M. Abraham, Gött, Nach., 1902, p. 20.

[†] Cf. p. 302.

or, by virtue of the fundamental equations for dielectrics,

$$[-\dot{\mathbf{B}}\,.\,\mathbf{D}] + [\dot{\mathbf{D}}\,.\,\mathbf{B}], \quad \text{or} \quad (\partial/\partial t)[\mathbf{D}\,.\,\mathbf{B}].$$

This result compels us to adopt one of three alternatives: either to modify the theory so as to reduce to zero the resultant force on an element of free aether; this expedient has not met with general favour;* or to assume that the force in question sets the aether in motion: this alternative was chosen by Helmholtz,† but is inconsistent with the theory of the aether which was generally received in the closing years of the century; or lastly, with Thomson,‡ to accept the principle that the aether is itself the vehicle of mechanical momentum, of amount $[\mathbf{D}\,.\,\mathbf{B}]$ per unit volume.

Maxwell's theory was now being developed in ways which could scarcely have been anticipated by its author. But although every year added something to the superstructure, the foundations remained much as Maxwell had laid them; the doubtful argument by which he had sought to justify the introduction of displacement-currents was still all that was offered in their defence. In 1884, however, the theory was established§ on a different basis by a pupil of Helmholtz', Heinrich Hertz (*b.* 1857, *d.* 1894).

The train of Hertz' ideas resembles that by which Ampère, on hearing of Oersted's discovery of the magnetic field produced by electric currents, inferred that electric currents should exert ponderomotive forces on each other. Ampère argued that a current, being competent to originate a magnetic field, must be equivalent to a magnet in other respects; and therefore that currents, like magnets, should exhibit forces of mutual attraction and repulsion.

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* It was, however, adopted by G. T. Walker, *Aberration and the Electromagnetic Field*, Camb., 1900.

† Berlin Sitzungsberichte, 1893, p. 649; Ann. d. Phys. liii (1894), p. 135. Helmholtz supposed the aether to behave as a frictionless incompressible fluid.

‡ Loc. cit.

§ Ann. d. Phys. xxiii (1884), p. 84: English version in Hertz's *Miscellaneous Papers*, translated by D. E. Jones and G. A. Schott, p. 273.

Ampère's reasoning rests on the assumption that the magnetic field produced by a current is in all respects of the same nature as that produced by a magnet; in other words, that only one kind of magnetic force exists. This principle of the "unity of magnetic force" Hertz now proposed to supplement by asserting that the electric force generated by a changing magnetic field is identical in nature with the electric force due to electrostatic charges; this second principle he called the "unity of electric force." Suppose, then, that a system of electric currents $\boldsymbol{\iota}$ exists in otherwise empty space. According to the older theory, these currents give rise to a vector-potential $\mathbf{a}_1$, equal to Pot $\boldsymbol{\iota}$;* and the magnetic force $\mathbf{H}_1$ is the curl of $\mathbf{a}_1$: while the electric force $\mathbf{E}_1$ at any point in the field, produced by the variation of the currents, is $-\dot{\mathbf{a}}_1$.

It is now assumed that the electric force so produced is indistinguishable from the electric force which would be set up by electrostatic charges, and therefore that the system of varying currents exerts ponderomotive forces on electrostatic charges; the principle of action and reaction then requires that electrostatic charges should exert ponderomotive forces on a system of varying currents, and consequently (again appealing to the principle of the unity of electric force) that two systems of varying currents should exert on each other ponderomotive forces due to the variations.

But just as Helmholtz,† by aid of the principle of conservation of energy, deduced the existence of an electromotive force of induction from the existence of the ponderomotive forces between electric currents (i.e. variable electric systems), so from the existence of ponderomotive forces between variable systems of currents (i.e. variable magnetic systems) we may infer that variations in the rate of change of a variable magnetic system give rise to induced magnetic forces in the surrounding space. The analytical formulae which determine these forces will be of the same kind as in the electric case; so that the induced magnetic force $\mathbf{H}'$ is given by an equation of the form

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* $\alpha = \text{Pot}\ \beta$ is used to denote the solution of the equation $\nabla^2\alpha + 4\pi\beta = 0$.

† Cf. p. 243.

$$\mathbf{H}' = (1/c^2)\,\dot{\mathbf{b}}_1,$$

where c denotes some constant, and $\mathbf{b}_1$, which is analogous to the vector-potential in the electric case, is a circuital vector whose curl is the electric force $\mathbf{E}_1$ of the variable magnetic system. The value of $\mathbf{b}_1$ is therefore $(1/4\pi)$ curl Pot $\mathbf{E}_1$: so we have

$$\mathbf{H}' = -\frac{1}{4\pi c^2}\frac{\partial^2}{\partial t^2}\,\text{curl Pot }\mathbf{a}_1.$$

This must be added to $\mathbf{H}_1$. Writing $\mathbf{H}_2$ for the sum, $\mathbf{H}_1 + \mathbf{H}'$, we see that $\mathbf{H}_2$ is the curl of $\mathbf{a}_2$, where

$$\mathbf{a}_2 = \mathbf{a}_1 - \frac{1}{4\pi c^2}\frac{\partial^2}{\partial t^2}\,\text{Pot }\mathbf{a}_1;$$

and the electric force $\mathbf{E}_2$ will then be $-\dot{\mathbf{a}}_2$.

This system is not, however, final; for we must now perform the process again with these improved values of the electric and magnetic forces and the vector-potential; and so we obtain for the magnetic force the value curl $\mathbf{a}_3$, and for the electric force the value $-\dot{\mathbf{a}}_3$, where

$$\begin{aligned}\mathbf{a}_3 &= \mathbf{a}_1 - \frac{1}{4\pi c^2}\frac{\partial^2}{\partial t^2}\,\text{Pot }\mathbf{a}_2 \\ &= \mathbf{a}_1 - \frac{1}{4\pi c^2}\frac{\partial^2}{\partial t^2}\,\text{Pot }\mathbf{a}_1 + \frac{1}{(4\pi c^2)^2}\frac{\partial^4}{\partial t^4}\,\text{Pot Pot }\mathbf{a}_1.\end{aligned}$$

This process must again be repeated indefinitely; so finally we obtain for the magnetic force $\mathbf{H}$ the value curl $\mathbf{a}$, and for the electric force $\mathbf{E}$ the value $-\dot{\mathbf{a}}$, where

$$\begin{aligned}\mathbf{a} = \mathbf{a}_1 &- \frac{1}{4\pi c^2}\frac{\partial^2}{\partial t^2}\,\text{Pot }\mathbf{a}_1 + \frac{1}{(4\pi c^2)^2}\frac{\partial^4}{\partial t^4}\,\text{Pot Pot }\mathbf{a}_1 \\ &- \frac{1}{(4\pi c^2)^3}\frac{\partial^6}{\partial t^6}\,\text{Pot Pot Pot }\mathbf{a}_1 + \dots\end{aligned}$$

It is evident that the quantity **a** thus defined satisfies the equation

$$\nabla^2\mathbf{a} = \nabla^2\mathbf{a}_1 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,\mathbf{a},$$

or

$$\nabla^2\mathbf{a} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,\mathbf{a} = -4\pi\mathbf{\imath}.$$

This equation may be written

$$\text{curl}\,\mathbf{H} = (1/c^2)\,\dot{\mathbf{E}} + 4\pi\mathbf{\imath},$$

while the equations $\mathbf{H} = \text{curl}\,\mathbf{a}$, $\mathbf{E} = -\dot{\mathbf{a}}$ give

$$\text{curl}\,\mathbf{E} = -\dot{\mathbf{H}}.$$

These are, however, the fundamental equations of Maxwell's theory in the form given in his memoir of 1868.*

That Hertz's deduction is ingenious and interesting will readily be admitted. That it is conclusive may scarcely be claimed: for the argument of Helmholtz regarding the induction of currents is not altogether satisfactory; and Hertz, in following his master, is on no surer ground.

In the course of a discussion† on the validity of Hertz's assumptions, which followed the publication of his paper, E. Aulinger‡ brought to light a contradiction between the principles of the unity of electric and of magnetic force and the electrodynamics of Weber. Consider an electrostatically charged hollow sphere, in the interior of which is a wire carrying a variable current. According to Weber's theory, the sphere would exert a turning couple on the wire; but according to Hertz's principles, no action would be exerted, since charging the sphere makes no difference to either the electric or the magnetic force in its interior. The experiment thus suggested would be a crucial test of the correctness of Weber's theory; it has the advantage of requiring nothing but closed currents and electrostatic charges at rest; but the quantities to be observed would be on the limits of observational accuracy.

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* Cf. p. 287.

† Lorberg, Ann. d. Phys. xxvii (1886), p. 666; xxxi (1887), p. 131. Boltzmann, *ibid.* xxix (1886), p. 598.

‡ Ann. d. Phys. xxvii (1886), p. 119.

After his attempt to justify the Maxwellian equations on theoretical grounds, Hertz turned his attention to the possibility of verifying them by direct experiment. His interest in the matter had first been aroused some years previously, when the Berlin Academy proposed as a prize subject "To establish experimentally a relation between electromagnetic actions and the polarization of dielectrics." Helmholtz suggested to Hertz that he should attempt the solution; but at the time he saw no way of bringing phenomena of this kind within the limits of observation. From this time forward, however, the idea of electric oscillations was continually present to his mind; and in the spring of 1886 he noticed an effect* which formed the starting-point of his later researches. When an open circuit was formed of a piece of copper wire, bent into the form of a rectangle, so that the ends of the wire were separated only by a short air-gap, and when this open circuit was connected by a wire with any point of a circuit through which the spark-discharge of an induction-coil was taking place, it was found that a spark passed in the air-gap of the open circuit. This was explained by supposing that the change of potential, which is propagated along the connecting wire from the induction-coil, reaches one end of the open circuit before it reaches the other, so that a spark passes between them; and the phenomenon therefore was regarded as indicating a finite velocity of propagation of electric potential along wires.†

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* Ann. d. Phys. xxxi (1887), p. 421. Hertz's *Electric Waves*, translated by D. E. Jones, p. 29.

† Unknown to Hertz, the transmission of electric waves along wires had been observed in 1870 by Wilhelm von Bezold, München Sitzungsberichte, i (1870), p. 113; Phil. Mag. xl (1870), p. 42. "If," he wrote at the conclusion of a series of experiments, "electrical waves be sent into a wire insulated at the end, they will be reflected at that end. The phenomena which accompany this process in alternating discharges appear to owe their origin to the interference of the advancing and reflected waves," and, "an electric discharge travels with the same rapidity in wires of equal length, without reference to the materials of which these wires are made."

The subject was investigated by O. J. Lodge and A. P. Chattock at almost the same time as Hertz's experiments were being carried out: mention was made of their researches at the meeting of the British Association in 1888.

Continuing his experiments, Hertz* found that a spark could be induced in the open or secondary circuit even when it was not in metallic connexion with the primary circuit in which the electric oscillations were generated; and he rightly interpreted the phenomenon by showing that the secondary circuit was of such dimensions as to make the free period of electric oscillations in it nearly equal to the period of the oscillations in the primary circuit; the disturbance which passed from one circuit to the other by induction would consequently be greatly intensified in the secondary circuit by resonance.

The discovery that sparks may be produced in the air-gap of a secondary circuit, provided it has the dimensions proper for resonance, was of great importance: for it supplied a method of detecting electrical effects in air at a distance from the primary disturbance; a suitable detector was in fact all that was needed in order to observe the propagation of electric waves in free space, and thereby decisively test the Maxwellian theory. To this work Hertz now addressed himself.†

The *radiator* or primary source of the disturbances studied by Hertz may be constructed of two sheets of metal in the same plane, each sheet carrying a stiff wire which projects towards the other sheet and terminates in a knob; the sheets are to be excited by connecting them to the terminals of an induction coil. The sheets may be regarded as the two coatings of a modified Leyden jar, with air as the dielectric between them; the electric field is extended throughout the air, instead of being confined to the narrow space between the coatings, as in the ordinary Leyden jar. Such a disposition ensures that the system shall lose a large part of its energy by radiation at each oscillation.

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* Loc. cit.

† Sir Oliver Lodge was about this time independently studying electric oscillations in air in connexion with the theory of lightning-conductors: cf. Lodge, Phil. Mag. xxvi (1888), p. 217. So long before as 1842, Joseph Henry, of Washington, had noticed that the inductive effects of the Leyden jar discharge could be observed at considerable distances, and had even suggested a comparison with "a spark from flint and steel in the case of light."

As in the jar discharge,* the electricity surges from one sheet to the other, with a period proportional to $(CL)^{\frac{1}{2}}$, where C denotes the electrostatic capacity of the system formed by the two sheets, and L denotes the self-induction of the connexion. The capacity and induction should be made as small as possible in order to make the period small. The detector used by Hertz was that already described, namely, a wire bent into an incompletely closed curve, and of such dimensions that its free period of oscillation was the same as that of the primary oscillation, so that resonance might take place.

Towards the end of the year 1887, when studying the sparks induced in the resonating circuit by the primary disturbance, Hertz noticed† that the phenomena were distinctly modified when a large mass of an insulating substance was brought into the neighbourhood of the apparatus; thus confirming the principle that the changing electric polarization which is produced when an alternating electric force acts on a dielectric is capable of displaying electromagnetic effects.

Early in the following year (1888) Hertz determined to verify Maxwell's theory directly by showing that electromagnetic actions are propagated in air with a finite velocity.‡ For this purpose he transmitted the disturbance from the primary oscillator by two different paths, viz., through the air and along a wire; and having exposed the detector to the joint influence of the two partial disturbances, he observed interference between them. In this way he found the ratio of the velocity of electric waves in air to their velocity when conducted by wires; and the latter velocity he determined by observing the distance between the nodes of stationary waves in the wire, and calculating the period of the primary oscillation. The velocity of propagation of electric disturbances in air was in this way shown to be finite and of the same order as the velocity of light.*

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* Cf. p. 253.

† Ann. d. Phys. xxxiv, p. 373. *Electric Waves* (English edition), p. 95.

‡ Ann. d. Phys. xxxiv (1888), p. 551. *Electric Waves* (English edition) p. 107.

Later in 1888 Hertz† showed that electric waves in air are reflected at the surface of a wall; stationary waves may thus be produced, and interference may be obtained between direct and reflected beams travelling in the same direction.

The theoretical analysis of the disturbance emitted by a Hertzian radiator according to Maxwell's theory was given by Hertz in the following year.‡

The effects of the radiator are chiefly determined by the free electric charges which, alternately appearing at the two sides, generate an electric field by their presence and a magnetic field by their motion. In each oscillation, as the charges on the poles of the radiator increase from zero, lines of electric force, having their ends on these poles, move outwards into the surrounding space. When the charges on the poles attain their greatest values, the lines cease to issue outwards, and the existing lines begin to retreat inwards towards the poles; but the outer lines of force contract in such a way that their upper and lower parts touch each other at some distance from the radiator, and the remoter portion of each of these lines thus takes the form of a loop; and when the rest of the line of force retreats inwards towards the radiator, this loop becomes detached and is propagated outwards as radiation. In this way the radiator emits a series of whirl-rings, which as they move grow thinner and wider; at a distance, the disturbance is approximately a plane wave, the opposite sides of the ring representing the two phases of the wave. When one of these rings has become detached from the radiator, the energy contained may subsequently be regarded as travelling outwards with it.

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* Hertz's experiments gave the value 45/28 for the ratio of the velocity of electric waves in air to the velocity of electric waves conducted by the wires, and 2×10^{10} cms. per sec. for the latter velocity. These numbers were afterwards found to be open to objection: Poincaré (Comptes Rendus, cxi (1890), p. 322) showed that the period calculated by Hertz was $\sqrt{2} \times$ the true period, which would make the velocity of propagation in air equal to that of light $\times \sqrt{2}$. Ernst Lecher (Wiener Berichte, May 8, 1890; Phil. Mag. xxx (1890), p. 128), experimenting on the velocity of propagation of electric vibrations in wires, found instead of Hertz's 2×10^{10} cms. per sec., a value within two per cent. of the velocity of light. E. Sarasin and L. De La Rive at Geneva (Archives des Sc. Phys. xxix (1893)) finally proved that the velocities of propagation in air and along wires are equal.

† Ann. d. Phys. xxxiv (1888), p. 610. *Electric Waves* (English edition), p. 124.

‡ *Ibid.*, xxxvi (1889), p. 1. *Electric Waves* (English edition), p. 137.

To discuss the problem analytically* we take the axis of the radiator as axis of z, and the centre of the spark-gap as origin. The field may be regarded as due to an electric doublet formed of a positive and an equal negative charge, displaced from each other along the axis of the vibrator, and of moment

$$Ae^{-p_1 t} \sin (2\pi ct/\lambda),$$

the factor $e^{-p_1 t}$ being inserted to represent the damping.

The simplest method of proceeding, which was suggested by Fitz Gerald,† is to form the retarded potentials ϕ and $\mathbf{a}$ of L. Lorenz.‡ These are determined in terms of the charges and their velocities by the equations

$$\phi = \Sigma \frac{(e)_{t-r/c}}{r}, \qquad a_z = \Sigma \frac{(e\dot{z})_{t-r/c}}{r},$$

whence it is readily shown that in the present case

$$\phi = -\partial F/\partial z, \qquad \mathbf{a} = (0,\ 0,\ \partial F/\partial t),$$

where

$$F = \frac{Ae^{-p_1(t-r/c)}}{r} \sin \frac{2\pi}{\lambda} (ct - r).$$

The electric and magnetic forces are then determined by the equations

$$\mathbf{E} = c^2 \operatorname{grad} \phi - \dot{\mathbf{a}}, \qquad \mathbf{H} = \operatorname{curl} \mathbf{a}.$$

It is found that the electric force may be regarded as compounded of a force ϕ_2, parallel to the axis of the vibrator and depending at any instant only on the distance from the vibrator, together with a force $\phi_1 \sin \theta$ acting in the meridian plane perpendicular to the radius from the centre, where ϕ_1 depends at any instant only on the distance from the vibrator, and θ denotes the angle which the radius makes with the axis of the oscillator. At points on the axis, and in the equatorial plane, the electric force is parallel to the axis. At a great distance from the oscillator, ϕ_2 is small compared with ϕ_1, so the wave is purely transverse. The magnetic force is directed along circles whose centres are on the axis of the radiator; and its magnitude may be represented in the form $\phi_3 \sin \theta$, where ϕ_3 depends only on r and t; at great distances from the radiator, $c\phi_3$ is approximately equal to ϕ_1.

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* Cf. Karl Pearson and A. Lee, Phil. Trans. cxciii (1899), p. 165.

† Brit. Assoc. Rep., Leeds (1890), p. 755.

‡ Cf. p. 298. The use of retarded potentials was also recommended in the following year by Poincaré, Comptes Rendus, cxiii (1891), p. 515.

If the activity of the oscillator be supposed to be continually maintained, so that there is no damping, we may replace p_1 by zero, and may proceed as in the case of the magnetic oscillator* to determine the amount of energy radiated. The mean outward flow of energy per unit time is found to be $\frac{1}{3}c^3A^2\,(2\pi/\lambda)^4$; from which it is seen that the rate of loss of energy by radiation increases greatly as the wave-length decreases.

The action of an electrical vibrator may be studied by the aid of mechanical models. In one of these, devised by Larmor,† the aether is represented by an incompressible elastic solid, in which are two cavities, corresponding to the conductors of the vibrator, filled with incompressible fluid of negligible inertia. The electric force is represented by the displacement of the solid. For such rapid alternations as are here considered, the metallic poles behave as perfect conductors; and the tangential components of electric force at their surfaces are zero. This condition may be satisfied in the model by supposing the lining of each cavity to be of flexible sheet-metal, so as to be incapable of tangential displacement; the normal displacement of the lining then corresponds to the surface-density of electric charge on the conductor.

In order to obtain oscillations in the solid resembling those of an electric vibrator, we may suppose that the two cavities

* Cf. p. 346.

† Proc. Camb. Phil. Soc. vii (1891), p. 165.

have the form of semicircular tubes forming the two halves of a complete circle. Each tube is enlarged at each of its ends, so as to present a front of considerable area to the corresponding front at the end of the other tube. Thus at each end of one diameter of the circle there is a pair of opposing fronts, which are separated from each other by a thin sheet of the elastic solid.

The disturbance may be originated by forcing an excess of liquid into one of the enlarged ends of one of the cavities. This involves displacing the thin sheet of elastic solid, which separates it from the opposing front of the other cavity, and thus causing a corresponding deficiency of liquid in the enlarged end behind this front. The liquid will then surge backwards and forwards in each cavity between its enlarged ends; and, the motion being communicated to the elastic solid, vibrations will be generated resembling those which are produced in the aether by a Hertzian oscillator.

In the latter part of the year 1888 the researches of Hertz* yielded more complete evidence of the similarity of electric waves to light. It was shown that the part of the radiation from an oscillator which was transmitted through an opening in a screen was propagated in a straight line, with diffraction effects. Of the other properties of light, polarization existed in the original radiation, as was evident from the manner in which it was produced; and polarization in other directions was obtained by passing the waves through a grating of parallel metallic wires; the component of the electric force parallel to the wires was absorbed, so that in the transmitted beam the electric vibration was at right angles to the wires. This effect obviously resembled the polarization of ordinary light by a plate of tourmaline. Refraction was obtained by passing the radiation through prisms of hard pitch.†

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* Ann. d. Phys. xxxvi (1889), p. 769; *Electric Waves* (English ed.), p. 172.

† O. J. Lodge and J. L. Howard in the same year showed that electric radiation might be refracted and concentrated by means of large lenses. Cf. Phil. Mag. xxvii (1889), p. 48.

The old question as to whether the light-vector is in, or at right angles to, the plane of polarization* now presented itself in a new aspect. The wave-front of an electric wave contains two vectors, the electric and magnetic, which are at right angles to each other. Which of these is in the plane of polarization? The answer was furnished by Fitz Gerald and Trouton,† who found on reflecting Hertzian waves from a wall of masonry that no reflexion was obtained at the polarizing angle when the vibrator was in the plane of reflexion. The inference from this is that the magnetic vector is in the plane of polarization of the electric wave, and the electric vector is at right angles to the plane of polarization. An interesting development followed in 1890, when O. Wiener‡ succeeded in photographing stationary waves of light. The stationary waves were obtained by the composition of a beam incident on a mirror with the reflected beam, and were photographed on a thin film of transparent collodion, placed close to the mirror and slightly inclined to it. If the beam used in such an experiment is plane-polarized, and is incident at an angle of 45°, the stationary vector is evidently that perpendicular to the plane of incidence; but Wiener found that under these conditions the effect was obtained only when the light was polarized in the plane of incidence; so that the chemical activity must be associated with the vector perpendicular to the plane of polarization—i.e., the electric vector.

In 1890 and the years immediately following appeared several memoirs relating to the fundamental equations of electro-magnetic theory. Hertz, after presenting§ the general content of Maxwell's theory for bodies at rest, proceeded* to extend the equations to the case in which material bodies are in motion in the field.

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* Cf. pp. 168 et sqq.

† Nature, xxxix (1889), p. 391.

‡ Ann. d. Phys. xl (1890), p. 203. Cf. a controversy regarding the results; Comptes Rendus, cxii (1891), pp. 186, 325, 329, 365, 383, 456; and Ann. d. Phys. xli (1890), p. 154; xliii (1891), p. 177; xlviii (1893), p. 119.

§ Gott. Nach. 1890, p. 106; Ann. d. Phys. xl (1890), p. 577; *Electric Waves* (English ed.), p. 195. In this memoir Hertz advocated the form of the equations which Maxwell had used in his paper of 1868 (cf. *supra*, p. 287) in preference to the earlier form, which involved the scalar and vector potentials.

In a really comprehensive and correct theory, as Hertz remarked, a distinction should be drawn between the quantities which specify the state of the aether at every point, and those which specify the state of the ponderable matter entangled with it. This anticipation has been fulfilled by later investigators; but Hertz considered that the time was not ripe for such a complete theory, and preferred, like Maxwell, to assume that the state of the compound system—matter plus aether—can be specified in the same way when the matter moves as when it is at rest; or, as Hertz himself expressed it, that "the aether contained within ponderable bodies moves with them."

Maxwell's own hypothesis with regard to moving systems† amounted merely to a modification in the equation

$$\dot{\mathbf{B}} = -\operatorname{curl}\mathbf{E},$$

which represents the law that the electromotive force in a closed circuit is measured by the rate of decrease in the number of lines of magnetic induction which pass through the circuit. This law is true whether the circuit is at rest or in motion; but in the latter case, the **E** in the equation must be taken to be the electromotive force in a stationary circuit whose position momentarily coincides with that of the moving circuit; and since an electromotive force $[\mathbf{w} \,.\, \mathbf{B}]$ is generated in matter by its motion with velocity **w** in a magnetic field **B**, we see that **E** is connected with the electromotive force **E′** in the moving ponderable body by the equation

$$\mathbf{E}' = \mathbf{E} + [\mathbf{w} \,.\, \mathbf{B}],$$

so that the equation of electromagnetic induction in the moving body is

$$\dot{\mathbf{B}} = -\operatorname{curl}\mathbf{E}' + \operatorname{curl}[\mathbf{w} \,.\, \mathbf{B}].$$

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* Ann. d. Phys. xli (1890), p. 369; *Electric Waves* (English ed.), p. 241.

The propagation of light through a moving dielectric had been discussed previously, on the basis of Maxwell's equations for moving bodies, by J. J. Thomson, Phil. Mag. ix (1880), p. 284; Proc. Camb. Phil. Soc. v (1885), p. 250.

† Cf. p. 288.

Maxwell made no change in the other electromagnetic equations, which therefore retained the customary forms

$$\mathbf{D} = \varepsilon \mathbf{E}'/4\pi c^2, \quad \text{div}\, \mathbf{D} = 0, \quad 4\pi(\iota + \dot{\mathbf{D}}) = \text{curl}\, \mathbf{H},$$

Hertz, however, impressed by the duality of electric and magnetic phenomena, modified the last of these equations by assuming that a magnetic force $4\pi[\mathbf{D} \cdot \mathbf{w}]$ is generated in a dielectric which moves with velocity $\mathbf{w}$ in an electric field; such a force would be the magnetic analogue of the electromotive force of induction. A term involving curl $[\mathbf{D} \cdot \mathbf{w}]$ is then introduced into the last equation.

The theory of Hertz resembles in many respects that of Heaviside,* who likewise insisted much on the duplex nature of the electromagnetic field, and was in consequence disposed to accept the term involving curl $[\mathbf{D} \cdot \mathbf{w}]$ in the equations of moving media. Heaviside recognized more clearly than his predecessors the distinction between the force $\mathbf{E}'$, which determines the flux $\mathbf{D}$, and the force $\mathbf{E}$, whose curl represents the electric current; and, in conformity with his principle of duality, he made a similar distinction between the magnetic force $\mathbf{H}'$, which determines the flux $\mathbf{B}$, and the force $\mathbf{H}$, whose curl represents the "magnetic current." This distinction, as Heaviside showed, is of importance when the system is acted on by "impressed forces," such as voltaic electromotive forces, or permanent magnetization; these latter must be included in $\mathbf{E}'$ and $\mathbf{H}'$, since they help to give rise to the fluxes $\mathbf{D}$ and $\mathbf{B}$; but they must not be included in $\mathbf{E}$ and $\mathbf{H}$, since their curls are not electric or magnetic currents; so that in general we have

$$\mathbf{E}' = \mathbf{E} + \mathbf{e}, \quad \mathbf{H}' = \quad + \mathbf{h},$$

where $\mathbf{e}$ and $\mathbf{h}$ denote the impressed forces.

Developing the theory by the aid of these conceptions, Heaviside was led to make a further modification. An *im-*

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* Heaviside's general theory was published in a series of papers in the Electrician, from 1885 onwards. His earlier work was republished in his *Electrical Papers* (2 vols., 1892), and his *Electromagnetic Theory* (2 vols., 1894). Mention may be specially made of a memoir in Phil. Trans. clxxxiii (1892), p. 423.

pressed force is best defined in terms of the energy which it communicates to the system; thus, if $\mathbf{e}$ be an impressed electric force, the energy communicated to unit volume of the electromagnetic system in unit time is $\mathbf{e} \times$ the electric current. In order that this equation may be true, it is necessary to regard the electric current in a moving medium as composed of the conduction-current, displacement-current, convection-current, and also of the term curl $[\mathbf{D} \,.\, \mathbf{w}]$, whose presence in the equation we have already noticed. This may be called the *current of dielectric convection*. Thus the total current is

$$\mathbf{S} = \dot{\mathbf{D}} + \mathbf{\imath} + \rho \mathbf{w} + \text{curl}\,[\mathbf{D} \,.\, \mathbf{w}],$$

where $\rho\mathbf{w}$ denotes the conduction-current; and the equation connecting current with magnetic force is

$$\text{curl}\,(\mathbf{H}' - \mathbf{h}_0) = 4\pi \mathbf{S},$$

where $\mathbf{h}_0$ denotes the impressed magnetic forces other than that induced by motion of the medium.

We must now consider the advances which were effected during the period following the publication of Maxwell's *Treatise* in some of the special problems of electricity and optics.

We have seen* that Maxwell accounted for the rotation of the plane of polarization of light in a medium subjected to a magnetic field $\mathbf{K}$ by adding to the kinetic energy of the aether, which is represented by $\frac{1}{2}\rho\dot{\mathbf{e}}^2$, a term $\frac{1}{2}\sigma(\dot{\mathbf{e}} \,.\, \text{curl}\, \partial\mathbf{e}/\partial\theta)$, where σ is a magneto-optic constant characteristic of the substance through which the light is transmitted, and $\partial/\partial\theta$ stands for $K_x\partial/\partial x + K_y\partial/\partial y + K_z\partial/\partial z$. This theory was developed further in 1879 by Fitz Gerald,† who brought it into closer connexion with the electromagnetic theory of light by identifying the curl of the displacement $\mathbf{e}$ of the aethereal particles with the electric displacement; the derivate of $\mathbf{e}$ with respect to the time then corresponds to the magnetic force. Being thus in possession of a definitely electromagnetic theory of the magnetic rotation of light, Fitz Gerald proceeded to extend it so as to take account of a closely related phenomenon. In 1876 J. Kerr* had shown experimentally that when plane-polarized light is regularly reflected from either pole of an iron electromagnet, the reflected ray has a component polarized in a plane at right angles to the ordinary reflected ray. Shortly after this discovery had been made known, Fitz Gerald† had proposed to explain it by means of the same term in the equations which accounts for the magnetic rotation of light in transparent bodies. His argument was that if the incident plane-polarized ray be resolved into two rays circularly polarized in opposite senses, the refractive index will have different values for these two rays, and hence the intensities after reflexion will be different; so that on recompounding them, two plane-polarized rays will be obtained—one polarized in the plane of incidence, and the other polarized at right angles to it.

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* Cf. p. 308.

† Phil. Trans., 1879, p. 691. Fitz Gerald's *Scient. Writings*, p. 45.

The analytical discussion of Kerr's phenomenon, which was given by Fitz Gerald in his memoir of 1879, was based on these ideas; the most essential features of the phenomenon were explained, but the investigation was in some respects imperfect.‡

A new and fruitful conception was introduced in 1879–1880, when H. A. Rowland§ suggested a connexion between the magnetic rotation of light and the phenomenon which had been discovered by his pupil Hall.|| Hall's effect may be regarded

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* Phil. Mag. (5) iii (1877), p. 321.

† Proc. R. S. xxv (1877), p. 447; Fitz Gerald's *Scient. Writings*, p. 9.

‡ Cf. Larmor's remarks in his *Report on the Action of Magnetism on Light*, Brit. Assoc. Rep., 1893; and his editorial comments in Fitz Gerald's *Scientific Writings*. Larmor traced to its source an inconsistency in the equations by which Fitz Gerald had represented the boundary-conditions at an interface between the media. Fitz Gerald had indeed made the mistake, similar to that which was so often made by the earlier writers on the elastic-solid theory of light, of forgetting that when a medium is assumed to be incompressible, the condition of incompressibility must be introduced into the variational equation of motion (as was done *supra*, p. 172). Larmor showed that when this correction was made, new terms (resembling the terms in p, *supra*, p. 172) made their appearance; and the inconsistency in the equations was thus removed.

§ Amer. Jour. Math. ii, p. 354, iii, p. 89; Phil. Mag. xi (1881), p. 254.

|| Cf. p. 321.

as a rotation of conduction-currents under the influence of a magnetic field; and if it be assumed that displacement-currents in dielectrics are rotated in the same way, the Faraday effect may evidently be explained. Considering the matter from the analytical point of view, the Hall effect may be represented by the addition of a term $k\,[\mathbf{K}\,.\,\mathbf{S}]$ to the electromotive force, where $\mathbf{K}$ denotes the impressed magnetic force, and $\mathbf{S}$ denotes the current: so Rowland assumed that in dielectrics there is an additional term in the electric force, proportional to $[\mathbf{K}\,.\,\dot{\mathbf{D}}]$, i.e. proportional to the rate of increase of $[\mathbf{K}\,.\,\mathbf{D}]$. Now it is universally true that the total electric force round a circuit is proportional to the rate of decrease of the total magnetic induction through the circuit: so the total magnetic induction through the circuit must contain a term proportional to the integral of $[\mathbf{K}\,.\,\mathbf{D}]$ taken round the circuit: and therefore the magnetic induction at any point must contain a term proportional to curl $[\mathbf{K}\,.\,\mathbf{D}]$. We may therefore write

$$\mathbf{B} = \mathbf{H} + \sigma \operatorname{curl} [\mathbf{K}\,.\,\mathbf{D}],$$

where σ denotes a constant. But if this be combined with the customary electromagnetic equations

$$\operatorname{curl} \mathbf{H} = 4\pi\dot{\mathbf{D}}, \qquad \operatorname{curl} \mathbf{E} = -\dot{\mathbf{B}}, \qquad \mathbf{D} = \epsilon\mathbf{E}/4\pi c^2,$$

and all the vectors except $\mathbf{B}$ be eliminated ($\mathbf{K}$ being treated as a constant), we obtain the equation

$$\ddot{\mathbf{B}} = (c^2/\varepsilon)\, \nabla^2\mathbf{B} + (\sigma/4\pi) \operatorname{curl} (\partial^2\mathbf{B}/\partial t\,\partial\theta),$$

where $\partial/\partial\theta$ stands for $(K_x\partial/\partial x + K_y\partial/\partial y + K_z\partial/\partial z)$; and this is identical with the equation which Maxwell had given* for the motion of the aether in magnetized media. It follows that the assumptions of Maxwell and of Rowland, different though they are physically, lead to the same analytical equations—at any rate so far as concerns propagation through a homogeneous medium.

The connexions of Hall's phenomenon with the magnetic rotation of light, and with the reflexion of light from magnetized metals, were extensively studied[*] in the years following the publication of Rowland's memoir: but it was not until the modern theory of electrons had been developed that a satisfactory representation of the molecular processes involved in magneto-optic phenomena was attained.

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* Cf. p. 308.

The allied phenomenon of rotary polarization in naturally active bodies was investigated in 1892 by Goldhammer.[†] It

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[*] The theory of Basset (Phil. Trans. clxxxii (1891), p. 371) was, like Rowland's, based on the idea of extending Hall's phenomenon to dielectric media. An objection to this theory was that the tangential component of the electromotive force was not continuous across the interface between a magnetized and an unmagnetized medium; but Basset subsequently overcame this difficulty (Nature, lii (1895), p. 618; liii (1895), p. 130; Amer. Jour. Math. xix (1897), p. 60)—the effect analogous to Hall's being introduced into the equation connecting electric displacement with electric force, so that the equation took the form

$$\mathbf{E} = (4\pi c^2/\epsilon)\,\mathbf{D} + \sigma\,[\mathbf{K}\,.\,\dot{\mathbf{D}}].$$

Basset, in 1893 (Proc. Camb. Phil. Soc. viii, p. 68), derived analytical expressions which represent Kerr's magneto-optic phenomenon by substituting a complex quantity for the refractive index in the formulae applicable to transparent magnetized substances.

The magnetic rotation of light and Kerr's phenomenon have been investigated also by R. T. Glazebrook, Phil. Mag. xi (1881), p. 397; by J. J. Thomson, *Recent Researches*, p. 482: by D. A. Goldhammer, Ann. d. Phys. xlvi (1892), p. 71; xlvii (1892), p. 345; xlviii (1893), p. 740; l (1893), p. 772: by P. Drude, Ann. d. Phys. xlvi (1892), p. 353; xlviii (1893), p. 122; xlix (1893), p. 690; lii (1894)) p. 496: by C. H. Wind, Verslagen Kon. Akad. Amsterdam, 29th Sept., 1894: by Reiff, Ann. d. Phys. lvii (1896), p. 281: by J. G. Leathem, Phil. Trans. cxc (1897), p. 89; Trans. Camb. Phil. Soc. xvii (1898), p. 16: and by W. Voigt in many memoirs, and in his treatise, *Magneto- und Elektro-optik.* Larmor's report presented to the British Association in 1893 has been already mentioned.

In most of the later theories the equations of propagation of light in magnetized metals are derived from the two fundamental electromagnetic equations

$$\text{curl}\ \mathbf{H} = 4\pi\mathbf{S}, \quad -\ \text{curl}\ \mathbf{E} = \dot{\mathbf{H}};$$

the total current $\mathbf{S}$ being assumed to consist of a part (the displacement-current) proportional to $\dot{\mathbf{E}}$, a part (the conduction-current) proportional to $\mathbf{E}$, and a part proportional to the vector-product of $\mathbf{E}$ and the magnetization.

Various mechanical models of media in which magneto-optic phenomena take place have been devised at different times. W. Thomson (Proc. Lond. Math. Soc. vi (1875)) investigated the propagation of waves of displacement along a stretched chain whose links contain rotating fly-wheels: cf. also Larmor, Proc. Lond. Math. Soc. xxi. (1890), p. 423; xxiii (1891), p. 127; F. Hasenöhrl, Wien Sitzungsberichte cvii, 2a (1898), p. 1015; W. Thomson (Kelvin), Phil. Mag. xlviii (1899), p. 236, and *Baltimore Lectures*; and Fitz Gerald, Electrician, Aug. 4, 1899, Fitz Gerald's *Scientific Writings*, p. 481.

[†] Journal de Physique (3) i, pp. 205, 345.

will be remembered* that in the elastic-solid theory of Boussinesq, the rotation of the plane of polarization of saccharine solutions had been represented by substituting the equation

$$\mathbf{e}' \equiv \mathbf{A}\mathbf{e} + \mathbf{B} \operatorname{curl} \mathbf{e}$$

in place of the usual equation

$$\mathbf{e}' = \mathbf{A}\mathbf{e}.$$

Goldhammer now proposed to represent rotatory power in the electromagnetic theory by substituting the equation

$$\mathbf{E} = (4\pi c^2/\varepsilon)\, \mathbf{D} + k \operatorname{curl} \mathbf{D},$$

in place of the customary equation

$$\mathbf{E} = (4\pi c^2/\varepsilon)\, \mathbf{D}:$$

the constant k being a measure of the natural rotatory power of the substance concerned. The remaining equations are as usual,

$$\operatorname{curl} \mathbf{H} = 4\pi \dot{\mathbf{D}}, \quad - \operatorname{curl} \mathbf{E} = \dot{\mathbf{H}}.$$

Eliminating **H** and **E**, we have

$$\ddot{\mathbf{D}} = (c^2/\varepsilon)\, \nabla^2 \mathbf{D} + (k/4\pi)\, \nabla^2 \operatorname{curl} \mathbf{D}.$$

For a plane wave which is propagated parallel to the axis of x, this equation reduces to

$$\begin{cases} \dfrac{\partial^2 D_y}{\partial t^2} = \dfrac{c^2}{\varepsilon} \dfrac{\partial^2 D_y}{\partial x^2} - \dfrac{k}{4\pi} \dfrac{\partial^3 D_z}{\partial x^3}, \\[2ex] \dfrac{\partial^2 D_z}{\partial t^2} = \dfrac{c^2}{\varepsilon} \dfrac{\partial^2 D_z}{\partial x^2} + \dfrac{k}{4\pi} \dfrac{\partial^3 D_y}{\partial x^3}; \end{cases}$$

and, as MacCullagh had shown in 1836,† these equations are competent to represent the rotation of the plane of polarization.

In the closing years of the nineteenth century, the general theory of aether and electricity assumed a new form. But before discussing the memoirs in which the new conception was unfolded, we shall consider the progress which had been made since the middle of the century in the study of conduction in liquid and gaseous media.

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* Cf. p. 186.

† Cf. p. 175.

CHAPTER XI.

CONDUCTION IN SOLUTIONS AND GASES, FROM FARADAY TO J. J. THOMSON.

THE hypothesis which Grothuss and Davy had advanced* to explain the decomposition of electrolytes was open to serious objection in more than one respect. Since the electric force was supposed first to dissociate the molecules of the electrolyte into ions, and afterwards to set them in motion toward the electrodes, it would seem reasonable to expect that doubling the electric force would double both the dissociation of the molecules and the velocity of the ions, and would therefore quadruple the electrolysis—an inference which is not verified by observation. Moreover it might be expected, on Grothuss' theory, that some definite magnitude of electromotive force would be requisite for the dissociation, and that no electrolysis at all would take place when the electromotive force was below this value, which again is contrary to experience.

A way of escape from these difficulties was first indicated, in 1850, by Alex. Williamson,† who suggested that in compound liquids decompositions and recombinations of the molecules are continually taking place throughout the whole mass of the liquid, quite independently of the application of an external electric force. An atom of one element in the compound is thus paired now with one and now with another atom of another element, and in the intervals between these alliances the atom may be regarded as entirely free. In 1857 this idea was made by

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* Cf. p. 78.

† Phil. Mag. xxxvii (1850), p. 350; Liebig's Annalen d. Chem. u. Pharm. lxxvii (1851) p. 37.

R. Clausius,[*] of Zurich, the basis of a theory of electrolysis. According to it, the electromotive force emanating from the electrodes does not effect the dissociation of the electrolyte into ions, since a degree of dissociation sufficient for the purpose already exists in consequence of the perpetual mutability of the molecules of the electrolyte. Clausius assumed that these ions are in opposite electric conditions; the applied electric-force therefore causes a general drift of all the ions of one kind towards the anode, and of all the ions of the other kind towards the cathode. These opposite motions of the two kinds of ions constitute the galvanic current in the liquid.

The merits of the Williamson-Clausius hypothesis were not fully recognized for many years; but it became the foundation of that theory of electrolysis which was generally accepted at the end of the century.

Meanwhile another aspect of electrolysis was receiving attention. It had long been known that the passage of a current through an electrolytic solution is attended not only by the appearance of the products of decomposition at the electrodes, but also by changes of relative strength in different parts of the solution itself. Thus in the electrolysis of a solution of copper sulphate, with copper electrodes, in which copper is dissolved off the anode and deposited on the cathode, it is found that the concentration of the solution diminishes near the cathode, and increases near the anode. Some experiments on the subject were made by Faraday[†] in 1835; and in 1844 it was further investigated by Frederic Daniell and W. A. Miller,[‡] who explained it by asserting that the cation and anion have not (as had previously been supposed) the same facility of moving to their respective electrodes; but that in many cases the cation appears to move but little, while the transport is effected chiefly by the anion.

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[*] Ann. d. Phys. ci (1857), p. 338; Phil. Mag. xv (1858), p. 94.

[†] *Exper. Res.* §§ 525–530.

[‡] Phil. Trans., 1844, p. 1. Cf. also Pouillet, Comptes Rendus xx (1845), p. 1544.

This idea was adopted by W. Hittorf, of Münster, who, in the years 1853 to 1859, published* a series of memoirs on the migration of the ions. Let the velocity of the anions in the solution be to the velocity of the cations in the ratio $v : u$. Then it is easily seen that if $(u + v)$ molecules of the electrolyte are decomposed by the current, and yielded up as ions at the electrodes, v of these molecules will have been taken from the fluid on the side of the cathode, and u of them from the fluid on the side of the anode. By measuring the concentration of the liquid round the electrodes after the passage of a current, Hittorf determined the ratio v/u in a large number of cases of electrolysis.†

The theory of ionic movements was advanced a further stage by F. W. Kohlrausch‡ (*b.* 1840, *d.* 1910), of Würzburg. Kohlrausch showed that although the ohmic specific conductivity k of a solution diminishes indefinitely as the strength of the solution is reduced, yet the ratio k/m, where m denotes the number of gramme-equivalents§ of salt per unit volume, tends to a definite limit when the solution is indefinitely dilute. This limiting value may be denoted by λ. He further showed that λ may be expressed as the sum of two parts, one of which depends on the cation, but is independent of the nature of the anion; while the other depends on the anion, but not on the cation—a fact which may be explained by supposing that, in very dilute solutions, the two ions move independently under the influence of the electric force. Let u and v denote the velocities of the cation and anion respectively, when the potential difference per cm. in the solution is unity: then the total current carried through a cube of unit volume is $mE(u+v)$, where E denotes the electric charge carried by one gramme-equivalent of ion.* Thus $mE(u+v)$ = total current $= k = m\lambda$, or $\lambda = E(u+v)$. The determination of v/u by the method of Hittorf, and of $(u+v)$ by the method of Kohlrausch, made it possible to calculate the absolute velocities of drift of the ions from experimental data.

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* Ann. d. Phys. lxxxix (1853), p. 177; xcviii (1856), p. 1; ciii (1858), p. 1; cvi (1859), pp. 337, 513.

† The ratio $v/(u+v)$ was termed by Hittorf the *transport number* of the anion.

‡ Ann d. Phys. vi (1879), pp. 1, 145. The chief results had been communicated to the Academy of Göttingen in 1876 and 1877.

§ A gramme-equivalent means a mass of the salt whose weight in grammes is the molecular weight divided by the valency of the ions.

Meanwhile, important advances in voltaic theory were being effected in connexion with a different class of investigations.

Suppose that two mercury electrodes are placed in a solution of acidulated water, and that a difference of potential, insufficient to produce continuous decomposition of the water, is set up between the electrodes by an external agency. Initially a slight electric current—the polarizing current,† as it is called—is observed; but after a short time it ceases; and after its cessation the state of the system is one of electrical equilibrium. It is evident that the polarizing current must in some way have set up in the cell an electromotive force equal and opposite to the external difference of potential; and it is also evident that the seat of this electromotive force must be at the electrodes, which are now said to be *polarized*.

An abrupt fall of electric potential at an interface between two media, such as the mercury and the solution in the present case, requires that there should be a field of electric force, of considerable intensity, within a thin stratum at the interface; and this must owe its existence to the presence of electric charges. Since there is no electric field outside the thin stratum, there must be as much vitreous as resinous electricity present; but the vitreous charges must preponderate on one side of the stratum, and the resinous charges on the other side; so that the system as a whole resembles the two coatings of a condenser with the intervening dielectric. In the case of the polarized mercury cathode in acidulated water, there must be on the electrode itself a negative charge: the surface of this electrode in the polarized state may be supposed to be either mercury, or mercury covered with a layer of hydrogen. In the solution adjacent to the electrode, there must be an excess of cations and a deficiency of anions, so as to constitute the other layer of the condenser: these cations may be either mercury cations dissolved from the electrode, or the hydrogen cations of the solution.

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* i.e. E is 96580 coulombs.

† The phenomenon of voltaic polarization was discovered by Ritter in 1803. Ritter explained it by comparing the action of the polarizing current to that of a current which is used to charge a condenser. Volta in 1805 put forward the alternative explanation, that the products of decomposition set up a reverse electromotive force.

It was shown in 1870 by Cromwell Fleetwood Varley* that a mercury cathode, thus polarized in acidulated water, shows a tendency to adopt a definite superficial form, as if the surface-tension at the interface between the mercury and the solution were in some way dependent on the electric conditions. The matter was more fully investigated in 1873 by a young French physicist, then preparing for his inaugural thesis, Gabriel Lippmann.† In Lippmann's instrumental disposition, which is called a *capillary electrometer*, mercury electrodes are immersed in acidulated water: the anode H_0 has a large surface, while the cathode H has a variable surface S small in comparison. When the external electromotive force is applied, it is easily seen that the fall of potential at the large electrode is only slightly affected, while the fall of potential at the small electrode is altered by polarization by an amount practically equal to the external electromotive force. Lippmann found that the constant of capillarity of the interface at the small electrode was a function of the external electromotive force, and therefore of the difference of potential between the mercury and the electrolyte.

Let V denote the external electromotive force: we may, without loss of generality, assume the potential of H_0 to be zero, so that the potential of H is $-V$. The state of the system may be varied by altering either V or S; we assume that these alterations may be performed independently, reversibly, and isothermally, and that the state of the large electrode H_0 is not altered thereby. Let de denote the quantity of electricity which passes through the cell from H_0 to H, when the state of the system is thus varied: then if E denote the available energy of the system, and γ the surface-tension at H, we have

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* Phil. Trans. clxi (1871), p. 129.

† Comptes Rendus lxxvi (1873), p. 1407; Phil. Mag. xlvii (1874), p. 281; Ann. de Chim. et de Phys. v (1875), p. 494, xii (1877), p. 265.

$$dE = \gamma dS + Vde,$$

γ being measured by the work required to increase the surface when no electricity flows through the circuit.

In order that equilibrium may be re-established between the electrode and the solution when the fall of potential at the cathode is altered, it will be necessary not only that some hydrogen cations should come out of the solution and be deposited on the electrode, yielding up their charges, but also that there should be changes in the clustering of the charged ions of hydrogen, mercury, and sulphion in the layer of the solution immediately adjacent to the electrode. Each of these circumstances necessitates a flow of electricity in the outer circuit: in the one case to neutralize the charges of the cations deposited, and in the other case to increase the surface-density of electric charge on the electrode, which forms the opposite sheet of the quasi-condenser. Let $Sf\ (V)$ denote the total quantity of electricity which has thus flowed in the circuit when the external electromotive force has attained the value V. Then evidently

$$de = d\ \{Sf(V)\};$$

so

$$dE = \{\gamma + Vf(V)\}dS + VSf'\ (V)\ dV.$$

Since this expression must be an exact differential, we have

$$\frac{d\gamma}{dV} + f\ (V) = 0;$$

so that $-\,d\gamma/dV$ is equal to that flux of electricity per unit of new surface formed, which will maintain the surface in a constant condition (V being constant) when it is extended. Integrating the previous equation, we have

$$E = S\left\{\gamma - V\frac{d\gamma}{dV}\right\}.$$

Lippmann found that when the external electromotive force was applied, the surface-tension increased at first, until, when the external electromotive force amounted to about one volt, the surface-tension attained a maximum value, after which it diminished. He found that $d^2\gamma/dV^2$ was sensibly independent of V, so that the curve which represents the relation between γ and V is a parabola.*

The theory so far is more or less independent of assumptions as to what actually takes place at the electrode: on this latter question many conflicting views have been put forward. In 1878 Josiah Willard Gibbs,† of Yale (*b.* 1839, *d.* 1903), discussed the problem on the supposition that the polarizing current is simply an ordinary electrolytic conduction-current, which causes a liberation of hydrogen from the ionic form at the cathode. If this be so, the amount of electricity which passes through the cell in any displacement must be proportional to the quantity of hydrogen which is yielded up to the electrode in the displacement; so that $d\gamma/dV$ must be proportional to the amount of hydrogen deposited per unit area of the electrode.‡

A different view of the physical conditions at the polarized electrode was taken by Helmholtz,§ who assumed that the ions of hydrogen which are brought to the cathode by the polarizing current do not give up their charges there, but remain in the vicinity of the electrode, and form one face of a quasi-condenser of which the other face is the electrode itself.* If σ denote the surface-density of electricity on either face of this quasi-condenser, we have, therefore,

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* Lippman, Comptes Rendus, xcv (1882), p. 686.

† Trans. Conn. Acad. iii (1876–1878), pp. 108, 343; Gibbs' *Scientific Papers*, i, p. 55.

‡ This is embodied in equation (690) of Gibbs' memoir.

§ Berlin Monatsber., 1881, p. 945; *Wiss. Abh.* i, p. 925; Ann. d. Phys. xvi. (1882), p. 31. Cf. also Planck, Ann. d. Phys. xliv (1891), p. 385.

$$de = - d(S\sigma)\,; \quad \text{so} \quad \sigma = d\gamma/dV.$$

This equation shows that when $d\gamma/dV$ is zero—i.e., when the surface-tension is a maximum—σ must be zero; that is to say, there must be no difference of potential between the mercury and the electrolyte. The external electromotive force is then balanced entirely by the discontinuity of potential at the other electrode H_0; and thus a method is suggested of measuring the latter discontinuity of potential. All previous measurements of differences of potential had involved the employment of more than one interface; and it was not known how the measured difference of potential should be distributed among these interfaces; so that the suggestion of a means of measuring single differences of potential was a distinct advance, even though the hypotheses on which the method was based were somewhat insecure.

A further consequence deduced by Helmholtz from this theory leads to a second method of determining the difference of potential between mercury and an electrolyte. If a mercury surface is rapidly extending, and electricity is not rapidly transferred through the electrolyte, the electric surface-density in the double layer must rapidly decrease, since the same quantity of electricity is being distributed over an increasing area. Thus it may be inferred that a rapidly extending mercury-surface in an electrolyte is at the same potential as the electrolyte.

This conception is realized in the *dropping-electrode,* in which a jet of mercury, falling from a reservoir into an electrolytic solution, is so adjusted that it breaks into drops when the jet touches the solution. According to Helmholtz's conclusion there is no difference of potential between the drops and the electrolyte; and therefore the difference of potential between the electrolyte and a layer of mercury underlying it in the same vessel is equal to the difference of potential between this layer of mercury and the mercury in the upper reservoir, which difference is a measurable quantity.

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* The conception of double layers of electricity at the surface of separation of two bodies had been already applied by Helmholtz to explain various other phenomena—e.g., the Volta contact-difference of potential of two metals, frictional electricity, and "electric endosmose," or the transport of fluid which occurs when an electric current is passed through two conducting liquids separated by a porous barrier. Cf. Helmholtz, Berlin Monatsberichte, February 27, 1879; Ann. d. Phys. vii (1879), p. 337; Helmholtz, *Wiss. Abh.* i, p. 855.

It will be seen that according to the theories both of Gibbs and of Helmholtz, and indeed according to all other theories on the subject,* $d\gamma/dV$ is zero for an electrode whose surface is

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* E.g., that of Warburg, Ann. d. Phys. xli (1890), p. 1. In this it is assumed that the electrolytic solution near the electrodes originally contains a salt of mercury in solution. When the external electromotive force is applied, a conduction-current passes through the electrolyte, which in the body of the electrolyte is carried by the acid and hydrogen ions. Warburg supposed that at the cathode the hydrogen ions react with the salt of mercury, reducing it to metallic mercury, which is deposited on the electrode. Thus a considerable change in concentration of the salt of mercury is caused at the cathode. At the anode, the acid ions carrying the current attack the mercury of the electrode, and thus increase the local concentration of the mercuric salt; but on account of the size of the anode this increase is trivial and may be neglected.

Warburg thus supposed that the electromotive force of the polarized cell is really that of a concentration cell, depending on the different concentrations of mercuric salt at the electrodes. He found $d\gamma/dV$ to be equal to the amount of mercuric salt at the cathode per unit area of cathode, divided by the electrochemical equivalent of mercury. The equation previously obtained is thus presented in a new physical interpretation.

Warburg connected the increase of the surface-tension with the fact that the surface-tension between mercury and a solution always increases when the concentration of the solution is diminished. His theory, of course, leads to no conclusion regarding the *absolute* potential difference between the mercury and the solution, as Helmholtz' does.

At an electrode whose surface is rapidly increasing—e.g., a dropping electrode—Warburg supposed that the surface-density of mercuric salt tends to zero, so $d\gamma/dV$ is zero.

The explanation of dropping electrodes favoured by Nernst, *Beilage zu den Ann. d. Phys.* lviii (1896), is that the difference of potential corresponding to the equilibrium between the mercury and the electrolyte is instantaneously established; but that ions are withdrawn from the solution in order to form the double layer necessary for this, and that these ions are carried down with the drops rapidly increasing—e.g., a dropping electrode; that is to say, the difference of potential between an ordinary mercury electrode and the electrolyte, when the surface-tension has its maximum value, is equal to the difference of potential between a dropping-electrode and the same electrolyte. This result has been experimentally verified by various investigators, who have shown that the applied electromotive force when the surface-tension has its maximum value in the capillary electrometer, is equal to the electromotive force of a cell having as electrodes a large mercury electrode and a dropping electrode.

Another memoir which belongs to the same period of Helmholtz' career, and which has led to important developments, was concerned with a special class of voltaic cells. The most usual type of cell is that in which the positive electrode is composed of a different metal from the negative electrode, and the evolution of energy depends on the difference in the chemical affinities of these metals for the liquids in the cell. But in the class of cells now considered* by Helmholtz, the two electrodes are composed of the same metal (say, copper); and the liquid (say, solution of copper sulphate) is more concentrated in the neighbourhood of one electrode than in the neighbourhood of the other. When the cell is in operation, the salt passes from the places of high concentration to the places of low concentration, so as to equalize its distribution; and this process is accompanied by the flow of a current in the outer circuit between the electrodes. Such cells had been studied experimentally by James Moser a short time previously† to Helmholtz' investigation.

The activity of the cell is due to the fact that the available energy of a solution depends on its concentration; the molecules

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of mercury, until the upper layer of the solution is so much impoverished that the double layer can no longer be formed. The impoverishment of the upper layer of the solution has actually been observed by Palmaer, Zeitsch. Phys. Chem. xxv (1898), p. 265; xxviii (1899), p. 257; xxxvi (1901), p. 664.

* Berlin Monatsber., 1877, p. 713; Phil. Mag. (5) v (1878), p. 348; reprinted with additions in Ann. d. Phys. iii (1878), p. 201.

† Ann. d. Phys. iii (1878), p. 216.

of salt, in passing from a high to a low concentration, are therefore capable of supplying energy, just as a compressed gas is capable of supplying energy when its degree of compression is reduced. To examine the matter quantitatively, let $nf(n/V)$ denote the term in the available energy of a solution, which is due to the dissolution of n gramme-molecules of salt in a volume V of pure solvent; the function f will of course depend also on the temperature. Then when dn gramme-molecules of solvent are evaporated from the solution, the decrease in the available energy of the system is evidently equal to the available energy of dn gramme-molecules of liquid solvent, less the available energy of dn gramme-molecules of the vapour of the solvent, together with $nf(n/V)$ less $nf\{n/(V-v\,dn)\}$, where v denotes the volume of one gramme-molecule of the liquid. But this decrease in available energy must be equal to the mechanical work supplied to the external world, which is $dn\,.\,p_1(v'-v)$, if p_1 denote the vapour-pressure of the solution at the temperature in question, and v' denote the volume of one gramme-molecule of vapour. We have therefore

$$\begin{aligned} dn\,.\,p_1(v'-v) = &- \text{available energy of } dn \text{ gramme-molecules of solvent vapour} \\ &+ \text{available energy of } dn \text{ gramme-molecules of liquid solvent} \\ &+ nf(n/V) - nf\{n/(V-v\,dn)\}. \end{aligned}$$

Subtracting from this the equation obtained by making n zero, we have

$$dn\,.\,(p_1-p_0)\,(v'-v) = nf(n/V) - nf\{n/(V-v\,dn)\},$$

where p_0 denotes the vapour-pressure of the pure solvent at the temperature in question; so that

$$(p_1-p_0)\,(v'-v) = -\,(n^2/V^2)f'(n/V)v.$$

Now, it is known that when a salt is dissolved in water, the vapour-pressure is lowered in proportion to the concentration of the salt—at any rate when the concentration is small: in fact, by the law of Raoult, $(p_0 - p_1)/p_0$ is approximately equal to nv/V; so that the previous equation becomes

$$p_0 V(v' - v) = nf'(n/V).$$

Neglecting v in comparison with v', and making use of the equation of state of perfect gases (namely,

$$p_0 v' = RT,$$

where T denotes the absolute temperature, and R denotes the constant of the equation of state), we have

$$f'(n/V) = RTV/n,$$

and therefore

$$f(n/V) = RT \log (n/V).$$

Thus in the available energy of one gramme-molecule of a dissolved salt, the term which depends on the concentration is proportional to the logarithm of the concentration; and hence, if in a concentration-cell one gramme-molecule of the salt passes from a high concentration c_2 at one electrode to a low concentration c_1 at the other electrode, its available energy is thereby diminished by an amount proportional to $\log (c_2/c_1)$. The energy which thus disappears is given up by the system in the form of electrical work; and therefore the electromotive force of the concentration-cell must be proportional to $\log (c_2/c_1)$. The theory of solutions and their vapour-pressure was not at the time sufficiently developed to enable Helmholtz to determine precisely the coefficient of $\log (c_2/c_1)$ in the expression.*

An important advance in the theory of solutions was effected in 1887, by a young Swedish physicist, Svante Arrhenius.†

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* The formula given by Helmholtz was that the electromotive force of the cell is equal to $b(1 - n) v \log (c_2/c_1)$, where c_2 and c_1 denote the concentrations of the solution at the electrodes, v denotes the volume of one gramme of vapour in equilibrium with the water at the temperature in question, n denotes the transport number for the cation (Hittorf's $1/n$), and b denotes $q \times$ the lowering of vapour-pressure when one gramme-equivalent of salt is dissolved in q grammes of water, where q denotes a large number.

† Zeitschrift für phys. Chem. i (1887), p. 631. Previous investigations, in which the theory was to some extent foreshadowed, were published in Bihang till Svenska Vet. Ak. Förh. viii (1884), Nos. 13 and 14.

Interpreting the properties discovered by Kohlrausch* in the light of the ideas of Williamson and Clausius regarding the spontaneous dissociation of electrolytes, Arrhenius inferred that in very dilute solutions the electrolyte is completely dissociated into ions, but that in more concentrated solutions the salt is less completely dissociated; and that as in all solutions the transport of electricity in the solution is effected solely by the movement of ions, the equivalent conductivity† must be proportional to the fraction which expresses the degree of ionization. By aid of these conceptions it became possible to estimate the dissociation quantitatively, and to construct a general theory of electrolytes.

Contemporary physicists and chemists found it difficult at first to believe that a salt exists in dilute solution only in the form of ions, e.g. that the sodium and chlorine exist separately and independently in a solution of common salt. But there is a certain amount of chemical evidence in favour of Arrhenius' conception. For instance, the tests in chemical analysis are really tests for the ions; iron in the form of a ferrocyanide, and chlorine in the form of a chlorate, do not respond to the characteristic tests for iron and chlorine respectively, which are really the tests for the iron and chlorine ions.

The general acceptance of Arrhenius' views was hastened by the advocacy of Ostwald, who brought to light further evidence in their favour. For instance, all permanganates in dilute solution show the same purple colour; and Ostwald considered their absorption-spectra to be identical;‡ this identity is easily accounted for on Arrhenius' theory, by supposing that the spectrum in question is that of the anion which corresponds to the acid radicle. The blue colour which is observed in dilute solutions of copper salts, even when the strong solution is not blue, may in the same way be ascribed to a blue copper cation. A striking instance of the same kind is afforded by ferric sulphocyanide; here the strong solution shows a deep red colour, due to the salt itself; but on dilution the colour disappears, the ions being colourless.

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* Cf. p. 374.

† I.e. the ohmic specific conductivity of the solution divided by the number of gramme-equivalents of salt per unit volume.

‡ Examination of the spectra with higher dispersion does not altogether confirm this conclusion.

If it be granted that ions can have any kind of permanent existence in a salt solution, it may be shown from thermodynamical considerations that the degree of dissociation must increase as the dilution increases, and that at infinite dilution there must be complete dissociation. For the available energy of a dilute solution of volume V, containing n_1 gramme-molecules of one substance, n_2 gramme-molecules of another, and so on, is (as may be shown by an obvious extension of the reasoning already employed in connexion with concentration-cells)*

$$\sum_r n_r \phi_r (T) + RT \sum_r n_r \log (n_r/V) + \text{the available energy}$$

possessed by the solvent before the introduction of the solutes, where $\phi_r (T)$ depends on T and on the nature of the r^{th} solute, but not on V, and R denotes the constant which occurs in the equation of state of perfect gases. When the system is in equilibrium, the proportions of the reacting substances will be so adjusted that the available energy has a stationary value for small virtual alterations δn_1, δn_2, ... of the proportions; and therefore

$$0 = \sum_r \delta n_r \,.\, \phi_r (T) + RT \sum_r \delta n_r \,.\, \log (n_r/V) + RT \Sigma \delta n_r.$$

Applying this to the case of an electrolyte in which the disappearance of one molecule of salt (indicated by the suffix $_1$) gives rise to one cation (indicated by the suffix $_2$) and one anion (indicated by the suffix $_3$), we have $\delta n_1 = - \delta n_2 = - \delta n_3$; so the equation becomes

$$0 = \phi_1 (T) - \phi_2 (T) - \phi_3 (T) + RT \log (n_1 V/n_2 n_3) - RT,$$

or

$$n_1 V/n_2 n_3 = \text{a function of } T \text{ only.}$$

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* Cf. pp. 382–383.

Since in a neutral solution the number of anions is equal to the number of cations, this equation may be written

$$\frac{n_2^2}{Vn_1} = \text{a function of } T \text{ only},$$

it shows that when V is very large (so that the solution is very dilute), n_2 is very large compared with n_1; that is to say, the salt tends towards a state of complete dissociation.

The ideas of Arrhenius contributed to the success of Walther Nernst* in perfecting Helmholtz' theory of concentration cells, and representing their mechanism in a much more definite fashion than had been done heretofore.

In an electrolytic solution let the drift velocity of the cations under unit electric force be u, and that of the anions be v, so that the fraction $u/(u+v)$ of the current is transported by the cations, and the fraction $v/(u+v)$ by the anions. If the concentration of the solution be c_1 at one electrode and c_2 at the other, it follows from the formula previously found for the available energy that one gramme-ion of cations, in moving from one electrode to the other, is capable of yielding up an amount† $RT \log c_1/c_2$ of energy; while one gramme-ion of anions going in the opposite direction must absorb the same amount of energy. The total quantity of work furnished when one gramme-molecule of salt is transferred from concentration c_2 to concentration c_1 is therefore

$$\frac{u - v}{u + v} RT \log \frac{c_1}{c_2}.$$

The quantity of electric charge which passes in the circuit when one gramme-molecule of the salt is transferred is proportional to the valency ν of the ions, and the work furnished is proportional to the product of this charge and the electromotive force E of the cell; so that in suitable units we have

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* Zeitschr. fur phys. Chem. ii (1888), p. 613; iv (1889), p. 129; Berlin Sitzungsberichte, 1888, p. 83; Ann. d. Phys. xlv (1892), p. 360. Cf. also Max Planck, Ann. d. Phys. xxxix (1890), p. 161; xl (1890), p. 561.

† The correct law of dependence of the available energy on the temperature was

$$E = \frac{RT}{\nu} \frac{u - v}{u + v} \log \frac{c_2}{c_1}.$$

A typical concentration-cell to which this formula may be applied may be constituted in the following way:—Let a quantity of zinc amalgam, in which the concentration of zinc is c_1, be in contact with a dilute solution of zinc sulphate, and let this in turn be in contact with a quantity of zinc amalgam of concentration c_2. When the two masses of amalgam are connected by a conducting wire outside the cell, an electric current flows in the wire from the weak to the strong amalgam,* while zinc cations pass through the solution from the strong amalgam to the weak. The electromotive force of such a cell, in which the current may be supposed to be carried solely by cations, is

$$\frac{RT}{\nu} \log \frac{c_2}{c_1}$$

Not content with the derivation of the electromotive force from considerations of energy, Nernst proceeded to supply a definite mechanical conception of the process of conduction in electrolytes. The ions are impelled by the electric force associated with the gradient of potential in the electrolyte. But this is not the only force which acts on them; for, since their available energy decreases as the concentration decreases, there must be a force assisting every process by which the concentration is decreased. The matter may be illustrated by the analogy of a gas compressed in a cylinder fitted with a piston; the available energy of the gas decreases as its degree of compression decreases; and therefore that movement of the piston which tends to decrease the compression is assisted by a force—the "pressure" of the gas on the piston. Similarly, if a solution were contained within a cylinder fitted with a piston which is permeable to the pure solvent but not to the solute, and if the whole were immersed in pure solvent, the available energy of the system would be decreased if the piston were to move outwards so as to admit more solvent into the solution; and therefore this movement of the piston would be assisted by a force—the "osmotic pressure of the solution," as it is called.*

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* It will hardly be necessary to remark that this supposed direction of the current is purely conventional.

Consider, then, the case of a single electrolyte supposed to be perfectly dissociated; its state will be supposed to be the same at all points of any plane at right angles to the axis of x. Let ν denote the valency of the ions, and V the electric potential at any point. Since† the available energy of a given quantity of a substance in very dilute solution depends on the concentration in exactly the same way as the available energy of a given quantity of a perfect gas depends on its density, it follows that the osmotic pressure p for each ion is determined in terms of the concentration and temperature by the equation of state of perfect gases

$$Mp = RTc,$$

where M denotes the molecular weight of the salt, and c the mass of salt per unit volume.

Consider the cations contained in a parallelepiped at the place x, whose cross-section is of unit area and whose length is dx. The mechanical force acting on them due to the electric field is $-(\nu c/M)\, dV/dx \,.\, dx$, and the mechanical force on them due to the osmotic pressure is $-dp/dx \,.\, dx$. If u denote the velocity of drift of the cations in a field of unit electric force, the total amount of charge which would be transferred by cations across unit area in unit time under the influence of the electric forces alone would be $-(u\nu c/M)\, dV/dx$; so, under the influence of both forces, it is

$$-\frac{u\nu c}{M}\left(\frac{dV}{dx}+\frac{RT}{c\nu}\frac{dc}{dx}\right).$$

Similarly, if v denote the velocity of drift of the anions in a unit electric field, the charge transferred across unit area in unit time by the anions is

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* Cf. van't Hoff, Svenska Vet.-Ak. Handlingar xxi (1886), No. 17; Zeitschrift für Phys. Chem. i (1887), p. 481.

† As follows from the expression obtained, *supra*, p. 383.

$$\frac{v\nu c}{M}\left(-\frac{dV}{dx}+\frac{RT}{c\nu}\frac{dc}{dx}\right).$$

We have therefore, if the total current be denoted by i,

$$i=-(u+v)\frac{\nu c}{M}\frac{dV}{dx}-(u-v)\frac{RT}{M}\frac{dc}{dx},$$

or

$$-\frac{dV}{dx}dx=\frac{M\,dx}{(u+v)\,\nu c}\,i+\frac{u-v}{u+v}\,\frac{RT}{\nu c}\,\frac{dc}{dx}\,dx.$$

The first term on the right evidently represents the product of the current into the ohmic resistance of the parallelepiped dx, while the second term represents the internal electromotive force of the parallelepiped. It follows that if r denote the specific resistance, we must have

$$u+v=M/r\nu c,$$

in agreement with Kohlrausch's equation;* while by integrating the expression for the internal electromotive force of the parallelepiped dx, we obtain for the electromotive force of a cell whose activity depends on the transference of electrolyte between the concentrations c_1 and c_2, the value

$$\frac{u-v}{u+v}\,\frac{RT}{\nu}\int\frac{1}{c}\,\frac{dc}{dx}\,dx,$$

or

$$\frac{u-v}{u+v}\,\frac{RT}{\nu}\log\frac{c_2}{c_1},$$

in agreement with the result already obtained.

It may be remarked that although the current arising from a concentration cell which is kept at a constant temperature is capable of performing work, yet this work is provided, not by any diminution in the total internal energy of the cell, but by the abstraction of thermal energy from neighbouring bodies. This indeed (as may be seen by reference to W. Thomson's general equation of available energy)* must be the case with any system whose available energy is exactly proportional to the absolute temperature.

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* Cf. p. 374.

The advances which were effected in the last quarter of the nineteenth century in regard to the conduction of electricity through liquids, considerable though these advances were, may be regarded as the natural development of a theory which had long been before the world. It was otherwise with the kindred problem of the conduction of electricity through gases: for although many generations of philosophers had studied the remarkable effects which are presented by the passage of a current through a rarefied gas, it was not until recent times that a satisfactory theory of the phenomena was discovered.

Some of the electricians of the earlier part of the eighteenth century performed experiments in vacuous spaces; in particular, Hauksbee† in 1705 observed a luminosity when glass is rubbed in rarefied air. But the first investigator of the continuous discharge through a rarefied gas seems to have been Watson,‡ who, by means of an electrical machine, sent a current through an exhausted glass tube three feet long and three inches in diameter. "It was," he wrote, "a most delightful spectacle, when the room was darkened, to see the electricity in its passage: to be able to observe not, as in the open air, its brushes or pencils of rays an inch or two in length, but here the coruscations were of the whole length of the tube between the plates, that is to say, thirty-two inches." Its appearance he described as being on different occasions "of a bright silver hue," "resembling very much the most lively coruscations of the *aurora borealis*," and "forming a continued arch of lambent flame." His theoretical explanation was that the electricity "is seen, without any preternatural force, pushing itself on through the vacuum by its own elasticity, in order to maintain the equilibrium in the machine"—a conception which follows naturally from the combination of Watson's one-fluid theory with the prevalent doctrine of electrical atmospheres.*

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* Cf. p. 241.

† Phil. Trans. xxiv (1705), p. 2165. Fra. Hauksbee, *Physico-Mechanical Experiments*, London, 1709.

‡ Phil. Trans. xlv (1748), p. 93, xlvii (1752), p. 362.

A different explanation was put forward by Nollet, who performed electrical experiments in rarefied air at about the same time as Watson,† and saw in them a striking confirmation of his own hypothesis of efflux and afflux of electric matter.‡ According to Nollet, the particles of the effluent stream collide with those of the affluent stream which is moving in the opposite direction; and being thus violently shaken, are excited to the point of emitting light.

Almost a century elapsed before anything more was discovered regarding the discharge in vacuous spaces. But in 1838 Faraday,§ while passing a current from the electrical machine between two brass rods in rarefied air, noticed that the purple haze or stream of light which proceeded from the positive pole stopped short before it arrived at the negative rod. The negative rod, which was itself covered with a continuous glow, was thus separated from the purple column by a narrow dark space: to this, in honour of its discoverer, the name *Faraday's dark space* has generally been given by subsequent writers.

That vitreous and resinous electricity give rise to different types of discharge had long been known; and indeed, as we have seen,|| it was the study of these differences that led Franklin to identify the electricity of glass with the superfluity of fluid, and the electricity of amber with the deficiency of it. But phenomena of this class are in general much more complex than might be supposed from the appearance which they present at a first examination; and the value of Faraday's discovery of the negative glow and dark space lay chiefly in the simple and definite character of these features of the discharge, which indicated them as promising subjects for further research. Faraday himself felt the importance of

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* Cf. ch. ii.

† Nollet, *Recherches sur l'Electricité*, 1749, troisième discours.

‡ Cf. p. 40.

§ Phil. Trans., 1838; *Exper. Res.* i, § 1526.

|| Cf. p. 44.

investigations in this direction. "The results connected with the different conditions of positive and negative discharge," he wrote,* "will have a far greater influence on the philosophy of electrical science than we at present imagine."

Twenty more years, however, passed before another notable advance was made. That a subject so full of promise should progress so slowly may appear strange; but one reason at any rate is to be found in the incapacity of the air-pumps then in use to rarefy gases to the degree required for effective study of the negative glow. The invention of Geissler's mercurial air-pump in 1855 did much to remove this difficulty; and it was in Geissler's exhausted tubes that Julius Plücker,† of Bonn, studied the discharge three years later.

It had been shown by Sir Humphrey Davy in 1821‡ that one form of electric discharge—namely, the arc between carbon poles—is deflected when a magnet is brought near to it. Plücker now performed a similar experiment with the vacuum discharge, and observed a similar deflexion. But the most interesting of his results were obtained by examining the behaviour of the negative glow in the magnetic field; when the negative electrode was reduced to a single point, the whole of the negative light became concentrated along the line of magnetic force passing through this point. In other words, the negative glow disposed itself as if it were constituted of flexible chains of iron filings attached at one end to the cathode.

Plücker noticed that when the cathode was of platinum, small particles were torn off it and deposited on the walls of the glass bulb. "It is most natural," he wrote, "to imagine that the magnetic light is formed by the incandescence of these platinum particles as they are torn from the negative electrode." He likewise observed that during the discharge the walls of the tube, near the cathode, glowed with a phosphorescent light, and remarked that the position of this light was altered when the magnetic field was changed. This led to another discovery; for in 1869 Plücker's pupil, W. Hittorf,[*] having placed a solid body between a point-cathode and the phosphorescent light, was surprised to find that a shadow was cast. He rightly inferred from this that the negative glow is formed of rays which proceed from the cathode in straight lines, and which cause the phosphorescence when they strike the walls of the tube.

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* *Exper. Res.*, § 1523.

† Ann. d. Phys. ciii (1858), pp. 88, 151; civ (1858), pp. 113, 622; cv (1858), p. 67; cvii (1859), p. 77. Phil. Mag. xvi (1858), pp. 119, 408; xviii (1859), pp. 1, 7.

‡ Phil. Trans., 1821, p. 425.

Hittorf's observation was amplified in 1876 by Eugen Goldstein,[†] who found that distinct shadows were cast, not only when the cathode was a single point, but also when it formed an extended surface, provided the shadow-throwing object was placed close to it. This clearly showed that the *cathode rays* (a term now for the first time introduced) are not emitted indiscriminately in all directions, but that each portion of the cathode surface emits rays which are practically confined to a single direction; and Goldstein found this direction to be normal to the surface. In this respect his discovery established an important distinction between the manner in which cathode rays are emitted from an electrode and that in which light is emitted from an incandescent surface.

The question as to the nature of the cathode rays attracted much attention during the next two decades. In the year following Hittorf's investigation, Cromwell Varley[‡] put forward the hypothesis that the rays are composed of "attenuated particles of matter, projected from the negative pole by electricity"; and that it is in virtue of their negative charges that these particles are influenced by a magnetic field.[§]

During some years following this, the properties of highly rarefied gases were investigated by Sir William Crookes. Influenced, doubtless, by the ideas which were developed in connexion with his discovery of the radiometer, Crookes,* like Varley, proposed to regard the cathode rays as a molecular torrent: he supposed the molecules of the residual gas, coming into contact with the cathode, to acquire from it a resinous charge, and immediately to fly off normally to the surface, by reason of the mutual repulsion exerted by similarly electrified bodies. Carrying the exhaustion to a higher degree, Crookes was enabled to study a dark space which under such circumstances appears between the cathode and the cathode glow; and to show that at the highest rarefactions this dark space (which has since been generally known by his name) enlarges until the whole tube is occupied by it. He suggested that the thickness of the dark space may be a measure of the mean length of free path of the molecules. "The extra velocity," he wrote, "with which the molecules rebound from the excited negative pole keeps back the more slowly moving molecules which are advancing towards that pole. The conflict occurs at the boundary of the dark space, where the luminous margin bears witness to the energy of the collisions."† Thus according to Crookes the dark space is dark and the glow bright because there are collisions in the latter and not in the former. The fluorescence or phosphorescence on the walls of the tube he attributed to the impact of the particles on the glass.

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[*] Ann. d. Phys. cxxxvi (1869), pp. 1, 197; translated, Annales de Chimie, xvi (1869), p. 487.

[†] Berlin Monatsberichte, 1876, p. 279.

[‡] Proc. Roy. Soc. xix (1871), p. 236.

[§] Priestley in 1766 had shown that a current of electrified air flows from the points of bodies which are electrified either vitreously or resinously: cf. Priestley's *History of Electricity*, p. 591.

Crookes spoke of the cathode rays as an "ultra-gaseous" or "fourth state" of matter. These expressions have led some later writers to ascribe to him the enunciation or prediction of a hypothesis regarding the nature of the particles projected from the cathode, which arose some years afterwards, and which we shall presently describe; but it is clear from Crookes' memoirs that he conceived the particles of the cathode rays to be ordinary gaseous molecules, carrying electric charges; and by

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* Phil. Trans. clxx (1879), pp. 135, 641; Phil. Mag. vii (1879), p. 57.

† Phil. Mag. vii (1879), p. 57.

"a new state of matter" he understood simply a state in which the free path is so long that collisions may be disregarded.

Crookes found that two adjacent pencils of cathode rays appeared to repel each other. At the time this was regarded as a direct confirmation of the hypothesis that the rays are streams of electrically charged particles; but it was shown later that the deflexion of the rays must be assigned to causes other than mutual repulsion.

How admirably the molecular-torrent theory accounts for the deviation of the cathode rays by a magnetic field was shown by the calculations of Eduard Riecke in 1881.* If the axis of z be taken parallel to the magnetic force H, the equations of motion of a particle of mass m, charge e, and velocity (u, v, w) are

$$m\,du/dt = evH, \qquad m\,dv/dt = -euH, \qquad m\,dw/dt = 0.$$

The last equation shows that the component of velocity of the particle parallel to the magnetic force is constant; the other equations give

$$u = A \sin (eHt/m), \qquad v = A \cos (eHt/m),$$

showing that the projection of the path on a plane at right angles to the magnetic force is a circle. Thus, in a magnetic field the particles of the molecular torrent describe spiral paths whose axes are the lines of magnetic force.

But the hypothesis of Varley and Crookes was before long involved in difficulties. Tait† in 1880 remarked that if the particles are moving with great velocities, the periods of the luminous vibrations received from them should be affected to a measurable extent in accordance with Doppler's principle. Tait tried to obtain this effect, but without success. It may, however, be argued that if, as Crookes supposed, the particles become luminous only when they have collided with other particles, and have thereby lost part of their velocity, the phenomenon in question is not to be expected.

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* Gött. Nach., 2 February, 1881; reprinted, Ann. d. Phys. xiii (1881), p. 191.

† Proc. Roy. Soc. Edinb. x (1880), p. 430.

The alternative to the molecular-torrent theory is to suppose that the cathode radiation is a disturbance of the aether. This view was maintained by several physicists,[*] and notably by Hertz,[†] who rejected Varley's hypothesis when he found experimentally that the rays did not appear to produce any external electric or magnetic force, and were apparently not affected by an electrostatic field. It was, however, pointed out by Fitz Gerald[‡] that external space is probably screened from the effects of the rays by other electric actions which take place in the discharge tube.

It was further urged against the charged-particle theory that cathode rays are capable of passing through films of metal which are so thick as to be quite opaque to ordinary light;[§] it seemed inconceivable that particles of matter should not be stopped by even the thinnest gold-leaf. At the time of Hertz's experiments on the subject, an attempt to obviate this difficulty was made by J.-J. Thomson,[||] who suggested that the metallic film when bombarded by the rays might itself acquire the property of emitting charged particles, so that the rays which were observed on the further side need not have passed through the film. It was Thomson who ultimately found the true explanation; but this depended in part on another order of ideas, whose introduction and development must now be traced.

The tendency, which was now general, to abandon the electron-theory of Weber in favour of Maxwell's theory involved certain changes in the conceptions of electric charge.

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[*] E.g. E. Wiedemann, Ann. d. Phys. x (1880), p. 202; translated, Phil. Mag. x (1880), p. 357. E. Goldstein, Ann. d. Phys. xii (1881), p. 249.

[†] Ann. d. Phys. xix (1883), p. 782.

[‡] Nature, November 5, 1896; Fitz Gerald's *Scientific Writings*, p. 433.

[§] The penetrating power of the rays had been noticed by Hittorf, and by E. Wiedemann and Ebert, Sitzber. d. phys.-med. Soc. zu Erlangen, 11th December, 1891. It was investigated more thoroughly by Hertz, Ann. d. Phys. xlv (1892), p. 28, and by Philipp Lenard, of Bonn, Ann. d. Phys. li (1894), p. 225; lii (1894), p. 23, who conducted a series of experiments on cathode rays which had passed out of the discharge tube through a thin window of aluminium.

[||] J. J. Thomson, *Recent Researches*, p. 126.

In the theory of Weber, electric phenomena were attributed to the agency of stationary or moving charges, which could most readily be pictured as having a discrete and atom-like existence. The conception of displacement, on the other hand, which lay at the root of the Maxwellian theory, was more in harmony with the representation of electricity as something of a continuous nature; and as Maxwell's views met with increasing acceptance, the atomistic hypothesis seemed to have entered on a period of decay. Its revival was due largely to the advocacy of Helmholtz,* who, in a lecture delivered to the Chemical Society of London in 1881, pointed out† that it was thoroughly in accord with the ideas of Faraday,‡ on which Maxwell's theory was founded. "If," he said, "we accept the hypothesis that the elementary substances are composed of atoms, we cannot avoid concluding that electricity also, positive as well as negative, is divided into definite elementary portions which behave like atoms of electricity."

When the conduction of electricity is considered in the light of this hypothesis, it seems almost inevitable to conclude that the process is of much the same character in gases as in electrolytes; and before long this view was actively maintained. It had indeed long been known that a compound gas might be decomposed by the electric discharge; and that in some cases the constituents are liberated at the electrodes in such a way as to suggest an analogy with electrolysis. The question had been studied in 1861 by Adolphe Perrot, who examined § the gases liberated by the passage of the electric spark through steam. He found that while the product of this action was a detonating mixture of hydrogen and oxygen, there was a decided preponderance of hydrogen at one pole and of oxygen at the other.

The analogy of gaseous conduction to electrolysis was applied by W. Giese,|| of Berlin, in 1882, in order to explain

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* Cf. also G. Johnstone Stoney, Phil. Mag., May, 1881.

† Journ. Chem. Soc. xxxix (1881), p. 277.

‡ Cf. p. 200.

§ Annales de Chimie (3), lxi, p. 161.

|| Ann. d. Phys. xvii (1882), pp. 1, 236, 519.

the conductivity of the hot gases of flames. "It is assumed," he wrote, "that in electrolytes, even before the application of an external electromotive force, there are present atoms or atomic groups—the ions, as they are called—which originate when the molecules dissociate; by these the passage of electricity through the liquid is effected, for they are set in motion by the electric field and carry their charges with them. We shall now extend this hypothesis by assuming that in gases also the property of conductivity is due to the presence of ions. Such ions may be supposed to exist in small numbers in all gases at the ordinary temperature and pressure; and as the temperature rises their numbers will increase."

Ideas similar to this were presented in a general theory of the discharge in rarefied gases, which was devised two years later by Arthur Schuster, of Manchester.* Schuster remarked that when hot liquids are maintained at a high potential, the vapours which rise from them are found to be entirely free from electrification; from which he inferred that a molecule striking an electrified surface in its rapid motion cannot carry away any part of the charge, and that one molecule cannot communicate electricity to another in an encounter in which both molecules remain intact. Thus he was led to the conclusion that dissociation of the gaseous molecules is necessary for the passage of electricity through gases.†

Schuster advocated the charged-particle theory of cathode rays, and by extending and interpreting an experiment of Hittorf's was able to adduce strong evidence in its favour. He placed the positive and negative electrodes so close to each other that at very low pressures the Crookes' dark space extended from the cathode to beyond the anode. In these circumstances it was found that the discharge from the positive electrode always passed to the nearest point of the inner boundary of the Crookes' dark space—which, of course, was in the opposite direction to the cathode. Thus, in the neighbourhood of the positive discharge, the current was flowing in two opposite directions at closely adjoining places; which could scarcely happen unless the current in one direction were carried by particles moving against the lines of force by virtue of their inertia.

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* Proc. Roy. Soc. xxxvii (1884), p. 317.

† In the case of an elementary gas, this would imply dissociation of the molecule into two atoms chemically alike, but oppositely charged; in electrolysis the dissociation is into two chemically unlike ions.

Continuing his researches, Schuster* showed in 1887 that a steady electric current may be obtained in air between electrodes whose difference of potential is but small, provided that an independent current is maintained in the same vessel; that is to say, a continuous discharge produces in the air such a condition that conduction occurs with the smallest electromotive forces. This effect he explained by aid of the hypothesis previously advanced; the ions produced by the main discharge become diffused throughout the vessel, and, coming under the influence of the field set up by the auxiliary electrodes, drift so as to carry a current between the latter.

A discovery related to this was made in the same year by Hertz,† in the course of the celebrated researches‡ which have been already mentioned. Happening to notice that the passage of one spark is facilitated by the passage of another spark in its neighbourhood, he followed up the observation, and found the phenomenon to be due to the agency of ultra-violet light emitted by the latter spark. It appeared in fact that the distance across which an electric spark can pass in air is greatly increased when light of very short wave-length is allowed to fall on the spark-gap. It was soon found§ that the effective light is that which falls on the negative electrode of the gap; and Wilhelm Hallwachs‖ extended the discovery by showing that when a sheet of metal is negatively electrified and exposed to ultra-violet light, the adjacent air is thrown into a state which permits the charge to leak rapidly away.

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* Proc. Roy. Soc. xlii (1887), p. 371. Hittorf had discovered that very small electromotive forces are sufficient to cause a discharge across a space through which the cathode radiation is passing.

† Berlin Ber., 1887, p. 487; Ann. d. Phys. xxxi (1887), p. 983; *Electric Waves* (English ed.), p. 63.

‡ Cf. p. 357.

§ By E. Wiedemann and Ebert, Ann. d. Phys. xxxiii (1888), p. 241.

‖ Ann. d. Phys. xxxiii (1888), p. 301.

Interest was now thoroughly aroused in the problem of conductivity in gases; and it was generally felt that the best hope of divining the nature of the process lay in studying the discharge at high rarefactions. "If a first step towards understanding the relations between aether and ponderable matter is to be made," said Lord Kelvin in 1893,* "it seems to me that the most hopeful foundation for it is knowledge derived from experiments on electricity in high vacuum."

Within the two following years considerable progress was effected in this direction. J. J. Thomson,† by a rotating-mirror method, succeeded in measuring the velocity of the cathode rays, finding it to be‡ $1 \cdot 9 \times 10^7$ cm./sec.; a value so much smaller than that of the velocity of light that it was scarcely possible to conceive of the rays as vibrations of the aether. A further blow was dealt at the latter hypothesis when Jean Perrin,§ having received the rays in a metallic cylinder, found that the cylinder became charged with resinous electricity. When the rays were deviated by a magnet in such a way that they could no longer enter the cylinder, it no longer acquired a charge. This appeared to demonstrate that the rays transport negative electricity.

With cathode rays is closely connected another type of radiation, which was discovered in December, 1895, by W. C. Röntgen.‖ The discovery seems to have originated in an accident: a photographic plate which, protected in the usual way, had been kept in a room in which vacuum-tube experiments were carried on, was found on development to show distinct markings. Experiments suggested by this showed that radiation, capable of affecting sensitive plates and of causing fluorescence in certain substances, is emitted by tubes in which the electric discharge is passing; and that the radiation proceeds from the place where the cathode rays strike the glass walls of the tube. The *X-rays,* as they were called by their discoverer, are propagated in straight lines, and can neither be refracted by any of the substances which refract light, nor deviated from their course by a magnetic field; they are moreover able to pass with little absorption through many substances which are opaque to ordinary and ultra-violet light—a property of which considerable use has been made in surgery.

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* Proc. Roy. Soc. liv (1893), p. 389.

† Phil. Mag. xxxviii (1894), p. 358.

‡ The value found by the same investigator in 1897 was much larger than this.

§ Comptes Rendus, cxxi (1895), p. 1130.

‖ Sitzungsber. der Würzburger Physikal.-Medic. Gesellschaft, 1895; reprinted, Ann. d. Phys. lxiv (1898), pp. 1, 12; translated, Nature, liii (1896), p. 274.

The nature of the new radiation was the subject of much speculation. Its discoverer suggested that it might prove to represent the long-sought-for longitudinal vibrations of the aether; while other writers advocated the rival claims of aethereal vortices, infra-red light, and "sifted" cathode rays. The hypothesis which subsequently obtained general acceptance was first propounded by Schuster* in the month following the publication of Röntgen's researches. It is, that the X-rays are transverse vibrations of the aether, of exceedingly small wave-length. A suggestion which was put forward later in the year by E. Wiechert† and Sir George Stokes,‡ to the effect that the rays are pulses generated in the aether when the glass of the discharge tube is bombarded by the cathode particles, is not really distinct from Schuster's hypothesis; for ordinary white light likewise consists of pulses, as Gouy§ had shown, and the essential feature which distinguishes the Röntgen pulses is that the harmonic vibrations into which they can be resolved by Fourier's analysis are of very short period.

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[*] Nature, January 23, 1896, p. 268. Fitz Gerald independently made the same suggestion in a letter to O. J. Lodge, printed in the Electrician xxxvii, p. 372.

[†] Ann. d. Phys. lix (1896), p. 321.

[‡] Nature, September 3, 1896, p. 427: Proc. Camb. Phil. Soc. ix (1896), p. 215; Mem. Manchester Lit. & Phil. Soc. xli (1896–7).

[§] Journ. de Phys. v (1886), p. 354.

The rapidity of the vibrations explains the failure of all attempts to refract the X-rays. For in the formula

$$\mu^2 = 1 + \frac{\sigma p^2}{\rho (p^2 - n^2)}$$

of the Maxwell-Sellmeier theory,[*] n denotes the frequency, and so is in this case extremely large; whence we have

$$\mu^2 = 1,$$

i.e., the refractive index of all substances for the X-rays is unity. In fact, the vibrations alternate too rapidly to have an effect on the sluggish systems which are concerned in refraction.

Some years afterwards H. Haga and C. H. Wind,[†] having measured the diffraction-patterns produced by X-rays, concluded that the wave-length of the vibrations concerned was of the order of one Ångstrom unit, that is about 1/6000 of the wave-length of the yellow light of sodium.

One of the most important properties of X-rays was discovered, shortly after the rays themselves had become known, by J. J. Thomson,[‡] who announced that when they pass through any substance, whether solid, liquid, or gaseous, they render it conducting. This he attributed, in accordance with the ionic theory of conduction, to "a kind of electrolysis, the molecule of the non-conductor being split up, or nearly split up, by the Röntgen rays."

The conductivity produced in gases by this means was at once investigated[§] more closely. It was found that a gas which had acquired conducting power by exposure to X-rays lost this quality when forced through a plug of glass-wool; whence it was inferred that the structure in virtue of which the gas conducts is of so coarse a character that it is unable to survive the passage through the fine pores of the plug. The conductivity was also found to be destroyed when an electric current was passed through the gas—a phenomenon for which a parallel may be found in electrolysis. For if the ions were removed from an electrolytic solution by the passage of a current, the solution would cease to conduct as soon as sufficient electricity had passed to remove them all; and it may be supposed that the conducting agents which are produced in a gas by exposure to X-rays are likewise abstracted from it when they are employed to transport charges.

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[*] Cf. p. 293.

[†] Proceedings of the Amsterdam Acad., March 25th, 1899 (English edition, i, p. 420), and September 27th, 1902 (English edition, v, p. 247).

[‡] Nature, February 27, 1896, p. 391.

[§] J. J. Thomson and E. Rutherford, Phil. Mag. xlii (1896), p. 392.

The same idea may be applied to explain another property of gases exposed to X-rays. The strength of the current through the gas depends both on the intensity of the radiation and also on the electromotive force; but if the former factor be constant, and the electromotive force be increased, the current does not increase indefinitely, but tends to attain a certain "saturation" value. The existence of this saturation value is evidently due to the inability of the electromotive force to do more than to remove the ions as fast as they are produced by the rays.

Meanwhile other evidence was accumulating to show that the conductivity produced in gases by X-rays is of the same nature as the conductivity of the gases from flames and from the path of a discharge, to which the theory of Giese and Schuster had already been applied. One proof of this identity was supplied by observations of the condensation of water-vapour into clouds. It had been noticed long before by John Aitken* that gases rising from flames cause precipitation of the aqueous vapour from a saturated gas; and R. von Helmholtz† had found that gases through which an electric discharge has been passed possess the same property. It was now shown by C. T. R. Wilson,‡ working in the Cavendish Laboratory at Cambridge, that the same is true of gases which have been exposed to X-rays. The explanation furnished by the ionic theory is that in all three cases the gas contains ions which act as centres of condensation for the vapour.

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* Trans. R. S. Edinb. xxx (1880), p. 337.

† Ann. d. Phys. xxxii (1887), p. 1.

‡ Proc. Roy. Soc., March 19, 1896; Phil. Trans., 1897, p. 265.

During the year which followed their discovery, the X-rays were so thoroughly examined that at the end of that period they were almost better understood than the cathode rays from which they derived their origin. But the obscurity in which this subject had been so long involved was now to be dispelled.

Lecturing at the Royal Institution on April 30th, 1897, J. J. Thomson advanced a new suggestion to reconcile the molecular-torrent hypothesis with Lenard's observations of the passage of cathode rays through material bodies. "We see from Lenard's table," he said, "that a cathode ray can travel through air at atmospheric pressure a distance of about half a centimetre before the brightness of the phosphorescence falls to about half its original value. Now the mean free path of the molecule of air at this pressure is about 10^{-5} cm., and if a molecule of air were projected it would lose half its momentum in a space comparable with the mean free path. Even if we suppose that it is not the same molecule that is carried, the effect of the obliquity of the collisions would reduce the momentum to half in a short multiple of that path.

"Thus, from Lenard's experiments on the absorption of the rays outside the tube, it follows on the hypothesis that the cathode rays are charged particles moving with high velocities that the size of the carriers must be small compared with the dimensions of ordinary atoms or molecules.* The assumption of a state of matter more finely subdivided than the atom of an element is a somewhat startling one; but a hypothesis that would involve somewhat similar consequences—viz. that the so-called elements are compounds of some primordial element —has been put forward from time to time by various chemists."

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* A similar suggestion was made by E. Wiechert, Verhandl. d. physik.-öcon. Gesellsch. in Königsberg, Jan. 1897.

Thomson's lecture drew from Fitz Gerald[*] the suggestion that "we are dealing with free electrons in these cathode rays"—a remark the point of which will become more evident when we come to consider the direction in which the Maxwellian theory was being developed at this time.

Shortly afterwards Thomson himself published an account[†] of experiments in which the only outstanding objections to the charged-particle theory were removed. The chief of these was Hertz' failure to deflect the cathode rays by an electrostatic field. Hertz had caused the rays to travel between parallel plates of metal maintained at different potentials; but Thomson now showed that in these circumstances the rays generate ions in the rarefied gas, which settle on the plates, and annul the electric force in the intervening space. By carrying the exhaustion to a much higher degree, he removed this source of confusion, and obtained the expected deflexion of the rays.

The electrostatic and magnetic deflexions taken together suffice to determine the ratio of the mass of a cathode particle to the charge which it carries. For the equation of motion of the particle is

$$m\ddot{\mathbf{r}} = e\mathbf{E} + e[\mathbf{v}\,.\,\mathbf{H}],$$

where $\mathbf{r}$ denotes the vector from the origin to the position of the particle; $\mathbf{E}$ and $\mathbf{H}$ denote the electric and magnetic forces; e the charge, m the mass, and $\mathbf{v}$ the velocity of the particle. By observing the circumstances in which the force $e\mathbf{E}$, due to the electric field, exactly balances the force $e[\mathbf{v}\,.\,\mathbf{H}]$, due to the magnetic field, it is possible to determine $\mathbf{v}$; and it is readily seen from the above equation that a measurement of the deflexion in the magnetic field supplies a relation between $\mathbf{v}$ and m/e; so both $\mathbf{v}$ and m/e may be determined. Thomson found the value of m/e to be independent of the nature of the rarefied gas: its amount was 10^{-7} (grammes/electromagnetic units of charge), which is only about the thousandth part of the value of m/e for the hydrogen atom in electrolysis. If the charge were supposed to be of the same order of magnitude as that on an electrolytic ion, it would be necessary to conclude that the particle whose mass was thus measured is much smaller than the atom, and the conjecture might be entertained that it is the primordial unit or corpuscle of which all atoms are ultimately composed.*

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[*] Electrician, May 21, 1897.

[†] Phil. Mag. xliv (1897), p. 298.

The nature of the resinously charged corpuscles which constitute cathode rays being thus far determined, it became of interest to inquire whether corresponding bodies existed carrying charges of vitreous electricity—a question to which at any rate a provisional answer was given by W. Wien† of Aachen in the same year. More than a decade previously E. Goldstein‡ had shown that when the cathode of a discharge-tube is perforated, radiation of a certain type passes outward through the perforations into the part of the tube behind the cathode. To this radiation he had given the name *canal rays*. Wien now showed that the canal rays are formed of positively charged particles, obtaining a value of m/e immensely larger than Thomson had obtained for the cathode rays, and indeed of the same order of magnitude as the corresponding ratio in electrolysis.

The disparity thus revealed between the corpuscles of cathode rays and the positive ions of Goldstein's rays excited great interest; it seemed to offer a prospect of explaining the curious differences between the relations of vitreous and of resinous electricity to ponderable matter. These phenomena had been studied by many previous investigators; in particular Schuster,§ in the Bakerian lecture of 1890, had remarked that "if the law of impact is different between the molecules of the gas and the positive and negative ions respectively, it follows that the rate of diffusion of the two sets of ions will in general be different," and had inferred from his theory of the discharge that "the negative ions diffuse more rapidly." This inference was confirmed in 1898 by John Zeleny,* who showed that of the ions produced in air by exposure to X-rays, the positive are decidedly less mobile than the negative.

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* The value of m/e for cathode rays was determined also in the same year by W. Kaufmann, Ann. d. Phys. lxi, p. 544.

† Verhandl. der physik. Gesells. zu Berlin, xvi (1897), p. 165; Ann. d. Phys. lxv (1898), p. 440.

‡ Berlin Sitzungsber., 1886, p. 691.

§ Proc. R.S. xlvii (1890), p. 526.

The magnitude of the electric charge on the ions of gases was not known with certainty until 1898, when a plan for determining it was successfully executed by J. J. Thomson.† The principles on which this celebrated investigation was based are very ingenious. By measuring the current in a gas which is exposed to Röntgen rays and subjected to a known electromotive force, it is possible to determine the value of the product nev, where n denotes the number of ions in unit volume of the gas, e the charge on an ion, and v the mean velocity of the positive and negative ions under the electromotive force. As v had been already determined,‡ the experiment led to a determination of ne; so if n could be found, the value of e might be deduced.

The method employed by Thomson to determine n was founded on the discovery, to which we have already referred, that when X-rays pass through dust-free air, saturated with aqueous vapour, the ions act as nuclei around which the water condenses, so that a cloud is produced by such a degree of saturation as would ordinarily be incapable of producing condensation. The size of the drops was calculated from measurements of the rate at which the cloud sank; and, by comparing this estimate with the measurement of the mass of water deposited, the number of drops was determined, and hence the number n of ions. The value of e consequently deduced was found to be independent of the nature of the gas in which the ions were produced, being approximately the same in hydrogen as in air, and being apparently in both cases the same as for the charge carried by the hydrogen ion in electrolysis.

Since the publication of Thomson's papers his general conclusions regarding the magnitudes of e and m/e for gaseous ions have been abundantly confirmed. It appears certain that electric charge exists in discrete units, vitreous and resinous, each of magnitude $1{\cdot}5 \times 10^{-19}$ coulombs approximately. Each ion, whether in an electrolytic liquid or in a gas, carries one (or an integral number) of these charges. An electrolytic ion also contains one or more atoms of matter; and a positive gaseous ion has a mass of the same order of magnitude as that of an atom of matter. But it is possible in many ways to produce in a gas negative ions which are not attached to atoms of matter; for these the inertia is only about one-thousandth of the inertia of an atom; and there is reason for believing that even this apparent mass is in its origin purely electrical.*

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* Phil. Mag. xlvi (1898), p. 120.

† Phil. Mag. xlvi (1898), p. 528.

‡ By E. Rutherford, Phil. Mag. xliv (1897), p. 422.

The closing years of the nineteenth century saw the foundation of another branch of experimental science which is closely related to the study of conduction in gases. When Röntgen announced his discovery of the X-rays, and described their power of exciting phosphorescence, a number of other workers commenced to investigate this property more completely. In particular, Henri Becquerel resolved to examine the radiations which are emitted by the phosphorescent double sulphate of uranium and potassium after exposure to the sun. The result was communicated to the French Academy on February 24th, 1896.† "Let a photographic plate," he said, "be wrapped in two sheets of very thick black paper, such that the plate is not affected by exposure to the sun for a day. Outside the paper place a quantity of the phosphorescent substance, and expose the whole to the sun for several hours. When the plate is developed, it displays a silhouette of the phosphorescent substance. So the latter must emit radiations which are capable of passing through paper opaque to ordinary light, and of reducing salts of silver."

At this time Becquerel supposed the radiation to have been excited by the exposure of the phosphorescent substance to the sun; but a week later he announced‡ that it persisted for an indefinite time after the substance had been removed from the sunlight, and after the luminosity which properly constitutes phosphorescence had died away; and he was thus led to conclude that the activity was spontaneous and permanent. It was soon found that those salts of uranium which do not phosphoresce—e.g., the uranous salts,—and the metal itself, all emit the rays; and it became evident that what Becquerel had discovered was a radically new physical property, possessed by the element uranium in all its chemical compounds.

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* Cf. p. 343.

† Comptes Rendus, cxxii (1896), p. 420.

‡ *Ibid.*, cxxii (March 2nd, 1896), p. 501.

Attempts were now made to trace this activity in other substances. In 1898 it was recognized in thorium and its compounds;[*] and in the same year P. Curie and Madame Sklodowska Curie announced to the French Academy the separation from the mineral pitchblende of two new highly active elements, to which they gave the names of polonium[†] and radium.[‡] A host of workers was soon engaged in studying the properties of the Becquerel rays. The discoverer himself had shown[§] in 1896 that these rays, like the X- and cathode rays, impart conductivity to gases. It was found in 1899 by Rutherford[||] that the rays from uranium are not all of the same kind, but that at least two distinct types are present; one of these, to which he gave the name α-rays, is readily absorbed; while another, which he named β-radiation, has a greater penetrating power. It was then shown by Giesel, Becquerel, and others, that part of the radiation is deflected by a magnetic field,[¶] and part is not.[**] After this Monsieur and Madame Curie[††] found that the deviable rays carry negative electric charges, and Becquerel* succeeded in deviating them by an electrostatic field. The deviable or β- rays were thus clearly of the same nature as cathode rays; and when measurements of the electric and magnetic deviations gave for the ratio m/e a value of the order 10^{-7}, the identity of the β-particles with the cathode-ray corpuscles was fully established.

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[*] By Schmidt, Ann. d. Phys., lxv (1898), p. 141; and by Madame Curie, Comptes Rendus, cxxvi (1898), p. 1101.

[†] Comptes Rendus, cxxvii (1898), p. 175.

[‡] *Ibid.*, cxxvii (1898), p. 1215.

[§] *Ibid.*, cxxii (1896), p. 559.

[||] Phil. Mag. (5), xlvii (1899), p. 109.

[¶] Giesel, Ann. d. Phys. lxix (1899), p. 834 (working with polonium); Becquerel, Comptes Rendus, cxxix (1899), p. 996 (working with radium); Meyer and v. Schweidler, Phys. Zeitschr. i (1899), p. 113 (working with polonium and radium).

[**] Becquerel, Comptes Rendus, cxxix (1889), p. 1205); cxxx (1900), pp. 206, 372. Curie, ibid. cxxx (1900), p. 73.

[††] Comptes Rendus, cxxx (1900), p. 647.

The subsequent history of the new branch of physics thus created falls outside the limits of the present work. We must now consider the progress which was achieved in the general theory of aether and electricity in the last decade of the nineteenth century.

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* Comptes Rendus, cxxx (1900), p. 809.

CHAPTER XII.

THE THEORY OF AETHER AND ELECTRONS IN THE CLOSING YEARS OF THE NINETEENTH CENTURY.

THE attempts of Maxwell* and of Hertz† to extend the theory of the electromagnetic field to the case in which ponderable bodies are in motion had not been altogether successful. Neither writer had taken account of any motion of the material particles relative to the aether entangled with them, so that in both investigations the moving bodies were regarded simply as homogeneous portions of the medium which fills all space, distinguished only by special values of the electric and magnetic constants. Such an assumption is evidently inconsistent with the admirable theory by which Fresnel‡ had explained the optical behaviour of moving transparent bodies; it was therefore not surprising that writers subsequent to Hertz should have proposed to replace his equations by others designed to agree with Fresnel's formulae. Before discussing these, however, it may be well to review briefly the evidence for and against the motion of the aether in and adjacent to moving ponderable bodies, as it appeared in the last decade of the nineteenth century.

The phenomena of aberration had been explained by Young§ on the assumption that the aether around bodies is unaffected by their motion. But it was shown by Stokes|| in 1845 that this is not the only possible explanation. For suppose that the motion of the earth communicates motion to the neighbouring portions of the aether; this may be regarded as superposed on the vibratory motion which the aethereal particles have

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* Cf p. 288. † Cf. p. 365. ‡ Cf. p. 116. § Cf. p. 115.

|| Phil. Mag. xxvii (1845), p. 9; xxviii (1846), p. 76; xxix (1846), p. 6.

when transmitting light: the orientation of the wave-fronts of the light will consequently in general be altered; and the direction in which a heavenly body is seen, being normal to the wave-fronts will thereby be affected. But if the aethereal motion is irrotational, so that the elements of the aether do not rotate, it is easily seen that the direction of propagation of the light in space is unaffected; the luminous disturbance is still propagated in straight lines from the star, while the normal to the wave-front at any point deviates from this line of propagation by the small angle u/c, where u denotes the component of the aethereal velocity at the point, resolved at right angles to the line of propagation, and c denotes the velocity of light. If it be supposed that the aether near the earth is at rest relatively to the earth's surface, the star will appear to be displaced towards the direction in which the earth is moving, through an angle measured by the ratio of the velocity of the earth to the velocity of light, multiplied by the sine of the angle between the direction of the earth's motion and the line joining the earth and star. This is precisely the law of aberration.

An objection to Stokes's theory has been pointed out by several writers, amongst others by H. A. Lorentz.* This is, that the irrotational motion of an incompressible fluid is completely determinate when the normal component of the velocity at its boundary is given: so that if the aether were supposed to have the same normal component of velocity as the earth, it would not have the same tangential component of velocity. It follows that no motion will in general exist which satisfies Stokes's conditions; and the difficulty is not solved in any very satisfactory fashion by either of the suggestions which, have been proposed to meet it. One of these is to suppose that the moving earth does generate a rotational disturbance, which, however, being radiated away with the velocity of light, does not affect the steadier irrotational motion; the other, which was advanced by Planck,[*] is that the two conditions of Stokes's theory—namely, that the motion of the aether is to be irrotational and that at the earth's surface its velocity is to be the same as that of the earth—may both be satisfied if the aether is supposed to be compressible in accordance with Boyle's law, and subject to gravity, so that round the earth it is compressed like the atmosphere; the velocity of light being supposed independent of the condensation of the aether.

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* Archives Néerl, xxi (1896), p. 103.

Lorentz,[†] in calling attention to the defects of Stokes's theory, proposed to combine the ideas of Stokes and Fresnel, by assuming that the aether near the earth is moving irrotationally (as in Stokes's theory), but that at the surface of the earth the aethereal velocity is not necessarily the same as that of ponderable matter, and that (as in Fresnel's theory) a material body imparts the fraction $(\mu^2 - 1)/\mu^2$ of its own motion to the aether within it. Fresnel's theory is a particular case of this new theory, being derived from it by supposing the velocity-potential to be zero.

Aberration is by no means the only astronomical phenomenon which depends on the velocity of propagation of light; we have indeed seen[‡] that this velocity was originally determined by observing the retardation of the eclipses of Jupiter's satellites. It was remarked by Maxwell[§] in 1879 that these eclipses furnish, theoretically at least, a means of determining the velocity of the solar system relative to the aether. For if the distance from the eclipsed satellite to the earth be divided by the observed retardation in time of the eclipse, the quotient represents the velocity of propagation of light in this direction, relative to the solar system; and this will differ from the velocity of propagation of light relative to the aether by the component, in this direction, of the sun's velocity relative to the aether. By taking observations when Jupiter is in different signs of the zodiac, it should therefore be possible to determine the sun's velocity relative to the aether, or at least that component of it which lies in the ecliptic.

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[*] Cf. Lorentz, Proc. Amsterdam Acad. (English ed.), i (1899), p. 443.

[†] Archives Néerl. xxi (1886), p. 103: cf. also Zittinsgsversl. Kon. Ak. Amsterdam, 1897–98, p. 266.

[‡] Cf. p. 22.

[§] Proc. R. S. xxx (1880), p. 108.

The same principles may be applied to the discussion of other astronomical phenomena. Thus the minimum of a variable star of the Algol type will be retarded or accelerated by an interval of time which is found by dividing the projection of the radius from the sun to the earth on the direction from the sun to the Algol variable by the velocity, relative to the solar system, of propagation of light from the variable; and thus the latter quantity may be deduced from observations of the retardation.*

Another instance in which the time taken by light to cross an orbit influences an observable quantity is afforded by the astronomy of double stars. Savary† long ago remarked that when the plane of the orbit of a double star is not at right angles to the line of sight, an inequality in the apparent motion must be caused by the circumstance that the light from the remoter star has the longer journey to make. Yvon Villarceau‡ showed that the effect might be represented by a constant alteration of the elliptic elements of the orbit (which alteration is of course beyond detection), together with a periodic inequality, which may be completely specified by the following statement: the apparent coordinates of one star relative to the other have the values which in the absence of this effect they would have at an earlier or later instant, differing from the actual time by the amount

$$\frac{m_1 - m_2}{m_1 + m_2} \cdot \frac{z}{c},$$

where m_1 and m_2 denote the masses of the stars, c the velocity of light, and z the actual distance of the two stars from each other at the time when the light was emitted, resolved along the line of sight. In the existing state of double-star astronomy, this effect would be masked by errors of observation.

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* The velocity of light was found from observations of Algol, by C. V. L. Charlier, Öfversigt af K. Vet.-Ak. Förhandl. xlvi (1889), p. 523.

† Conn. des Temps, 1830.

‡ Additions à la Connaissance des Temps, 1878: an improved deduction was given by H. Seeliger, Sitzungsberichte d. K. Ak. zu München, xix (1889), p. 19.

Villarceau also examined the consequences of supposing that the velocity of light depends on the velocity of the source by which it is emitted. If, for instance, the velocity of light from a star occulted by the moon were less than the velocity of light reflected by the moon, then the apparent position of the lunar disk would be more advanced in its movement than that of the star, so that at emersion the star would first appear at some distance outside the lunar disk, and at immersion the star would be projected on the interior of the disk at the instant of its disappearance. The amount by which the image of the star could encroach on that of the disk on this account could not be so much as $0''\cdot71$; encroachment to the extent of more than $1''$ has been observed, but is evidently to be attributed for the most part to other causes.

Among the consequences of the finite velocity of propagation of light which are of importance in astronomy, a leading place must be assigned to the principle enunciated in 1842 by Christian Doppler,* that the motion of a source of light relative to an observer modifies the period of the disturbance which is received by him. The phenomenon resembles the depression of the pitch of a note when the source of sound is receding from the observer. In either case, the period of the vibrations perceived by the observer is $(c + v) / c \times$ the natural period, where v denotes the velocity of separation of the source and observer, and c denotes the velocity of propagation of the disturbance. If, e.g., the velocity of separation is equal to the orbital velocity of the earth, the D lines of sodium in the spectrum of the source will be displaced towards the red, as compared with lines derived from a terrestrial sodium flame, by about one-tenth of the distance between them. The application of this principle to the determination of the relative velocity of stars in the line of sight, which has proved of great service in astrophysical research, was suggested by Fizeau in 1848.*

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* Abhandl. der K. Böhm. Ges. der Wissensch. (5) ii (1842), p. 465.

Passing now from the astronomical observatory, we must examine the information which has been gained in the physical laboratory regarding the effect of the earth's motion on optical phenomena. We have already† referred to the investigations by which the truth of Fresnel's formula was tested. An experiment of a different type was suggested in 1852 by Fizeau,‡ who remarked that, unless the aether is carried along by the earth, the radiation emitted by a terrestrial source should have different intensities in different directions. It was, however, shown long afterwards by Lorentz§ that such an experiment would not be expected on theoretical grounds to yield a positive result; the amount of radiant energy imparted to an absorbing body is independent of the earth's motion. A few years later Fizeau investigated‖ another possible effect. If a beam of polarized light is sent obliquely through a glass plate, the azimuth of polarization is altered to an extent which depends, amongst other things, on the refractive index of the glass. Fizeau performed this experiment with sunlight, the light being sent through the glass in the direction of the terrestrial motion, and in the opposite direction; the readings seemed to differ in the two cases, but on account of experimental difficulties the result was indecisive.

Some years later, the effect of the earth's motion on the rotation of the plane of polarization of light propagated along the axis of a quartz crystal was investigated by Mascart.¶ The result was negative, Mascart stating that the rotation could not have been altered by more than the (1/40,000)th part when the orientation of the apparatus was reversed from that of the terrestrial motion to the opposite direction. This was afterwards confirmed by Lord Rayleigh,[*] who found that the alteration, if it existed, could not amount to (1/100,000)th part.

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* An apparatus for demonstrating the Doppler-Fizeau effect in the laboratory was constructed by Belopolsky, Astrophys. Journal xiii (1901), p. 15.

† Cf. pp. 117–120.

‡ Ann. d. Phys. xcii (1854), p. 652.

§ Proc. Amsterdam Acad. (English edition), iv (1902) p. 678.

‖ Annales de Chim. (3) lxviii (1860), p. 129; Ann. d. Phys. cxiv (1861), p. 554.

¶ Annales de l'Ec. Norm. (2) i (1872), p. 157.

In terrestrial methods of determining the velocity of light the ray is made to retrace its path, so that any velocity which the earth might possess with respect to the luminiferous medium would affect the time of the double passage only by an amount proportional to the square of the constant of aberration.[†] In 1881, however, A. A. Michelson[‡] remarked that the effect, though of the second order, should be manifested by a measurable difference between the times for rays describing equal paths parallel and perpendicular respectively to the direction of the earth's motion. He produced interference-fringes between two pencils of light which had traversed paths perpendicular to each other; but when the apparatus was rotated through a right angle, so that the difference would be reversed, the expected displacement of the fringes could not be perceived. This result was regarded by Michelson himself as a vindication of Stokes's theory,[§] in which the aether in the neighbourhood of the earth is supposed to be set in motion. Lorentz[‖], however, showed that the quantity to be measured had only half the value supposed by Michelson, and suggested that the negative result of the experiment might be explained by that combination of Fresnel's and Stokes's theories which was developed in his own memoir[¶]; since, if the velocity of the aether near the earth were (say) half the earth's velocity, the displacement of Michelson's fringes would be insensible.

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[*] Phil. Mag. iv. (1902), p. 215.

[†] The constant of aberration is the ratio of the earth's orbital velocity to the velocity of light; cf. *supra*, p. 100.

[‡] Amer. Journ. Sci. xxii (1881), p. 20. His method was afterwards improved: cf. Michelson and Morley, Amer. Journ. Sci. xxxiv (1887), p. 333; Phil. Mag. xxiv (1887), p. 449.

[§] Cf. p. 411.

[‖] Arch. Néerl. xxi (1886), p. 103. On the Michelson-Morley experiment cf. also Hicks, Phil. Mag. iii (1902), p. 9.

[¶] Cf. p. 413.

A sequel to the experiment of Michelson and Morley was performed in 1897, when Michelson* attempted to determine by experiment whether the relative motion of earth and aether varies with the vertical height above the terrestrial surface. No result, however, could be obtained to indicate that the velocity of light depends on the distance from the centre of the earth; and Michelson concluded that if there were no choice but between the theories of Fresnel and Stokes, it would be necessary to adopt the latter, and to suppose that the earth's influence on the aether exends to many thousand kilometres above its surface. By this time, however, as will subsequently appear, a different explanation was at hand.

Meanwhile the perplexity of the subject was increased by experimental results which pointed in the opposite direction to that of Michelson. In 1892 Sir Oliver Lodge† observed the interference between the two portions of a bifurcated beam of light, which were made to travel in opposite directions round a closed path in the space between two rapidly rotating steel disks. The observations showed that the velocity of light is not affected by the motion of adjacent matter to the extent of (1/200)th part of the velocity of the matter. Continuing his investigations, Lodge‡ strongly magnetized the moving matter (iron in this experiment), so that the light was propagated across a moving magnetic field; and electrified it so that the path of the beams lay in a moving electrostatic field; but in no case was the velocity of the light appreciably affected.

We must now trace the steps by which theoretical physicists not only arrived at a solution of the apparent contradictions furnished by experiments with moving bodies, but so extended the domain of electrical science that it became necessary to enlarge the boundaries of space and time to contain it.

The first memoir in which the new conceptions were unfolded was published by H. A. Lorentz§ in 1892. The theory of Lorentz was, like those of Weber, Riemann, and Clausius,[*] a theory of electrons; that is to say, all electrodynamical phenomena were ascribed to the agency of moving electric charges, which were supposed in a magnetic field to experience forces proportional to their velocities, and to communicate these forces to the ponderable matter with which they might be associated.[†]

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* Amer. Journ. Sci. (4) iii (1897), p. 475.

† Phil. Trans. clxxxiv (1893), p. 727.

‡ *Ibid.*, clxxxix (1897), p. 149.

§ Archives Néerl. xxv (1892), p. 363: the theory is given in ch. iv, pp. 432 *et sqq*.

In spite of the fact that the earlier theories of electrons had failed to fulfil the expectations of their authors, the assumption that all electric and magnetic phenomena are due to the presence or motion of individual electric charges was one to which physicists were at this time disposed to give a favourable consideration ; for, as we have seen,[‡] evidence of the atomic nature of electricity was now contributed by the study of the conduction of electricity through liquids and gases. Moreover, the discoveries of Hertz[§] had shown that a molecule which is emitting light must contain some system resembling a Hertzian vibrator; and the essential process in a Hertzian vibrator is the oscillation of electricity to and fro. Lorentz himself from the outset of his career[||] had supposed the interaction of ponderable matter with the electric field to be effected by the agency of electric charges associated with the material atoms.

The principal difference by which the theory now advanced by Lorentz is distinguished from the theories of Weber,

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[*] Cf. pp. 226, 231, 262.

[†] Some writers have inclined to use the term ' electron-theory ' as if it were specially connected with Sir Joseph Thomson's justly celebrated discovery (cf. p. 407, *supra*) that all negative electrons have equal charges. But Thomson's discovery, though undoubtedly of the greatest importance as a guide to the structure of the universe, has hitherto exercised but little influence on general electromagnetic theory. The reason for this is that in theoretical investigations it is customary to denote the changes of electrons by symbols, $e_1, e_2, \ldots$; and the equality or non-equality of these makes no difference to the equations. To take an illustration from Celestial Mechanics, it would clearly make no difference in the general equations of the planetary theory if the masses of the planets happened to be all equal.

[‡] Cf. chapter xi.

[§] Cf. pp. 357–363.

[||] Verh. d. Ak. v. Wetenschappen, Amsterdam, Deel xviii (1878).

Riemann, and Clausius, and from Lorentz' own earlier work, lies in the conception which is entertained of the propagation of influence from one electron to another. In the older writings, the electrons were assumed to be capable of acting on each other at a distance, with forces depending on their charges, mutual distances, and velocities; in the present memoir, on the other hand, the electrons were supposed to interact not directly with each other, but with the medium in which they were embedded. To this medium were ascribed the properties characteristic of the aether in Maxwell's theory.

The only respect in which Lorentz' medium differed from Maxwell's was in regard to the effects of the motion of bodies. Impressed by the success of Fresnel's beautiful theory of the propagation of light in moving transparent substances,* Lorentz designed his equations so as to accord with that theory, and showed that this might be done by drawing a distinction between matter and aether, and assuming that a moving ponderable body cannot communicate its motion to the aether which surrounds it, or even to the aether which is entangled in its own particles; so that no part of the aether can be in motion relative to any other part. Such an aether is simply space endowed with certain dynamical properties.

The general plan of Lorentz' investigation was to reduce all the complicated cases of electromagnetic action to one simple and fundamental case, in which the field contains only free aether with solitary electrons dispersed in it; the theory which he adopted in this fundamental case was a combination of Clausius' theory of electricity with Maxwell's theory of the aether.

Suppose that $e\,(x, y, z)$ and $e'(x', y', z')$ are two electrons. In the theory of Clausius,† the kinetic potential of their mutual action is

$$\frac{ee'}{r}\,(\dot{x}\dot{x}' + \dot{y}\dot{y}' + \dot{z}\dot{z}' - c^2);$$

so when any number of electrons are present, the part of the kinetic potential which concerns any one of them—say, *e*—may be written

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* Cf. pp. 116 *et sqq.*

† Cf. p. 262.

$$L_e = e\,(a_x \dot{x} + a_y \dot{y} + a_z \dot{z} - c^2 \phi),$$

where **a** and ϕ denote potential functions, defined by the equations

$$\mathbf{a} = \iiint \frac{\rho' \mathbf{v}'}{r}\,dx'\,dy'\,dz', \qquad \phi = \iiint \frac{\rho'}{r}\,dx'\,dy'\,dz';$$

ρ denoting the volume-density of electric charge, and **v** its velocity, and the integration being taken over all space.

We shall now reject Clausius' assumption that electrons act instantaneously at a distance, and replace it by the assumption that they act on each other only through the mediation of an aether which fills all space, and satisfies Maxwell's equations. This modification may be effected in Clausius' theory without difficulty; for, as we have seen,[*] if the state of Maxwell's aether at any point is defined by the electric vector **d** and magnetic vector **h**,[†] these vectors may be expressed in terms of potentials **a** and ϕ by the equations

$$\mathbf{d} = c^2 \operatorname{grad} \phi - \dot{\mathbf{a}}, \qquad \mathbf{h} = \operatorname{curl} \mathbf{a};$$

and the functions **a** and ϕ may in turn be expressed in terms of the electric charges by the equations

$$\mathbf{a} = \iiint \{(\overline{\rho \mathbf{v}_x})'/r\}\,dx'\,dy'\,dz', \qquad \phi = \iiint \{(\overline{\rho})'/r\}\,dx'\,dy'\,dz',$$

where the bars indicate that the values of $(\rho \mathbf{v}_x)'$ and $(\rho)'$ refer to the instant $(t - r/c)$. Comparing these formulae with those given above for Clausius' potentials, we see that the only change which it is necessary to make in Clausius' theory is that of retarding the potentials in the way indicated by L. Lorenz.[‡] The electric and magnetic forces, thus defined in terms of the position and motion of the charges, satisfy the Maxwellian equations

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[*] Cf. pp. 298, 299.

[†] We shall use the small letters **d** and **h** in place of **E** and **H**, when we are concerned with Lorentz' fundamental case, in which the system consists solely of free aether and isolated electrons.

[‡] Cf. p. 298.

$$\begin{cases} \operatorname{div} \mathbf{d} = 4\pi c^2 \rho, \\ \operatorname{div} \mathbf{h} = 0, \\ \operatorname{curl} \mathbf{d} = -\dot{\mathbf{h}}, \\ \operatorname{curl} \mathbf{h} = \dot{\mathbf{d}}/c^2 + 4\pi\rho\mathbf{v}. \end{cases}$$

The theory of Lorentz is based on these four aethereal equations of Maxwell, together with the equation which determines the ponderomotive force on a charged particle; this, which we shall now derive, is the contribution furnished by Clausius' theory.

The Lagrangian equations of motion of the electron e are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0,$$

and two similar equations, where L denotes the total kinetic potential due to all causes, electric and mechanical. The ponderomotive force exerted on the electron by the electromagnetic field has for its x-component

$$\frac{\partial L_e}{\partial x} - \frac{d}{dt}\left(\frac{\partial L_e}{\partial \dot{x}}\right),$$

or

$$e\left(\frac{\partial a_x}{\partial x}\dot{x} + \frac{\partial a_y}{\partial x}\dot{y} + \frac{\partial a_z}{\partial x}\dot{z} - c^2\frac{\partial \phi}{\partial x}\right) - e\frac{da_x}{dt};$$

which, since

$$\frac{da_x}{dt} = \frac{\partial a_x}{\partial t} + \frac{\partial a_x}{\partial x}\dot{x} + \frac{\partial a_x}{\partial y}\dot{y} + \frac{\partial a_x}{\partial z}\dot{z},$$

reduces to

$$-e\left(c^2\frac{\partial \phi}{\partial x} + \frac{\partial a_x}{\partial t}\right) + e\dot{z}\left(\frac{\partial a_z}{\partial x} - \frac{\partial a_x}{\partial z}\right) + e\dot{y}\left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right),$$

or

$$ed_x + e(\dot{y}h_z - \dot{z}h_y),$$

so that the force in question is

$$e\mathbf{d} + e[\mathbf{v} \, . \, \mathbf{h}].$$

This was Lorentz' expression for the ponderomotive force on an electrified corpuscle of charge e moving with velocity $\mathbf{v}$ in a field defined by the electric force $\mathbf{d}$ and magnetic force $\mathbf{h}$.

In Lorentz' fundamental case, which has thus been examined, account has been taken only of the ultimate constituents of which the universe is supposed to be composed, namely, corpuscles and the aether. We must now see how to build up from these the more complex systems which are directly presented to our experience.

The electromagnetic field in ponderable bodies, which to our senses appears in general to vary continuously, would present a different aspect if we were able to discern molecular structure; we should then perceive the individual electrons by which the field is produced, and the rapid fluctuations of electric and magnetic force between them. As it is, the values furnished by our instruments represent averages taken over volumes which, though they appear small to us, are large compared with molecular dimensions.* We shall denote an average value of this kind by a bar placed over the corresponding symbol.

Lorentz supposed that the phenomena of electrostatic charge and of conduction-currents are due to the presence or motion of simple electrons such as have been considered above. The part of $\overline{\rho}$ arising from these is the measurable density of electrostatic charge; this we shall denote by ρ_1. If $\mathbf{w}$ denote the velocity of the ponderable matter, and if the velocity $\mathbf{v}$ of the electrons be written $\mathbf{w} + \mathbf{u}$, then the quantity $\overline{\rho\mathbf{v}}$, so far as it arises from electrons of this type, may be written $\rho_1 \mathbf{w} + \overline{\rho\mathbf{u}}$. The former of these terms represents the convection-current, and the latter the conduction-current.

Consider next the phenomena of dielectrics. Following Faraday, Thomson, and Mossotti,† Lorentz supposed that each dielectric molecule contains corpuscles charged vitreously and also corpuscles charged resinously. These in the absence of an external field are so arranged as to neutralize each other's electric fields outside the molecule. For simplicity we may suppose that in each molecule only one corpuscle, of charge e, is capable of being displaced from its position; it follows from what has been assumed that the other corpuscles in the molecule exert the same electrostatic action as a charge $-e$ situated at the original position of this corpuscle. Thus if e is displaced to an adjacent position, the entire molecule becomes equivalent to an electric doublet, whose moment is measured by the product of e and the displacement of e. The molecules in unit volume, taken together, will in this way give rise to a (vector) electric moment per unit volume, **P**, which may be compared to the (vector) intensity of magnetization in Poisson's theory of magnetism.* As in that theory, we may replace the doublet-distribution **P** of the scalar quantity ρ by a volume-distribution of ρ, determined by the equation†

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* These principles had been enunciated, and to some extent developed, by J. Willard Gibbs in 1882–3: Amer. Journ. Sci. xxiii, pp. 262, 460, xxv, p. 107; Gibbs' *Scientific Papers*, ii, pp. 182, 195, 211.

† Cf. pp. 210, 211.

$$\bar{\rho} = -\operatorname{div} \mathbf{P}.$$

This represents the part of $\bar{\rho}$ due to the dielectric molecules.

Moreover, the scalar quantity ρw_x has also a doublet-distribution, to which the same theorem may be applied; the average value of the part of ρw_x, due to dielectric molecules, is therefore determined by the equation

$$\overline{\rho w_x} = -\operatorname{div}(w_x \mathbf{P}) = -w_x \operatorname{div} \mathbf{P} - (\mathbf{P} \, . \, \nabla)\, w_x,$$

or

$$\overline{\rho \mathbf{w}} = -\operatorname{div} \mathbf{P} \, . \, \mathbf{w} - (\mathbf{P} \, . \, \nabla)\, \mathbf{w}.$$

We have now to find that part of $\overline{\rho \mathbf{u}}$ which is due to dielectric molecules. For a single doublet of moment **p** we have, by differentiation,

$$\iiint \rho \mathbf{u} \, dx \, dy \, dz = d\mathbf{p}/dt,$$

where the integration is taken throughout the molecule; so that

$$\iiint \rho \mathbf{u} \, dx \, dy \, dz = (d/dt)\,(V\mathbf{P}),$$

where the integration is taken throughout a volume V, which encloses a large number of molecules, but which is small compared with measurable quantities; and this equation may be written

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* Cf. p. 64.

† We assume all transitions gradual, so as to avoid surface-distributions.

$$\overline{\rho \mathbf{u}} = \frac{1}{V} \frac{d}{dt} (V\mathbf{P}).$$

Now, if $\dot{\mathbf{P}}$ refers to differentiation at a fixed point of space (as opposed to a differentiation which accompanies the moving body), we have

$$(d/dt)\,\mathbf{P} = \dot{\mathbf{P}} + (\mathbf{w} \,.\, \nabla)\,\mathbf{P},$$

and $$(d/dt)\, V = V \operatorname{div} \mathbf{w};$$

so that

$$\begin{aligned} \overline{\rho \mathbf{u}} &= \dot{\mathbf{P}} + (\mathbf{w} \,.\, \nabla)\,\mathbf{P} + \operatorname{div} \mathbf{w} \,.\, \mathbf{P} \\ &= \dot{\mathbf{P}} + \operatorname{curl} [\mathbf{P} \,.\, \mathbf{w}] + \operatorname{div} \mathbf{P} \,.\, \mathbf{w} + (\mathbf{P} \,.\, \nabla)\,\mathbf{w}, \end{aligned}$$

and therefore

$$\overline{\rho \mathbf{u}} + \overline{\rho \mathbf{w}} = \dot{\mathbf{P}} + \operatorname{curl} [\mathbf{P} \,.\, \mathbf{w}].$$

This equation determines the part of $\overline{\rho \mathbf{v}}$ which arises from the dielectric molecules.

The general equations of the aether thus become, when the averaging process is performed,

$$\begin{aligned} \operatorname{div} \overline{\mathbf{d}} &= 4\pi c^2 \rho_1 - 4\pi c^2 \operatorname{div} \mathbf{P}, \qquad \operatorname{div} \overline{\mathbf{h}} = 0, \\ \operatorname{curl} \overline{\mathbf{d}} &= -\dot{\overline{\mathbf{h}}}, \\ \operatorname{curl} \overline{\mathbf{h}} &= (1/c^2)\, \dot{\overline{\mathbf{d}}} + 4\pi \left\{ \begin{array}{c} \text{convection-current} + \text{conduction-current} \\ + \dot{\mathbf{P}} + \operatorname{curl} [\mathbf{P} \,.\, \mathbf{w}] \end{array} \right\}. \end{aligned}$$

In order to assimilate these to the ordinary electromagnetic equations, we must evidently write

$$\begin{aligned} \overline{\mathbf{d}} &= \mathbf{E}, \text{ the electric force;} \\ (1/4\pi c^2)\,\mathbf{E} + \mathbf{P} &= \mathbf{D}, \text{ the electric induction;} \\ \overline{\mathbf{h}} &= \mathbf{H}, \text{ the magnetic vector.} \end{aligned}$$

The equations then become (writing ρ for ρ_1, as there is no longer any need to use the subscript),

$$\begin{aligned} \operatorname{div} \mathbf{D} &= \rho, & -\operatorname{curl} \mathbf{E} &= \dot{\mathbf{H}}, \\ \operatorname{div} \mathbf{H} &= 0, & \operatorname{curl} \mathbf{H} &= 4\pi \mathbf{S}, \end{aligned}$$

where

$$\mathbf{S} = \text{conduction-current} + \text{convection-current} + \dot{\mathbf{D}} + \operatorname{curl} [\mathbf{P} \,.\, \mathbf{w}].$$

The term $\dot{\mathbf{D}}$ in **S** evidently represents the displacement-current of Maxwell; and the term curl [**P** . **w**] will be recognized as a modified form of the term curl [**D** . **w**], which was first introduced into the equations by Hertz.* It will be remembered that Hertz supposed this term to represent the generation of a magnetic force within a dielectric which is in motion in an electric field; and that Heaviside,† by adducing considerations relative to the energy, showed that the term ought to be regarded as part of the total current, and inferred from its existence that a dielectric which moves in an electric field is the seat of an electric current, which produces a magnetic field in the surrounding space. The modification introduced by Lorentz consisted in replacing **D** by **P** in the vector-product; this implied that the moving dielectric does not carry along the aethereal displacement, which is represented by the term $\mathbf{E}/4\pi c^2$ in **D**, but only carries along the charges which exist at opposite ends of the molecules of the ponderable dielectric, and which are represented by the term **P**. The part of the total current represented by the term curl [**P** . **w**] is generally called the *current of dielectric convection.*

That a magnetic field is produced when an uncharged dielectric is in motion at right angles to the lines of force of a constant electrostatic field had been shown experimentally in 1888 by Röntgen.‡ His experiment consisted in rotating a dielectric disk between the plates of a condenser; a magnetic field was produced, equivalent to that which would be produced by the rotation of the "fictitious charges" on the two faces of the dielectric, i.e., charges which bear the same relation to the dielectric polarization that Poisson's equivalent surface-density of magnetism§ bears to magnetic polarization. If U denote the difference of potential between the opposite coatings of the condenser, and ε the specific inductive capacity of the dielectric, the surface-density of electric charge on the coatings is proportional to $\pm \epsilon U$, and the fictitious charge on the surfaces of the dielectric is proportional to $\mp (\epsilon - 1) U$. It is evident from this that if a plane condenser is charged to a given difference of potential, and is rotated in its own plane, the magnetic field produced is proportional to ϵ if (as in Rowland's experiment*) the coatings are rotated while the dielectric remains at rest, but is in the opposite direction, and is proportional to $(\epsilon - 1)$ if (as in Röntgen's experiment) the dielectric is rotated while the coatings remain at rest. If the coatings and dielectric are rotated together, the magnetic action (being the sum of these) should be independent of ϵ—a conclusion which was verified later by Eichenwald.†

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* Cf. p. 366.

† Cf. p. 367.

‡ Ann. d. Phys. xxxv (1888), p. 264; xl (1890), p. 93.

§ Cf. p. 64.

Hitherto we have taken no account of the possible magnetization of the ponderable body. This would modify the equations in the usual manner,‡ so that they finally take the form

$$\operatorname{div} \mathbf{D} = \rho, \qquad \text{(I)}$$

$$\operatorname{div} \mathbf{B} = 0, \qquad \text{(II)}$$

$$\operatorname{curl} \mathbf{H} = 4\pi \mathbf{S}, \qquad \text{(III)}$$

$$-\operatorname{curl} \mathbf{E} = \dot{\mathbf{B}}, \qquad \text{(IV)},$$

where **S** denotes the total current formed of the displacement-current, the convection-current, the conduction-current, and the current of dielectric convection. Moreover, since

$$\mathbf{S} = \overline{\rho \mathbf{v}} + \dot{\mathbf{d}}/4\pi c^2,$$

we have

$$\operatorname{div} \mathbf{S} = \operatorname{div} \overline{\rho \mathbf{v}} + (1/4\pi c^2) \operatorname{div} (\partial \bar{\mathbf{d}}/\partial t)$$

$$= \operatorname{div} \overline{\rho \mathbf{v}} + \partial \bar{\rho}/\partial t,$$

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* Cf. p. 339.

† Ann. d. Phys. xi (1903), p. 421 ; xiii (1904), p. 919. Eichenwald performed other experiments of a similar character, e.g. he observed the magnetic field due to the changes of polarization in a dielectric which was moved in a non-homogeneous electric field.

‡ It is possible to construct a purely electronic theory of magnetization, a magnetic molecule being supposed to contain electrons in orbital revolution. It then appears that the vector which represents the average value of **h** is not **H**, but **B**.

which vanishes by virtue of the principle of conservation of electricity. Thus

$$\text{div } \mathbf{S} = 0, \qquad \text{(V)}$$

or the total current is a circuital vector. Equations (I) to (V) are the fundamental equations of Lorentz' theory of electrons.

We have now to consider the relation by which the polarization **P** of dielectrics is determined. If the dielectric is moving with velocity **w**, the ponderomotive force on unit electric charge moving with it is (as in all theories)*

$$\mathbf{E}' = \mathbf{E} + [\mathbf{w} \, . \, \mathbf{B}]. \qquad (1)$$

In order to connect **P** with **E**′, it is necessary to consider the motion of the corpuscles. Let e denote the charge and m the mass of a corpuscle, (ξ, η, ζ) its displacement from its position of equilibrium, $k^2(\xi, \eta, \zeta)$ the restitutive force which retains it in the vicinity of this point; then the equations of motion of the corpuscle are

$$m\ddot{\xi} + k^2\xi = eE_x',$$

and similar equations in η and ζ. When the corpuscle is set in motion by light of frequency n passing through the medium, the displacements and forces will be periodic functions of nt—say,

$$\xi = Ae^{nt\sqrt{-1}}, \qquad E'_x = E_0 e^{nt\sqrt{-1}}.$$

Substituting these values in the equations of motion, we obtain

$$A(k^2 - mn^2) = eE_0, \quad \text{and therefore} \quad \xi(k^2 - mn^2) = eE'_x.$$

Thus, if N denote the number of polarizable molecules per unit volume, the polarization is determined by the equation

$$\mathbf{P} = Ne(\xi, \eta, \zeta) = Ne^2\mathbf{E}'/(k^2 - mn^2).$$

In the particular case in which the dielectric is at rest, this equation gives

$$\mathbf{D} = (1/4\pi c^2)\mathbf{E} + \mathbf{P} = (1/4\pi c^2)\mathbf{E} + Ne^2\mathbf{E}/(k^2 - mn^2).$$

But, as we have seen,† **D** bears to **E** the ratio $\mu^2/4\pi c^2$, where μ

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* Cf. p. 365.

† Cf. p. 281.

denotes the refractive index of the dielectric; and therefore the refractive index is determined in terms of the frequency by the equation

$$\mu^2 = 1 + 4\pi e^2 c^2 N/(k^2 - mn^2).$$

This formula is equivalent to that which Maxwell and Sellmeier* had derived from the elastic-solid theory. Though superficially different, the derivations are alike in their essential feature, which is the assumption that the molecules of the dielectric contain systems which possess free periods of vibration, and which respond to the oscillations of the incident light. The formula may be derived on electromagnetic principles without any explicit reference to electrons; all that is necessary is to assume that the dielectric polarization has a free period of vibration.†

When the luminous vibrations are very slow, so that n is small, μ^2 reduces to the dielectric constant ϵ‡; so that the theory of Lorentz leads to the expression

$$\epsilon = 1 + 4\pi N e^2 c^2/k^2$$

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* Cf. p. 293.

† A theory of dispersion, which, so far as its physical assumptions and results are concerned, resembles that described above, was published in the same year (1892) by Helmholtz, Berl. Ber., 1892, p. 1093, Ann d. Phys. xlviii (1893), pp. 389, 723. In this, as in Lorentz' theory, the incident light is supposed to excite sympathetic vibrations in the electric doublets which exist in the molecules of transparent bodies. Helmholtz' equations were, however, derived in a different way from those of Lorentz, being deduced from the Principle of Least Action. The final result is, as in Lorentz' theory, represented (when the effect of damping is neglected) by the Maxwell-Sellmeier formula. Helmholtz' theory was developed further by Reiff, Ann. d. Phys. lv (1895), p. 82.

In a theory of dispersion given by Planck, Berl. Ber., 1902, p. 470, the damping of the oscillations is assumed to be due to the loss of energy by radiation: so that no new constant is required in order to express it.

Lorentz, in his lectures on the *Theory of Electrons* (Leipzig, 1909), p. 141, suggested that the dissipative term in the equations of motion of dielectric electrons might be ascribed to the destruction of the regular vibrations of the electrons within a molecule by the collisions of the molecule with other molecules.

Some interesting references to the ideas of Hertz on the electromagnetic explanation of dispersion will be found in a memoir by Drude, Ann. d. Phys. (6) i (1900), p. 437.

‡ Cf. p. 283.

for the specific inductive capacity in terms of the number and circumstances of the electrons.*

Returning now to the case in which the dielectric is supposed to be in motion, the equation for the polarization may be written

$$4\pi c^2 \mathbf{P} = (\mu^2 - 1)\mathbf{E}'; \qquad (2)$$

from this equation, Fresnel's formula for the velocity of light in a moving dielectric may be deduced. For, let the axis of z be taken parallel to the direction of motion of the dielectric, which is supposed to be also the direction of propagation of the light; and, considering a plane-polarized wave, take the axis of x parallel to the electric vector, so that the magnetic vector must be parallel to the axis of y. Then equation (III) above becomes

$$-\partial H_y/\partial z = 4\pi \dot{D}_x + 4\pi w\, \partial P_x/\partial z;$$

equation (IV) becomes (assuming B equal to H, as is always the case in optics),

$$-\partial E_x/\partial z = \dot{H}_y.$$

The equation which defines the electric induction gives

$$D_x = (1/4\pi c^2) E_x + P_x;$$

and equations (1) and (2) give

$$4\pi c^2 P_x = (\mu^2 - 1)(E_x - wH_y).$$

Eliminating D_x, P_x, and H_y, we have

$$c^2 \frac{\partial^2 E_x}{\partial z^2} = \frac{\partial^2 E_x}{\partial t^2} + \left(\mu^2 - 1\right)\left(\frac{\partial}{\partial t} + w\frac{\partial}{\partial z}\right)^2 E_x;$$

or, neglecting w^2/c^2,

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{\mu^2}{c^2}\frac{\partial^2 E_x}{\partial t^2} + \frac{2w(\mu^2 - 1)}{c^2}\frac{\partial^2 E_x}{\partial t\, \partial z}.\dagger$$

Substituting $E_x = e^{n(t - z/V)\sqrt{-1}}$, so that V denotes the velocity of light in the moving dielectric with respect to the fixed aether we have

$$c^2 = \mu^2 V^2 - 2w(\mu^2 - 1)V,$$

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* Cf. p. 211.

† This equation was first given as a result of the theory of electrons by Lorentz in the last chapter of his memoir of 1892, Arch. Néerl. xxv, p. 525. It was also given by Larmor, Phil. Trans., clxxxv (1894), p. 821.

or (neglecting w^2/c^2)

$$V = \frac{c}{\mu} + \frac{\mu^2 - 1}{\mu^2} w,$$

which is the formula of Fresnel.* The hypothesis of Fresnel, that a ponderable body in motion carries with it the excess of aether which it contains as compared with space free from matter, is thus seen to be transformed in Lorentz' theory into the supposition that the polarized molecules of the dielectric, like so many small condensers, increase the dielectric constant, and that it is (so to speak) this augmentation of the dielectric constant which travels with the moving matter. One evident objection to Fresnel's theory, namely, that it required the relative velocity of aether and matter to be different for light of different colours, is thus removed; for the theory of Lorentz only requires that the dielectric constant should have different values for light of different colours, and of this a satisfactory explanation is provided by the theory of dispersion.

The correctness of Lorentz' hypothesis, as opposed to that of Hertz (in which the whole of the contained aether was supposed to be transported with the moving body), was afterwards confirmed by various experiments. In 1901 R. Blondlot† drove a current of air through a magnetic field, at right angles to the lines of magnetic force. The air-current was made to pass between the faces of a condenser, which were connected by a wire, so as to be at the same potential. An electromotive force $\mathbf{E}'$ would be produced in the air by its motion in the magnetic field; and, according to the theory of Hertz, this should produce an electric induction $\mathbf{D}$ of amount $(\epsilon/4\pi c^2)\,\mathbf{E}'$ (where ϵ denotes the specific inductive capacity of the air, which is practically unity); so that, according to Hertz, the faces of the condenser should become charged. According to Lorentz' theory, on the other hand, the electric induction $\mathbf{D}$ is determined by the equation

$$4\pi c^2 \mathbf{D} = \mathbf{E} + (\epsilon - 1)\,\mathbf{E}'$$

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* Cf. p. 117.

† Comptes Rendus cxxxiii (1901), p. 778.

where **E** denotes the electric force on a charge at rest, which is zero in the present case. Thus, according to Lorentz' theory, the charges on the faces would have only $(\epsilon - 1)/\epsilon$ of the values which they would have in Hertz' theory; that is, they would be practically zero. The result of Blondlot's experiment was in favour of the theory of Lorentz.

An experiment of a similar character was performed in 1905 by H. A. Wilson.* In this, the space between the inner and outer coatings of a cylindrical condenser was filled with the dielectric ebonite. When the coatings of such a condenser are maintained at a definite difference of potential, charges are induced on them; and if the condenser be rotated on its axis in a magnetic field whose lines of force are parallel to the axis, these charges will be altered, owing to the additional polarization which is produced in the dielectric molecules by their motion in the magnetic field. As before, the value of the additional charge according to the theory of Lorentz is $(\epsilon - 1)/\epsilon$ times its value as calculated by the theory of Hertz. The result of Wilson's experiments was, like that of Blondlot's, in favour of Lorentz.

The reconciliation of the electromagnetic theory with Fresnel's law of the propagation of light in moving bodies was a distinct advance. But the theory of the motionless aether was hampered by one difficulty: it was, in its original form, incompetent to explain the negative result of the experiment of Michelson and Morley.† The adjustment of theory to observation in this particular was achieved by means of a remarkable hypothesis which must now be introduced.

In the issue of "Nature" for June 16th, 1892,‡ Lodge mentioned that Fitz Gerald had communicated to him a new suggestion for overcoming the difficulty. This was, to suppose that the dimensions of material bodies are slightly altered when they are in motion relative to the aether. Five months afterwards, this hypothesis of Fitz Gerald's was adopted by

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* Phil. Trans. cciv (1905), p. 121.

† Cf. p 417.

‡ Nature, xlvi (1892), p. 165.

Lorentz, in a communication to the Amsterdam Academy;[*] after which it won favour in a gradually widening circle, until eventually it came to be generally taken as the basis of all theoretical investigations on the motion of ponderable bodies through the aether.

Let us first see how it explains Michelson's result. On the supposition that the aether is motionless, one of the two portions into which the original beam of light is divided should accomplish its journey in a time less than the other by w^2l/c^3, where w denotes the velocity of the earth, c the velocity of light, and l the length of each arm. This would be exactly compensated if the arm which is pointed in the direction of the terrestrial motion were shorter than the other by an amount $w^2l/2c^2$; as would be the case if the linear dimensions of moving bodies were always contracted in the direction of their motion in the ratio of $(1 - w^2/2c^2)$ to unity. This is Fitz Gerald's hypothesis of contraction. Since for the earth the ratio w/c is only

$$\frac{30 \text{ km./sec.}}{300{,}000 \text{ km./sec.}},$$

the fraction w^2/c^2 is only one hundred-millionth.

Several further contributions to the theory of electrons in a motionless aether were made in a short treatise[†] which was published by Lorentz in 1895. One of these related to the explanation of an experimental result obtained some years previously by Th. des Coudres,[‡] of Leipzig. Des Coudres had observed the mutual inductance of coils in different circumstances of inclination of their common axis to the direction of the earth's motion, but had been unable to detect any effect depending on the orientation. Lorentz now showed that this could be explained by considerations similar to those which

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[*] Verslagen d. Kon. Ak. van Wetenschappen, 1892–3, p. 74 (November 26th 1892).

[†] *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern*, von H. A. Lorentz; Leiden, E. J. Brill. It was reprinted by Teubner, of Leipzig, in 1906.

[‡] Ann. d. Phys. xxxviii (1889), p. 73.

Budde and Fitz Gerald* had advanced in a similar case; a conductor carrying a constant electric current and moving with the earth would exert a force on electric charges at relative rest in its vicinity, were it not that this force induces on the surface of the conductor itself a compensating electrostatic charge, whose action annuls the expected effect.

The most satisfactory method of discussing the influence of the terrestrial motion on electrical phenomena is to transform the fundamental equations of the aether and electrons to axes moving with the earth. Taking the axis of x parallel to the direction of the earth's motion, and denoting the velocity of the earth by w, we write

$$x = x_1 + wt, \quad y = y_1, \quad z = z_1,$$

so that (x_1, y_1, z_1) denote coordinates referred to axes moving with the earth. Lorentz completed the change of coordinates by introducing in place of the variable t a "local time" t_1, defined by the equation

$$t = t_1 + wx_1/c^2.$$

It is also necessary to introduce, in place of $\mathbf{d}$ and $\mathbf{h}$, the electric and magnetic forces relative to the moving axes: these are†

$$\mathbf{d}_1 = \mathbf{d} + [\mathbf{w}\,.\,\mathbf{h}]$$

$$\mathbf{h}_1 = \mathbf{h} + (1/c^2)\,[\mathbf{d}\,.\,\mathbf{w}];$$

and in place of the velocity $\mathbf{v}$ of an electron referred to the original fixed axes, we must introduce its velocity $\mathbf{v}_1$ relative to the moving axes, which is given by the equation

$$\mathbf{v}_1 = \mathbf{v} - \mathbf{w}.$$

The fundamental equations of the aether and electrons, referred to the original axes, are

$$\operatorname{div} \mathbf{d} = 4\pi c^2 \rho, \qquad \operatorname{curl} \mathbf{d} = -\dot{\mathbf{h}},$$

$$\operatorname{div} \mathbf{h} = 0, \qquad \operatorname{curl} \mathbf{h} = (1/c^2)\,\dot{\mathbf{d}} + 4\pi\rho\mathbf{v},$$

$$\mathbf{F} = \mathbf{d} + [\mathbf{v}\,.\,\mathbf{h}],$$

where $\mathbf{F}$ denotes the ponderomotive force on a particle carrying a unit charge.

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* Cf. p. 263.

† Cf. pp. 365, 366.

By direct transformation from the original to the new variables it is found that, when quantities of order w^2/c^2 and wv/c^2 are neglected, these equations take the form

$$\operatorname{div}_1 \mathbf{d}_1 = 4\pi c^2 \rho, \qquad \operatorname{curl}_1 \mathbf{d}_1 = -\,\partial \mathbf{h}_1/\partial t_1,$$
$$\operatorname{div}_1 \mathbf{h}_1 = 0, \qquad \operatorname{curl}_1 \mathbf{h}_1 = (1/c^2)\,\partial \mathbf{d}_1/\partial t_1 + 4\pi\rho \mathbf{v}_1,$$
$$\mathbf{F} = \mathbf{d}_1 + [\mathbf{v}_1 . \mathbf{h}_1],$$

where $\operatorname{div}_1 \mathbf{d}_1$ stands for

$$\frac{\partial d_{x_1}}{\partial x_1} + \frac{\partial d_{y_1}}{\partial y_1} + \frac{\partial d_{z_1}}{\partial z_1}.$$

Since these have the same form as the original equations, it follows that when terms depending on the square of the constant of aberration are neglected, all electrical phenomena may be expressed with reference to axes moving with the earth by the same equations as if the axes were at rest relative to the aether.

In the last chapter of the *Versuch* Lorentz discussed those experimental results which were as yet unexplained by the theory of the motionless aether. That the terrestrial motion exerts no influence on the rotation of the plane of polarization in quartz* might be explained by supposing that two independent effects, which are both due to the earth's motion, cancel each other; but Lorentz left the question undecided. Five years later Larmor† criticized this investigation, and arrived at the conclusion that there should be no first-order effect; but Lorentz‡ afterwards maintained his position against Larmor's criticism.

Although the physical conceptions of Lorentz had from the beginning included that of atomic electric charges, the analytical equations had hitherto involved ρ, the volume-density of electric charge; that is, they had been conformed to the hypothesis of a continuous distribution of electricity in space. It might hastily be supposed that in order to obtain an analytical theory of electrons, nothing more would be required than to modify the formulae by writing e (the charge of an electron) in place of $\rho\, dx\, dy\, dz$. That this is not the case was shown* a few years after the publication of the *Versuch.*

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* Cf. p. 416.

† Larmor, *Aether and Matter*, 1900.

‡ Proc. Amsterdam Acad. (English ed.), iv (1902), p. 669.

Consider, for example, the formula for the scalar potential at any point in the aether,

$$\phi = \iiint (\overline{\rho'}/r)\, dx'dy'dz',$$

where the bar indicates that the quantity underneath it is to have its retarded value.†

This integral, in which the integration is extended over all elements of space, must be transformed before the integration can be taken to extend over moving elements of charge. Let de' denote the sum of the electric charges which are accounted for under the heading of the volume-element $dx'dy'dz'$ in the above integral. This quantity de' is not identical with $\overline{\rho}'dx'dy'dz'$. For, to take the simplest case, suppose that it is required to compute the value of the potential-function for the origin at the time t, and that the charge is receding from the origin along the axis of x with velocity u. The charge which is to be ascribed to any position x is the charge which occupies that position at the instant $t - x/c$; so that when the reckoning is made according to intervals of space, it is necessary to reckon within a segment $(x_2 - x_1)$ not the electricity which at any one instant occupies that segment, but the electricity which at the instant $(t - x_2/c)$ occupies a segment $(x_2 - x'_1)$, where x'_1 denotes the point from which the electricity streams to x_1 in the interval between the instants $(t - x_2/c)$ and $(t - x_1/c)$. We have evidently

$$x_1 - x'_1 = u(x_2 - x_1)/c, \quad \text{or} \quad x_2 - x'_1 = (x_2 - x_1)(1 + u/c).$$

For this case we should therefore have

$$de' = \frac{x_2 - x'_1}{x_2 - x_1}\overline{\rho}'\, dx'\, dy'\, dz' = \left(1 + \frac{u}{c}\right)\overline{\rho}'\, dx'\, dy'\, dz'.$$

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* E. Wiechert, Arch. Néerl. (2) v (1900), p. 549. Cf. also A. Liénard, L'Éclairage élect. xvi (1898), pp. 5, 53, 106.

† Cf. p. 298.

In the general case, it is only necessary to replace u by the component of velocity of the electric charge in the direction of the radius vector from the point at which the potential is to be computed. This component may be written $v \cos (v \overset{\wedge}{.} r)$, where r is measured positively from the point in question to the charge, and v denotes the velocity of the charge. Thus

$$cde' = \{c + v \cos (v \overset{\wedge}{.} r)\} \bar{\rho}' \, dx' \, dy' \, dz',$$

and therefore

$$\phi = c \int \frac{de'}{c\bar{r} + (\bar{\mathbf{r}} \,.\, \bar{\mathbf{v}})},$$

where the integration is extended over all the charges in the field, and the bars over the letters imply that the position of the charge considered is that which it occupied at the instant $t - \bar{r}/c$. In the same way the vector-potential may be shown to have the value

$$\mathbf{a} = c \int \frac{\bar{\mathbf{v}} de'}{c\bar{r} + (\bar{\mathbf{r}} \,.\, \bar{\mathbf{v}})} \,.$$

Meanwhile the unsettled problem of the relative motion of earth and aether was provoking a fresh series of experimental investigations. The most interesting of these was due to Fitz Gerald,* who shortly before his death in February, 1901, commenced to examine the phenomena manifested by a charged electrical condenser, as it is carried through space in consequence of the terrestrial motion. On the assumption that a moving charge develops a magnetic field, there will be associated with the condenser a magnetic force at right angles to the lines of electric force and to the direction of the motion: magnetic energy must therefore be stored in the medium, when the plane of the condenser includes the direction of the drift; but when the plane of the condenser is at right angles to the terrestrial motion, the effects of the opposite charges neutralize each other. Fitz Gerald's original idea was that, in order to supply the magnetic energy, there must be a mechanical drag on the condenser at the moment of charging, similar to that which would be produced if the mass of a body at the surface of the earth were suddenly to become greater. Moreover, it was conjectured that the condenser, when freely suspended, would tend to move so as to assume the longitudinal orientation, which is that of maximum kinetic energy*: the transverse position would therefore be one of unstable equilibrium.

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* Fitz Gerald's *Scientific Writings*, p. 557.

For both effects a search was made by Fitz Gerald's pupil Trouton:† in the experiments designed to observe the turning couple, a condenser was suspended in a vertical plane by a fine wire, and charged. If the plane of the condenser were that of the meridian, about noon there should be no couple tending to alter the orientation, because the drift of aether due to the earth's motion would be at right angles to this plane; at any other hour, a couple should act. The effect to be detected was extremely small; for the magnetic force due to the motion of the charges would be of order w/c, where w denotes the velocity of the earth; so the magnetic energy of the system, which depends on the square of the force, would be of order $(w/c)^2$; and the couple, which depends on the derivate of this with respect to the azimuth, would therefore be likewise of the second order in (w/c).

No couple could be detected. As the energy of the magnetic field must be derived from some source, there seems to be no escape from the conclusion that the electrostatic energy of a charged condenser is diminished by the fraction $(w/c)^2$ of its amount when the condenser is moving with velocity w at right angles to its lines of electrostatic force. To explain this diminution, it is necessary to admit Fitz Gerald's hypothesis of contraction. The negative result of the experiment may be taken to indicate‡ that the kinetic potential of the system, when the Fitz Gerald contraction is taken into account as a constraint, is independent of the orientation of the plates with respect to the direction of the terrestrial motion.

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* Larmor, in Fitz Gerald's *Scientific Papers*, p 566.

† F. T. Trouton. Trans. Roy. Dub. Soc., April, 1902; F. T. Trouton and H. R. Noble, Phil. Trans. ccii (1903), p. 165.

‡ Cf. P. Langevin, Comptes Rendus, cxl (1905), p. 1171.

It may be remarked that the existence of the couple, had it been observed, would have demonstrated the possibility of drawing on the energy of the earth's motion for purposes of terrestrial utility.

The Fitz Gerald contraction of matter as it moves through the aether might conceivably be supposed to affect in some way the optical properties of the moving matter; for instance, transparent substances might become doubly refracting. Experiments designed to test this supposition were performed by Lord Rayleigh in 1902,* and by D. B. Brace in 1904†; but no double refraction comparable with the proportion $(w/c)^2$ of the single refraction could be detected. The Fitz Gerald contraction of a material body cannot therefore be of the same nature as the contraction which would be produced in the body by pressure, but must be accompanied by such concomitant changes in the relations of the molecules to the aether that an isotropic substance does not lose its simply refracting character.

By this time, indeed, the hypothesis of contraction, which originally had no direct connexion with electric theory, had assumed a new aspect. Lorentz, as we have seen,‡ had obtained the equations of a moving electric system by applying a transformation to the fundamental equations of the aether. In the original form of this transformation, quantities of higher order than the first in w/c were neglected. But in 1900 Larmor§ extended the analysis so as to include small quantities of the second order, and thereby discovered a remarkable connexion between the equations of transformation and the equations which represent Fitz Gerald's contraction. After this Lorentz* went further still, and obtained the transformation in a form which is exact to all orders of the small quantity w/c. In this form we shall now consider it.

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* Phil. Mag. iv (1902), p. 678.

† Phil. Mag. vii (1904), p. 317.

‡ Cf. p. 434. Cf. also Lorentz, Proc. Amsterdam Acad. (English ed.), i (1899), p. 427.

§ Larmor, *Aether and Matter*, p. 173.

The fundamental equations of the aether are

$$\operatorname{div} \mathbf{d} = 4\pi c^2 \rho, \qquad \operatorname{curl} \mathbf{d} = -\dot{\mathbf{h}},$$

$$\operatorname{div} \mathbf{h} = 0, \qquad \operatorname{curl} \mathbf{h} = \dot{\mathbf{d}}/c^2 + 4\pi\rho\mathbf{v}.$$

It is desired to find a transformation from the variables $x, y, z, t, \rho, \mathbf{d}, \mathbf{h}, \mathbf{v}$, to new variables $x_1, y_1, z_1, t_1, \rho_1, \mathbf{d}_1, \mathbf{h}_1, \mathbf{v}_1$, such that the equations in terms of these new variables may take the same form as the original equations, namely:

$$\operatorname{div}_1 \mathbf{d}_1 = 4\pi c^2 \rho_1, \qquad \operatorname{curl}_1 \mathbf{d}_1 = -\partial \mathbf{h}_1/\partial t_1,$$

$$\operatorname{div}_1 \mathbf{h}_1 = 0, \qquad \operatorname{curl}_1 \mathbf{h}_1 = (1/c^2)\, \partial \mathbf{d}_1/\partial t_1 + 4\pi \rho_1 \mathbf{v}_1.$$

Evidently one particular class of such transformations is that which corresponds to rotations of the axes of coordinates about the origin: these may be described as the linear homogeneous transformations of determinant unity which transform the expression $(x^2 + y^2 + z^2)$ into itself.

These particular transformations are, however, of little interest, since they do not change the variable t. But in place of them consider the more general class formed of all those linear homogeneous transformations of determinant unity in the variables x, y, z, ct, which transform the expression $(x^2 + y^2 + z - c^2t^2)$ into itself: we shall show that these transformations have the property of transforming the differential equations into themselves.

All transformations of this class may be obtained by the combination and repetition (with interchange of letters) of one of them, in which two of the variables—say, y and z—are unchanged. The equations of this typical transformation may easily be derived by considering that the equation of the rectangular hyperbola

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* Proc. Amsterdam Acad. (English ed.), vi, p. 809. Lorentz' work was completed in respect to the formulae which connect ρ_1, v_1, with ρ, v, by Einstein, Ann. d. Phys., xvii (1905), p. 891. It should be added that the transformation in question had been applied to the equation of vibratory motions many years before by Voigt, Gött. Nach. 1887, p. 41.

$$x^2 - (ct)^2 = 1$$

(in the plane of the variables x, ct) is unaltered when any pair of conjugate diameters are taken as new axes, and a new unit of length is taken proportional to the length of either of these diameters. The equations of transformation are thus found to be

$$\begin{aligned} x &= x_1 \cosh a + ct_1 \sinh a, & y &= y_1, \\ t &= t_1 \cosh a + (x_1/c) \sinh a, & z &= z_1, \end{aligned}$$

where a denotes a constant. The simpler equations previously given by Lorentz* may evidently be derived from these by writing w/c for $\tanh a$, and neglecting powers of w/c above the first. By an obvious extension of the equations given by Lorentz for the electric and magnetic forces, it is seen that the corresponding equations in the present transformation are

$$\left\{\begin{aligned} d_x &= d_{x_1}, \\ d_y &= d_{y_1} \cosh a + ch_{z_1} \sinh a, \\ d_z &= d_{z_1} \cosh a - ch_{y_1} \sinh a, \end{aligned}\right. \qquad \left\{\begin{aligned} h_x &= h_{x_1}, \\ h_y &= h_{y_1} \cosh a - (1/c) d_{z_1} \sinh a, \\ h_z &= h_{z_1} \cosh a + (1/c) d_{y_1} \sinh a. \end{aligned}\right.$$

The connexion between ρ and ρ_1 may be obtained in the following way. It is assumed that if a charge e is attached to a particle which occupies the position (ξ, η, ζ) at the instant t, an equal charge will be attached to the corresponding point (ξ_1, η_1, ζ_1) at the corresponding instant t_1 in the transformed system; so that a charge e' attached to an adjacent particle $(\xi + \Delta\xi, \eta + \Delta\eta, \zeta + \Delta\zeta)$ at the instant t will give rise in the derived system to a charge e' at the place

$$\left(\xi_1 + \frac{\partial \xi_1}{\partial \xi}\Delta\xi + \frac{\partial \xi_1}{\partial \eta}\Delta\eta + \frac{\partial \xi_1}{\partial \zeta}\Delta\zeta,\ \eta_1 + \frac{\partial \eta_1}{\partial \xi}\Delta\xi + \frac{\partial \eta_1}{\partial \eta}\Delta\eta + \frac{\partial \eta_1}{\partial \zeta}\Delta\zeta, \right.$$
$$\left. \zeta_1 + \frac{\partial \zeta_1}{\partial \xi}\Delta\xi + \frac{\partial \zeta_1}{\partial \eta}\Delta\eta + \frac{\partial \zeta_1}{\partial \zeta}\Delta\zeta\right)$$

at the instant

$$\left(t_1 + \frac{\partial t_1}{\partial \xi}\Delta\xi + \frac{\partial t_1}{\partial \eta}\Delta\eta + \frac{\partial t_1}{\partial \zeta}\Delta\zeta\right);$$

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* Cf. p. 434.

that is to say, at the place

$$(\xi_1 + \Delta\xi \cosh \alpha, \quad \eta_1 + \Delta\eta, \quad \zeta_1 + \Delta\zeta)$$

at the instant $(t_1 - \sinh \alpha \,.\, \Delta\xi/c)$. Thus at the instant t_1, this charge will occupy the position

$$(\xi_1 + \Delta\xi \cosh \alpha + \sinh \alpha \,.\, \Delta\xi \,.\, v_{x_1}/c, \quad \eta_1 + \Delta\eta + \sinh \alpha \,.\, \Delta\xi \,.\, v_{y_1}/c,$$
$$\zeta_1 + \Delta\zeta + \sinh \alpha \,.\, \Delta\xi \,.\, v_{z_1}/c).$$

The charges corresponding to those in the original system which were at the instant t contained in a volume $\Delta\xi\ \Delta\eta\ \Delta\zeta$ will therefore in the derived system at the instant t_1 occupy a volume

$$\begin{vmatrix} \cosh \alpha + \sinh \alpha \,.\, v_{x_1}/c \,. & 0 & 0 \\ \sinh \alpha \,.\, v_{y_1}/c & 1 & 0 \\ \sinh \alpha \,.\, v_{z_1}/c & 0 & 1 \end{vmatrix} \,.\ \Delta\xi\ \Delta\eta\ \Delta\zeta,$$

or,

$$(\cosh \alpha + \sinh \alpha \,.\, v_{x_1}/c)\ \Delta\xi\ \Delta\eta\ \Delta\zeta.$$

Thus if ρ_1 denote the volume-density of electric charge in the transformed system, we shall have

$$\rho_1 (\cosh \alpha + \sinh \alpha \,.\, v_{x_1}/c) = \rho;$$

this equation expresses the connexion between ρ_1 and ρ. We have moreover

$$v_x = \frac{\dfrac{\partial x}{\partial x_1} v_{x_1} + \dfrac{\partial x}{\partial y_1} v_{y_1} + \dfrac{\partial x}{\partial z_1} v_{z_1} + \dfrac{\partial x}{\partial t_1}}{\dfrac{\partial t}{\partial x_1} v_{x_1} + \dfrac{\partial t}{\partial y_1} v_{y_1} + \dfrac{\partial t}{\partial z_1} v_{z_1} + \dfrac{\partial t}{\partial t_1}}$$

$$= c \tanh \alpha + \frac{v_{x_1} \operatorname{sech} \alpha}{\cosh \alpha + v_{x_1} c^{-1} \sinh \alpha},$$

and similarly

$$v_y = \frac{v_{y_1}}{\cosh \alpha + v_{x_1} c^{-1} \sinh \alpha},$$

and

$$v_z = \frac{v_{z_1}}{\cosh \alpha + v_{x_1} c^{-1} \sinh \alpha}.$$

When the original variables are by direct substitution replaced by the new variables in the differential equations, the latter take the form

$$\operatorname{div}_1 \mathbf{d}_1 = 4\pi c^2\rho_1, \qquad \operatorname{curl}_1 \mathbf{d}_1 = -\,\partial\mathbf{h}_1/\partial t_1,$$

$$\operatorname{div}_1 \mathbf{h}_1 = 0, \qquad \operatorname{curl}_1 \mathbf{h}_1 = (1/c^2)\,\partial\mathbf{d}_1/\partial t_1 + 4\pi\rho_1\mathbf{v}_1\,;$$

that is to say, the fundamental equations of the aether retain their form unaltered, when the variables are subjected to the transformation which has been specified.

We are now in a position to show the connexion of this transformation with Fitz Gerald's hypothesis of contraction. Suppose that two material particles are moving along the axis of x with velocity $w = c \tanh a$. From the relation

$$v_x = c \tanh a + \frac{v_{x_1} \operatorname{sech} a}{\cosh a + v_{x_1} c^{-1} \sinh a},$$

it follows that v_{x_1} is zero for each of the particles, which implies that they are at rest relative to the new axes. Let x_1 and x'_1 denote their coordinates with respect to this latter system; then the coordinates of one particle at the instant t_1, referred to the original axes, will be given by the equations

$$x = x_1 \cosh a + ct_1 \sinh a, \qquad t = t_1 \cosh a + x_1 c^{-1} \sinh a\,;$$

and the coordinates of the other particle will be given by

$$x' = x'_1 \cosh a + ct_1 \sinh a, \qquad t' = t_1 \cosh a + x'_1 c^{-1} \sinh a\,;$$

so that at time t the latter particle will have the coordinate x'', where

$$\begin{aligned} x'' &= x' + w\,(t - t') \\ &= x'_1 \cosh a + ct_1 \sinh a + (x - x'_1) \sinh^2 a \operatorname{sech} a, \end{aligned}$$

which gives

$$x'' - x = (x'_1 - x_1)\,(1 - w^2/c^2)^{\frac{1}{2}}.$$

This equation shows that the distance between the particles in the system of measurement furnished by the original axes, with reference to which the particles were moving with velocity w, bears the ratio $(1 - w^2/c^2)^{\frac{1}{2}} : 1$ to their distance in the system of measurement furnished by the transformed axes, with reference to which the particles are at rest. But according to FitzGerald's hypothesis of contraction, when a material body is in motion relative to the aether, in a direction parallel to the axis of x, its dimensions parallel to this direction contract in precisely this ratio; so that the equation of the body, in terms of the coordinates x_1, y_1, z_1, which move with it, is unaltered. Thus the hypothesis of FitzGerald may be expressed by the statement that *the equations of the figures of ponderable bodies are covariant with respect to those transformations for which the fundamental equations of the aether are covariant.*

The covariance holds with respect to all linear homogeneous transformations in the variables (x, y, z, t), of determinant unity, which transform the expression $(x^2 + y^2 + z^2 - c^2t^2)$ into itself. This group comprises an infinite number of transformations; so that there are an infinite number of sets of variables resembling (x_1, y_1, z_1, t_1), of which any one set (x_r, y_r, z_r, t_r) can be derived from any other set (x_s, y_s, z_s, t_s) by a transformation of the group; among the sets we must of course include the original set of coordinates (x, y, z, t). But hitherto we have proceeded on the assumption that the original set (x, y, z, t) is entitled to a primacy among all the other sets, since the axes (x, y, z) have been supposed to possess the special property of having no motion relative to the aether, and the time represented by the variable t has been understood to be a definite physical quantity. The other sets of variables (x_r, y_r, z_r, t_r) have been regarded merely as symbols convenient for use in problems relating to moving bodies, but not as corresponding to physical entities in the same degree as (x, y, z, t). We must now inquire whether this view is justified.

The question amounts to asking whether absolute position in space, or at any rate absolute fixity relative to the aether, is something which can be brought within the bounds of human knowledge.

It is well known that the science of dynamics, as founded on Newton's laws of motion, does not supply any criterion by which rest may be distinguished from uniform motion; for if the laws of motion are applicable when the position of bodies is referred to any particular set of axes, they will be equally applicable when position is referred to any other set of axes which have a uniform motion of translation relative to these.

The older theories of electrostatics, magnetism, and electrodynamics, which are based on the conception of action at a distance, are concerned only with relative configurations and motions, and are therefore useless in the search for a basis of absolute reckoning.

But the existence of an aether, which is postulated in the undulatory theory of light, seems at first sight to involve the conceptions of rest and motion relative to it, and thus to afford a means of specifying absolute position. Suppose, for instance, that a disturbance is generated at any point in free aether; this disturbance will spread outwards in the form of a sphere; and the centre of this sphere will for all subsequent time occupy an unchanged position relative to the aether. In this way, or in many other ways, we might hope to determine, by electrical or optical experiments, the velocity of the earth relative to the aether.

The failure of such experiments as had been tried led FitzGerald* to suggest that the dimensions of material bodies undergo contraction when the bodies are in motion relative to the aether. By the transformation of Lorentz and Larmor, as we have seen, this hypothesis came to be expressed in a new form; namely that the equation of the figure of the body, referred to a frame of reference moving with it, is always the same, but that frames of reference which are in motion relative to each other are based on different standards of length and time. This way of regarding the matter brings into prominence the fundamental questions involved. Before speaking of lengths and velocities, it is necessary to examine the nature of systems of measurement of space and time.

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* Cf. p. 432.

Of the events with which Natural Philosophy is concerned, each is perceived to happen at some definite location at some definite moment. When a material object has been observed to occupy a certain position at a certain instant, the same object may again be observed at a subsequent instant; but it is impossible to determine whether the object is or is not in the same position, since there is no obvious means of preserving the identity of any location from one moment to another. The physicist, however, finds it convenient to construct a framework of axes in space and time for the purpose of fitting his experiences into an orderly arrangement; and the question at issue is whether experience furnishes the means of determining a framework completely and uniquely by absolute properties, or whether the selection inevitably rests on arbitrary choice and accidental circumstance.

In attempting to answer this question, it may first be observed that the choice is always made so as to simplify the description of natural phenomena as much as possible; thus, the variable which is to measure time is so chosen that its increment in the interval between any two consecutive beats of a pendulum is the same as its increment in the interval between any other two consecutive beats. If the selection of the four variables (x, y, z, t) is well made, it should be possible to express the laws of nature by statements of a simple character, e.g., that a body isolated from the influence of external agents moves through equal intervals of space in equal intervals of time.

Accepting, then, the principle that the framework of axes is to be chosen so as to furnish the simplest possible expression of the natural laws, it becomes of importance to determine which of the natural laws are entitled, by reason of their primary importance, to receive the greatest consideration.

Now many indications point to the probability that the various types of forces which are observed in ponderable bodies—forces of cohesion, of chemical union, and so forth—are ultimately electric in their nature. Such an assumption would have the great advantage of explaining the contraction postulated by Fitz Gerald, since it would represent the contraction as actually produced by the motion. But if this assumption be correct, the theory of electricity and aether is without doubt the fundamental theory of Natural Philosophy; and the framework of space and time should be chosen with a view chiefly to the expression of electrical phenomena. This may most naturally be done by stipulating that the wave-fronts of disturbances generated in free aether shall, in the system of length and time adopted, be accounted spheres whose centres are at the origins of disturbance and whose radii are proportional to the times elapsed since their initiation. Referred to axes of (x, y, z, t) which satisfy these conditions, the fundamental equations of the electric field assume the form which has been taken as the basis of all our theoretical investigations.

Imagine now a distant star which is moving with a uniform velocity w or $c \tanh a$ relative to this framework (x, y, z, t). The theorem of transformation shows that there exists another framework (x_1, y_1, z_1, t_1), with respect to which the star is at rest, and in which moreover the condition laid down regarding the wave-surface is satisfied. This framework is peculiarly fitted for the representation of the phenomena which happen on the star; whose inhabitants would therefore naturally adopt it as their system of space and time. Beings, on the other hand, who dwell on a body which is at rest with respect to the axes (x, y, z, t) would prefer to use the latter system; and from the point of view of the universe at large, either of these systems is as good as the other. The equations of motion of the aether are the same with respect to both sets of coordinates, and therefore neither can claim to possess the only property which could confer a primacy—namely, an absolute relation to the aether.*

To sum up, we may say that the phenomena whose study is the object of Natural Philosophy take place each at a definite location at a definite moment; the whole constituting a four-dimensional world of space and time. To construct a set of axes of space and time is equivalent to projecting this four-dimensional world into a three-dimensional world of space and a one-dimensional world of time; and this projection may be performed in an infinite number of ways, each of which is distinguished from the others only by characteristics merely arbitrary and accidental.*

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* This was first clearly expressed by Einstein, Ann. d. Phys. xvii (1905), p. 891.

In order to represent natural phenomena without introducing this contingent element, it would be necessary to abandon the customary three-dimensional system of coordinates, and to operate in four dimensions. Analysis of this kind has been devised, and has been applied to the theory of the aether; but its development belongs to the twentieth century, and consequently falls outside the scope of the present work.

From what has been said, it will be evident that, in the closing years of the nineteenth century, electrical investigation was chiefly concerned with systems in motion. The theory of electrons was, however, applied with success in other directions, and notably to the explanation of a new experimental discovery.

The last recorded observation of Faraday† was an attempt to detect changes in the period, or in the state of polarization, of the light emitted by a sodium flame, when the flame was placed in a strong magnetic field. No result was obtained; but the conviction that an effect of this nature remained to be discovered was felt by many of his successors. Tait‡ examined the influence of a magnetic field on the selective absorption of light; impelled thereto, as he explained, by theoretical considerations. For from the phenomenon of magnetic rotation it may be inferred§ that rays circularly polarized in opposite senses are propagated with different velocities in the magnetized medium; and therefore if only those rays are absorbed which have a certain definite wave-length in the medium, the period of the ray absorbed from a beam of circularly polarized white light will not be the same when the polarization is right-handed as when it is left-handed. "Thus," wrote Tait, "what was originally a single dark absorption-line might become a double line."

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* Cf. H. Minkowski, *Raum und Zeit.*: Leipzig, 1909.

† Bence Jones' *Life of Faraday*, ii, p. 449.

‡ Proc. R.S. Edinb. ix (1875), p. 118.

§ Cf. pp. 174, 216.

The effect anticipated under different forms by Faraday and Tait was discovered, towards the end of 1896, by P. Zeeman.* Repeating Faraday's procedure, he placed a sodium flame between the poles of an electromagnet, and observed a widening of the D-lines in the spectrum when the magnetizing current was applied.

A theoretical explanation of the phenomenon was immediately furnished to Zeeman by Lorentz.† The radiation is supposed to be emitted by electrons which describe orbits within the sodium atoms. If e denote the charge of an electron of mass m, the ponderomotive force which acts on it by virtue of the external magnetic field is $e\,[\dot{\mathbf{r}}\,.\,\mathbf{K}]$, where $\mathbf{K}$ denotes the magnetic force and $\mathbf{r}$ denotes the displacement of the electron from its position of equilibrium; and therefore, if the force which restrains the electron in its orbit be $\kappa^2\mathbf{r}$, the equation of motion of the electron is

$$m\ddot{\mathbf{r}} + \kappa^2\mathbf{r} = e\,[\dot{\mathbf{r}}\,.\,\mathbf{K}].$$

The motion of the electron may (as is shown in treatises on dynamics) be represented by the superposition of certain particular solutions called *principal oscillations*, whose distinguishing property is that they are periodic in the time. In order to determine the principal oscillations, we write $\mathbf{r}_0 e^{nt\sqrt{-1}}$ for $\mathbf{r}$, where $\mathbf{r}_0$ denotes a vector which is independent of the time, and n denotes the frequency of the principal oscillation: substituting in the equation, we have

$$(\kappa^2 - mn^2)\,\mathbf{r}_0 = en\sqrt{-1}\,[\mathbf{r}_0\;\;\mathbf{K}].$$

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* Zittingsverslagen der Akad. v. Wet. te Amsterdam v (1896), pp. 181, 242; vi (1897), pp. 13, 99; Phil. Mag. (5) xliii (1897), p. 226.

† Phil. Mag. xliii (1897), p. 232.

This equation may be satisfied either (1) if $\mathbf{r}_0$ is parallel to $\mathbf{K}$, in which case it reduces to

$$\kappa^2 - mn^2 = 0,$$

so that n has the value $\kappa m^{-\frac{1}{2}}$, or (2) if $\mathbf{r}_0$ is at right angles to $\mathbf{K}$, in which case by squaring both sides of the equation we obtain the result

$$(\kappa^2 - mn^2)^2 = e^2 n^2 K^2,$$

which gives for n the approximate values $\kappa m^{-\frac{1}{2}} \pm eK/2m$.

When there is no external magnetic field, so that $\mathbf{K}$ is zero, the three values of n which have been obtained all reduce to $\kappa m^{-\frac{1}{2}}$, which represents the frequency of vibration of the emitted light before the magnetic field is applied. When the field is applied, this single frequency is replaced by the three frequencies $\kappa m^{-\frac{1}{2}}$, $\kappa m^{-\frac{1}{2}} + eK/2m$, $\kappa m^{-\frac{1}{2}} - eK/2m$; that is to say, the single line in the spectrum is replaced by three lines close together. The apparatus used by Zeeman in his earliest experiments was not of sufficient power to exhibit this triplication distinctly, and the effect was therefore described at first as a widening of the spectral lines.*

We have seen above that the principal oscillation of the electron corresponding to the frequency $\kappa m^{-\frac{1}{2}}$ is performed in a direction parallel to the magnetic force $\mathbf{K}$. It will therefore give rise to radiation resembling that of a Hertzian vibrator, and the electric vector of the radiation will be parallel to the lines of force of the external magnetic field. It follows that when the light received in the spectroscope is that which has been emitted in a direction at right angles to the magnetic field, this constituent (which is represented by the middle line of the triplet in the spectrum) will appear polarized in a plane at right angles to the field; but when the light received in the spectroscope is that which has been emitted in the direction of the magnetic force, this constituent will be absent.

We have also seen that the principal oscillations of the electron corresponding to the frequencies $\kappa m^{-\frac{1}{2}} \pm eK/2m$ are performed in a plane at right angles to the magnetic field **K**. In order to determine the nature of these two principal oscillations, we observe that it is possible for the electron to describe a circular orbit in this plane, if the radius of the orbit be suitably chosen; for in a circular motion the forces $\kappa^2\mathbf{r}$ and $e[\dot{\mathbf{r}}\,.\,\mathbf{K}]$ would be directed towards the centre of the circle; and it would therefore be necessary only to adjust the radius so that these furnish the exact amount of centripetal force required. Such a motion, being periodic, would be a principal oscillation. Moreover, since the force $e\,[\dot{\mathbf{r}}\,.\,\mathbf{K}]$ changes sign when the sense of the movement in the circle is reversed, it is evident that there are two such circular orbits, corresponding to the two senses in which the electron may circulate; these must, therefore, be no other than the two principal oscillations of frequencies $\kappa m^{-\frac{1}{2}} \pm eK/2m$. When the light received in the spectroscope is that which has been emitted in a direction at right angles to the external magnetic field, the circles are seen edgewise, and the light appears polarized in a plane parallel to the field; but when the light examined is that which has been emitted in a direction parallel to the external magnetic force, the radiations of frequencies $\kappa m^{-\frac{1}{2}} \pm eK/2m$ are seen to be circularly polarized in opposite senses. All these theoretical conclusions have been verified by observation.

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* Later observations, with more powerful apparatus, have shown that the primitive spectral line is frequently replaced by more than three components.

It was found by Cornu* and by C. G. W. König† that the more refrangible component (i.e., the one whose period is shorter than that of the original radiation) has its circular vibration in the same sense as the current in the electromagnet. From this it may be inferred that the vibration must be due to a resinously charged electron; for let the magnetizing current and the electron be supposed to circulate round the axis of z in the direction in which a right-handed screw must turn in order to progress along the positive direction of the axis of z; then the magnetic force is directed positively along the axis of z, and, in order that the force on the electron may be directed inward to the axis of z (so as to shorten the period), the charge on the electron must be negative.

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* Comptes Rendus, cxxv (1897), p. 555.

† Ann. d. Phys. lxii (1897), p. 240.

The value of e/m for this negative electron may be determined by measurement of the separation between the components of the triplet in a magnetic field of known strength; for, as we have seen, the difference of the frequencies of the outer components is eK/m. The values of e/m thus determined agree well with the estimations* of e/m for the corpuscles of cathode rays.

The phenomenon discovered by Zeeman is closely related to the magnetic rotation of the plane of polarization of light.† Both effects may be explained by supposing that the molecules of material bodies contain electric systems which possess natural periods of vibration, the simplest example of such a system being an electron which is attracted to a fixed centre with a force proportional to the distance. Zeeman's effect represents the influence of an external magnetic field on the *free* oscillations of these electric systems, while Faraday's effect represents the influence of the external magnetic field on the *forced* oscillations which the systems perform under the stimulus of incident light. The latter phenomenon may be analysed without difficulty on these principles, the equation of motion of one of the electrons being taken in the form

$$m\ddot{\mathbf{r}} + \kappa^2\mathbf{r} = e\mathbf{E} + e[\dot{\mathbf{r}} \, . \, \mathbf{H}],$$

where m denotes the mass and e the charge of the electron, $\mathbf{r}$ its distance from the centre of force, $\kappa^2\mathbf{r}$ the restitutive force, $\mathbf{E}$ and $\mathbf{H}$ the electric and magnetic forces. When the electron performs forced oscillations under the influence of light of frequency n, this equation becomes

$$(\kappa^2 - mn^2)\,\mathbf{r} = e\mathbf{E} + e[\dot{\mathbf{r}} \, . \, \mathbf{H}].$$

The influence of the magnetic force on the motion of the electron is small compared with the influence of the electric force, i.e. the second term on the right is small compared with the first term; so in the second term we may replace $\mathbf{r}$ by its value as found from the first term, namely, $e\mathbf{E}/(\kappa^2 - mn^2)$. The equation thus becomes

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* Cf. p. 405.

† Cf. pp. 213–216, 307–309, 367–370.

$$\mathbf{r} = \frac{e\mathbf{E}}{\kappa^2 - mn^2} + \frac{e^2}{(\kappa^2 - mn^2)^2}[\dot{\mathbf{E}} . \mathbf{H}].$$

If **P** denote* the electric moment per unit volume, we have

P = e**r** × the number of such systems in unit volume of the medium;

so **P** must be of the form

$$\frac{\varepsilon - 1}{4\pi c^2}\mathbf{E} + \sigma[\dot{\mathbf{E}} . \mathbf{H}],$$

where ε evidently represents the dielectric constant of the medium, and σ is the coefficient which measures the magnetic rotatory power. In the magneto-optic term we may replace **H** by **K**, the external magnetic force, since this is large compared with the magnetic force of the luminous vibrations. Thus if **D** denote the electric induction, we have

$$\mathbf{D} = \varepsilon\mathbf{E}/4\pi c^2 + \sigma[\dot{\mathbf{E}} . \mathbf{K}].$$

Combining this with the usual electromagnetic equations,

$$\text{curl } \mathbf{H} = 4\pi\dot{\mathbf{D}},$$

$$\text{curl } \mathbf{E} = -\dot{\mathbf{H}},$$

we have

$$-\text{curl curl } \mathbf{E} = \varepsilon\ddot{\mathbf{E}}/c^2 + 4\pi\sigma[\dddot{\mathbf{E}} . \mathbf{K}].$$

When a plane wave of light is propagated through the medium in the direction of the lines of magnetic force, and the axis of x is taken parallel to this direction, the equation gives

$$\begin{cases} \dfrac{\partial^2 E_y}{\partial x^2} = \dfrac{\varepsilon}{c^2}\dfrac{\partial^2 E_y}{\partial t^2} + 4\pi\sigma K\dfrac{\partial^3 E_z}{\partial t^3}, \\ \dfrac{\partial^2 E_z}{\partial x^2} = \dfrac{\varepsilon}{c^2}\dfrac{\partial^2 E_z}{\partial t^2} - 4\pi\sigma K\dfrac{\partial^3 E_y}{\partial t^3}; \end{cases}$$

and these equations, as we have seen,† are competent to explain the rotation of the plane of polarization.

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* Cf. p. 428.

† Cf. p 215.

From the occurrence of the factor $(\kappa^2 - mn^2)$ in the denominator of the expression for the magneto-optic constant σ, it may be inferred that the magnetic rotation will be very large for light whose period is nearly the same as a free period of vibration of the electrons. A large rotation is in fact observed* when plane-polarized light, whose frequency differs but little from the frequencies of the D-lines, is passed through sodium vapour in a direction parallel to the lines of magnetic force.

The optical properties of metals may be explained, according to the theory of electrons, by a slight extension of the analysis which applies to the propagation of light in transparent substances. It is, in fact, only necessary to suppose that some of the electrons in metals are free instead of being bound to the molecules: a supposition which may be embodied in the equations by assuming that an electric force **E** gives rise to a polarization **P**, where

$$\mathbf{E} = \alpha\ddot{\mathbf{P}} + \beta\dot{\mathbf{P}} + \gamma\mathbf{P};$$

the term in α represents the effect of the inertia of the electrons; the term in β represents their ohmic drift; and the term in γ represents the effect of the restitutive forces where these exist. This equation is to be combined with the customary electromagnetic equations

$$\text{curl}\,\mathbf{H} = \dot{\mathbf{E}}/c^2 + 4\pi\dot{\mathbf{P}}, \qquad -\text{curl}\,\mathbf{E} = \dot{\mathbf{H}}.$$

In discussing the propagation of light through the metal, we may for convenience suppose that the beam is plane-polarized and propagated parallel to the axis of z, the electric vector being parallel to the axis of x. Thus the equations of motion reduce to

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* The phenomenon was first observed by D. Macaluso and O. M. Corbino, Comptes Rendus, cxxvii (1898), p. 548, Rend. Lincei (5) vii (2) (1898), p. 293. The theoretical explanation was supplied by W. Voigt, Gött. Nach., 1898, p. 349, Ann. d. Phys. lxvii (1899), p. 345. Cf. also P. Zeeman, Proc. Amst. Acad. v (1902), p. 41, and J. J. Hallo, Arch. Néerl. (2) x (1905), p. 148.

Voigt also predicted that if plane-polarized light, of period nearly the same as that of the D radiation, were passed through sodium vapour in a magnetic field, in a direction perpendicular to the lines of magnetic force, the velocity of propagation would be found to depend on the orientation of the plane of polarization, so that the sodium vapour would behave as a uniaxal crystal. This prediction was confirmed experimentally by Voigt and Wiechert: cf. Voigt, Gött. Nach., 1898, p. 355: Ann. d. Phys. lxvii. (1899), p. 345. Cf. also A. Cotton, Comptes Rendus, cxxviii (1899), p. 294, and J. Geest, Arch. Néerl. (2), x (1905), p. 291.

$$\begin{cases} \dfrac{\partial^2 E_x}{\partial z^2} = \dfrac{1}{c^2}\dfrac{\partial^2 E_x}{\partial t^2} + 4\pi \dfrac{\partial^2 P_x}{\partial t^2}, \\ E_x = a\dfrac{\partial^2 P_x}{\partial t^2} + \beta\dfrac{\partial P_x}{\partial t} + \gamma P. \end{cases}$$

For E_x and P_x we may substitute exponential functions of

$$n\sqrt{-1}\,(t - z\mu/c),$$

where n denotes the frequency of the light, and μ the quasi-index of refraction of the metal: the equations then give at once

$$(\mu^2 - 1)\,(-an^2 + \beta n\sqrt{-1} + \gamma) = 4\pi c^2.$$

Writing $\nu\,(1 - \kappa\sqrt{-1})$ for μ, so that ν is inversely proportional to the velocity of light in the medium, and κ denotes the coefficient of absorption, and equating separately the real and imaginary parts of the equation, we obtain

$$\begin{cases} \nu^2\,(1 - \kappa^2) = 1 + \dfrac{4\pi c^2\,(\gamma - an^2)}{\beta^2 n^2 + (\gamma - an^2)^2}; \\ \nu^2\kappa = \dfrac{2\pi c^2\beta n}{\beta^2 n^2 + (\gamma - an^2)^2}. \end{cases}$$

When the wave-length of the light is very large, the inertia represented by the constant a has but little influence, and the equations reduce to those of Maxwell's original theory* of the propagation of light in metals. The formulae were experimentally confirmed for this case by the researches of E. Hagen and H. Rubens† with infra-red light; a relation being thus established between the ohmic conductivity of a metal and its optical properties with respect to light of great wave-length.

When, however, the luminous vibrations are performed more rapidly, the effect of the inertia becomes predominant; and if the constants of the metal are such that, for a certain range of values of n, $\nu^2\kappa$ is small, while $\nu^2(1-\kappa^2)$ is negative, it is evident that, for this range of values of n, ν will be small and κ large, i.e., the properties of the metal will approach those of ideal silver.* Finally, for indefinitely great values of n, $\nu^2\kappa$ is small and $\nu^2(1-\kappa^2)$ is nearly unity, so that ν tends to unity and κ to zero: an approximation to these conditions is realized in the X-rays.†

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* Cf. p. 290.

† Berlin Sitzungsber., 1903, pp. 269, 410; Ann. d. Phys. xi (1903), p. 873; Phil. Mag. vii (1904), p. 157.

In the last years of the nineteenth century, attempts were made to form more definite conceptions regarding the behaviour of electrons within metals. It will be remembered that the original theory of electrons had been proposed by Weber‡ for the purpose of explaining the phenomena of electric currents in metallic wires. Weber, however, made but little progress towards an electric theory of metals; for being concerned chiefly with magneto-electric induction and electromagnetic ponderomotive force, he scarcely brought the metal into the discussion at all, except in the assumption that electrons of opposite signs travel with equal and opposite velocities relative to its substance. The more comprehensive scheme of his successors half a century afterwards aimed at connecting in a unified theory all the known electrical properties of metals, such as the conduction of currents according to Ohm's law, the thermo-electric effects of Seebeck, Peltier, and W. Thomson, the galvano-magnetic effect of Hall, and other phenomena which will be mentioned subsequently.

The later investigators, indeed, ranged beyond the group of purely electrical properties, and sought by aid of the theory of electrons to explain the conduction of heat. The principal ground on which this extension was justified was an experimental result obtained in 1853 by G. Wiedemann and R. Franz,§ who found that at any temperature the ratio of the thermal conductivity of a body to its ohmic conductivity is approximately the same for all metals, and that the value of this ratio is proportional to the absolute temperature. In fact, the conductivity of a pure metal for heat is almost independent of the temperature; while the electric conductivity varies in inverse proportion to the absolute temperature, so that a pure metal as it approaches the absolute zero of temperature tends to assume the character of a perfect conductor. That the two conductivities are closely related was shown to be highly probable by the experiments of Tait, in which pieces of the same metal were found to exhibit variations in ohmic conductivity exactly parallel to variations in their thermal conductivity.

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* Cf. p. 179.

† Models illustrating the selective reflexion and absorption of light by metallic bodies and by gases were discussed by H. Lamb, Mem. and Proc. Manchester Lit. and Phil. Soc. xlii (1898), p. 1; Proc. Lond. Math. Soc. xxxii (1900), p. 11; Trans. Camb. Phil. Soc. xviii (1900), p. 348.

‡ Cf. p. 226.

§ Ann. d. Phys. lxxxix (1853), p. 497.

The attempt to explain the electrical and thermal properties of metals by aid of the theory of electrons rests on the assumption that conduction in metals is more or less similar to conduction in electrolytes; at any rate, that positive and negative charges drift in opposite directions through the substance of the conductor under the influence of an electric field. It was remarked in 1888 by J. J. Thomson,* who must be regarded as the founder of the modern theory, that the differences which are perceived between metallic and electrolytic conduction may be referred to special features in the two cases, which do not affect their general resemblance. In electrolytes the carriers are provided only by the salt, which is dispersed throughout a large inert mass of solvent; whereas in metals it may be supposed that every molecule is capable of furnishing carriers. Thomson, therefore, proposed to regard the current in metals as a series of intermittent discharges, caused by the rearrangement of the constituents of molecular systems—a conception similar to that by which Grothuss† had pictured conduction in electrolytes. This view would, as he showed, lead to a general explanation of the connexion between thermal and electrical conductivities.

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* J. J. Thomson, *Applications of Dynamics to Physics and Chemistry*, 1888, p. 296. Cf. also Giese, Ann. d. Phys. xxxvii (1889), p. 576.

† Cf. p. 78.

Most of the later writers on metallic conduction have preferred to take the hypothesis of Arrhenius* rather than that of Grothuss as a pattern; and have therefore supposed the interstices between the molecules of the metal to be at all times swarming with electric charges in rapid motion. In 1898 E. Riecke† effected an important advance by examining the consequences of the assumption that the average velocity of this random motion of the charges is nearly proportional to the square root of the absolute temperature T. P. Drude‡ in 1900 replaced this by the more definite assumption that the kinetic energy of each moving charge is equal to the average kinetic energy of a molecule of a perfect gas at the same temperature, and may therefore be expressed in the form qT, where q denotes a universal constant.

In the same year J. J. Thomson§ remarked that it would accord with the conclusions drawn from the study of ionization in gases to suppose that the vitreous and resinous charges play different parts in the process of conduction: the resinous charges may be conceived of as carried by simple negative corpuscles or electrons, such as constitute the cathode rays: they may be supposed to move about freely in the interstices between the atoms of the metal. The vitreous charges, on the other hand, may be regarded as more or less fixed in attachment to the metallic atoms. According to this view the transport of electricity is due almost entirely to the motion of the negative charges.

An experiment which was performed at this time by Riecke‖ lent some support to Thomson's hypothesis. A cylinder of aluminium was inserted between two cylinders of copper in a circuit, and a current was passed for such a time that the amount of copper deposited in an electrolytic arrangement would have amounted to over a kilogramme. The weight of each of the three cylinders, however, showed no measurable change; from which it appeared unlikely that metallic conduction is accompanied by the transport of metallic ions.

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* Cf. p. 384.

† Gött. Nach., 1898, pp. 48, 137. Ann. d. Phys. lxvi (1898), pp. 353, 545, 1199; ii. (1900), p. 835.

‡ Ann. d. Phys. (4) i (1900), p. 566; iii (1900), p. 369; vii (1902), p. 687.

§ Rapports prés. au Congrès de Physique, Paris, 1900, iii, p. 138.

‖ Phys. Zeitsch. iii (1901), p. 639.

The ideas of Thomson, Riecke, and Drude were combined by Lorentz* in an investigation which, as it is the most complete, will here be given as the representative of all of them.

It is supposed that the atoms of the metal are fixed, and that in the interstices between them a large number of resinous electrons are in rapid motion. The mutual collisions of the electrons are disregarded, so that their collisions with the fixed atoms alone come under consideration; these are regarded as analogous to collisions between moving and fixed elastic spheres.

The flow of heat and electricity in the metal is supposed to take place in a direction parallel to the axis of x, so that the metal is in the same condition at all points of any plane perpendicular to this direction; and the flow is supposed to be steady, so that the state of the system is independent of the time.

Consider a slab of thickness dx and of unit area; and suppose that the number of electrons in this slab whose x-components of velocity lie between u and $u + du$, whose y-components of velocity lie between v and $v + dv$, and whose z-components of velocity lie between w and $w + dw$, is

$$f(u, v, w, x)\, dx\, du\, dv\, dw.$$

One of these electrons, supposing it to escape collision, will in the interval of time dt travel from (x, y, z) to $(x + u\, dt, y + v dt, z + w dt)$: and its x-component of velocity will at the end of the interval be increased by an amount $e\, E dt/m$, if m and e denote its mass and charge, and E denotes the electric force. Suppose that the number of electrons lost to this group by collisions in the interval dt is $a\, dx\, du\, dv\, dw\, dt$, and that the number added to the group by collisions in the same interval is $b\,dx\,du\,dv\,dw\,dt$. Then we have

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* Amsterdam Proceedings (English edition) vii (1904-1905), pp. 438, 585, 684

$$f(u, v, w, x) + (b - a)\,dt = f(u + eE\,dt/m, v, w, x + u\,dt),$$

and therefore

$$b - a = \frac{eE}{m}\frac{\partial f}{\partial u} + u\frac{\partial f}{\partial x}.$$

Now, the law of distribution of velocities which Maxwell postulated for the molecules of a perfect gas at rest is expressed by the equation

$$f = \pi^{-\frac{3}{2}} \alpha^{-3} N e^{-\frac{r^2}{\alpha^2}},$$

where N denotes the number of moving corpuscles in unit volume, r denotes the resultant velocity of a corpuscle (so that $r^2 = u^2 + v^2 + w^2$), and α denotes a constant which specifies the average intensity of agitation, and consequently the temperature. It is assumed that the law of distribution of velocities among the electrons in a metal is nearly of this form; but a term must be added in order to represent the general drifting of the electrons parallel to the axis of x. The simplest assumption that can be made regarding this term is that it is of the form

$$u \times \text{a function of } r \text{ only};$$

we shall, therefore, write

$$f = N\pi^{-\frac{3}{2}} \alpha^{-3} e^{-\frac{r^2}{\alpha^2}} + u\chi(r).$$

The value of $\chi(r)$ may now be determined from the equation

$$b - a = \frac{eE}{m}\frac{\partial f}{\partial u} + u\frac{\partial f}{\partial x};$$

for on the left-hand side, the Maxwellian term

$$\pi^{-\frac{3}{2}} \alpha^{-3} N e^{-\frac{r^2}{\alpha^2}}$$

would give a zero result, since b is equal to a in Maxwell's system; thus $b - a$ must depend solely on the term $u\chi(r)$; and an examination of the circumstances of a collision, in the manner of the kinetic theory of gases, shows that $(b - a)$ must have the form $-ur\chi(r)/l$, where l denotes a constant which is closely related to the mean free path of the electrons. In the terms on the right-hand side of the equation, on the other hand, Maxwell's term gives a result different from zero; and in comparison with this we may neglect the terms which arise from $u\chi(r)$. Thus we have

$$-\frac{ur\chi(r)}{l} = \left(\frac{eE}{m}\frac{\partial}{\partial u} + u\frac{\partial}{\partial x}\right)\frac{N}{\pi^{\frac{3}{2}}\alpha^3}e^{-\frac{r^2}{\alpha^2}},$$

or

$$u\chi(r) = \frac{lu}{\pi^{\frac{3}{2}}r}\cdot e^{-\frac{r^2}{\alpha^2}}\cdot\left\{\frac{2eNE}{ma^5} - \frac{d}{dx}\left(\frac{N}{\alpha^3}\right) - \frac{2Nr^2}{\alpha^6}\frac{da}{dx}\right\};$$

and thus the law of distribution of velocities is determined.

The electric current i is determined by the equation

$$i = e\iiint uf(u, v, w)\,du\,dv\,dw,$$

where the integration is extended over all possible values of the components of velocity of the electrons. The Maxwellian term in $f(u, v, w)$ furnishes no contribution to this integral, so we have

$$i = e\iiint u^2\chi(r)\,du\,dv\,dw.$$

When the integration is performed, this formula becomes

$$i = \frac{2le}{3\pi^{\frac{1}{2}}}\left(\frac{2eNE}{ma} - \alpha\frac{dN}{dx} - N\frac{d\alpha}{dx}\right),$$

or

$$E = \frac{3\pi^{\frac{1}{2}}m}{4le^2}\frac{\alpha}{N}i + \frac{m}{2e}\left(\frac{a^2}{N}\frac{dN}{dx} + \alpha\frac{da}{dx}\right).$$

The coefficient of i in this equation must evidently represent the ohmic specific resistance of the metal; so if γ denote the specific conductivity, we have

$$\gamma = \frac{4le^2}{3\pi^{\frac{1}{2}}m}\frac{N}{\alpha}.$$

Let the equation be next applied to the case of two metals A and B in contact at the same temperature T, forming an open circuit in which there is no conduction of heat or electricity (so that i and da/dx are zero). Integrating the equation

$$E = \frac{m}{2e}\frac{a^2}{N}\frac{dN}{dx}$$

across the junction of the metals, we have

$$\text{Discontinuity of potential at junction} = \frac{ma^2}{2e}\log\frac{N_B}{N_A};$$

or since $\frac{3}{4}ma^2$, which represents the average kinetic energy of an electron, is by Drude's assumption equal to qT, where q denotes a universal constant, we have

$$\text{Discontinuity of potential at junction} = \frac{2}{3}\frac{q}{e}\,T\log\frac{N_B}{N_A}.$$

This may be interpreted as the difference of potential connected with the Peltier* effect at the junction of two metals; the product of the difference of potential and the current measures the evolution of heat at the junction. The Peltier discontinuity of potential is of the order of a thousandth of a volt, and must be distinguished from Volta's contact-difference of potential, which is generally much larger, and which, as it presumably depends on the relation of the metals to the medium in which they are immersed, is beyond the scope of the present investigation.

Returning to the general equations, we observe that the flux of energy W is parallel to the axis of x, and is given by the equation

$$W = \tfrac{1}{2}m\iiint ur^2 f(u, v, w)\,du\,dv\,dw,$$

where the integration is again extended over all possible values of the components of velocity; performing the integration, we have

$$W = \frac{2ml}{3\pi^{\frac{1}{2}}}\left(\frac{2ea}{m}NE - a^3\frac{dN}{dx} - 3Na^2\frac{da}{dx}\right);$$

or, substituting for E from the equation already found,

$$W = \frac{ma^2}{e}i - \frac{4ml}{3\pi^{\frac{1}{2}}}Na^2\frac{da}{dx}.$$

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* Cf. p. 264.

Consider now the case in which there is conduction of heat without conduction of electricity. The flux of energy will in this case be given by the equation

$$W = -\kappa \frac{dT}{dx},$$

where κ denotes the thermal conductivity of the metal expressed in suitable units; or

$$W = -\kappa . \frac{3ma}{2q} \frac{da}{dx}.$$

If it be assumed that the conduction of heat in metals is effected by motion of the electrons, this expression may be compared with the preceding; thus we have

$$\kappa = \tfrac{8}{9} \pi^{-\frac{1}{2}} l a q N;$$

and comparing this with the formula already found for the electric conductivity, we have

$$\frac{\kappa}{\gamma} = \tfrac{8}{9} T \left(\frac{q}{e}\right)^2,$$

an equation which shows that the ratio of the thermal to the electric conductivity is of the form $T \times$ a constant which is the same for all metals. This result accords with the law of Wiedemann and Franz.

Moreover, the value of q is known from the kinetic theory of gases; and the value of e has been determined by J. J. Thomson* and his followers; substituting these values in the formula for κ/γ, a fair agreement is obtained with the values of κ/γ determined experimentally.

It was remarked by J. J. Thomson that if, as is postulated in the above theory, a metal contains a great number of free electrons in temperature equilibrium with the atoms, the specific heat of the metal must depend largely on the energy required in order to raise the temperature of the electrons. Thomson considered that the observed specific heats of metals are smaller than is compatible with the theory, and was thus led to investigate* the consequences of his original hypothesis† regarding the motion of the electrons, which differs from the one just described in much the same way as Grothuss' theory of electrolysis differs from Arrhenius'. Each electron was now supposed to be free only for a very short time, from the moment when it is liberated by the dissociation of an atom to the moment when it collides with, and is absorbed by, a different atom. The atoms were conceived to be paired in doublets, one pole of each doublet being negatively, and the other positively, electrified. Under the influence of an external electric field the doublets orient themselves parallel to the electric force, and the electrons which are ejected from their negative poles give rise to a current predominantly in this direction. The electric conductivity of the metal may thus be calculated. In order to comprise the conduction of heat in his theory, Thomson assumed that the kinetic energy with which an electron leaves an atom is proportional to the absolute temperature; so that if one part of the metal is hotter than another, the temperature will be equalized by the interchange of corpuscles. This theory, like the other, leads to a rational explanation of the law of Wiedemann and Franz.

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* Cf. p. 407.

The theory of electrons in metals has received support from the study of another phenomenon. It was known to the philosophers of the eighteenth century that the air near an incandescent metal acquires the power of conducting electricity. "Let the end of a poker," wrote Canton,‡ "when red-hot, be brought but for a moment within three or four inches of a small electrified body, and its electrical power will be almost, if not entirely, destroyed."

The subject continued to attract attention at intervals§;

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* J. J. Thomson, *The Corpuscular Theory of Matter*; London, 1907.

† Cf. p. 457.

‡ Phil. Trans. lii (1762), p. 457.

§ Cf. E. Becquerel, Annales de Chimie xxxix (1853), p. 355; Guthrie, Phil. Mag. xlvi (1873), p. 254; also various memoirs by Elster and Geitel in the Annalen d. Phys. from 1882 onwards. The phenomenon is very noticeable, as Edison showed (Engineering, December 12, 1884, p. 553), when a filament of carbon is heated to incandescence in a rarefied gas. In recent years it has been found that ions are emitted when magnesia, or any of the oxides of the alkaline earth metals, is heated to a dull red heat.

and as the process of conduction in gases came to be better understood, the conductivity produced in the neighbourhood of incandescent metals was attributed to the emission of electrically charged particles by the metals. But it was not until the development of J. J. Thomson's theory of ionization in gases that notable advances were made. In 1899, Thomson* determined the ratio of the charge to the mass of the resinously charged ions emitted by a hot filament of carbon in rarefied hydrogen, by observing their deflexion in a magnetic field. The value obtained for the ratio was nearly the same as that which he had found for the corpuscles of cathode rays; whence he concluded that the negative ions emitted by the hot carbon were negative electrons.

The corresponding investigation† for the positive leak from hot bodies yielded the information that the mass of the positive ions is of the same order of magnitude as the mass of material atoms. There are reasons for believing that these ions are produced from gas which has been absorbed by the superficial layer of the metal.‡

If, when a hot metal is emitting ions in a rarefied gas, an electromotive force be established between the metal and a neighbouring electrode, either the positive or the negative ions are urged towards the electrode by the electric field, and a current is thus transmitted through the intervening space. When the metal is at a higher potential than the electrode, the current is carried by the vitreously charged ions: when the electrode is at the higher potential, by those with resinous charges. In either case, it is found that when the electromotive force is increased indefinitely, the current does not increase indefinitely likewise, but acquires a certain "saturation" value. The obvious explanation of this is that the supply of ions available for carrying the current is limited.

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* Phil. Mag. xlviii (1899), p. 547.

† J. J. Thomson, Proc. Camb. Phil. Soc. xv (1909), p. 64; O. W. Richardson, Phil. Mag. xvi. (1908), p. 740.

‡ Cf. Richardson, Phil. Trans. ccvii (1906), p. 1.

When the temperature of the metal is high, the ions emitted are mainly negative; and it is found* that in these circumstances, when the surrounding gas is rarefied, the saturation-current is almost independent of the nature of the gas or of its pressure. The leak of resinous electricity from a metallic surface in a rarefied gas must therefore depend only on the temperature and on the nature of the metal; and it was shown by O. W. Richardson† that the dependence on the temperature may be expressed by an equation of the form

$$i = AT^{\frac{1}{2}}e^{-\frac{b}{T}},$$

where i denotes the saturation-current per unit area of surface (which is proportional to the number of ions emitted in unit time), T denotes the absolute temperature, and A and b are constants.‡

In order to account for these phenomena, Richardson§ adopted the hypothesis which had previously been proposed‖ for the explanation of metallic conductivity; namely, that a metal is to be regarded as a sponge-like structure of comparatively large fixed positive ions and molecules, in the interstices of which negative electrons are in rapid motion. Since the electrons do not all escape freely at the surface, he postulated a superficial discontinuity of potential, sufficient to restrain most of them. Thus, let N denote the number of free electrons in unit volume of the metal; then in a parallelepiped whose height measured at right angles to the surface is dx, and whose base is of unit area, the number of electrons whose

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* Cf. J. A. McClelland, Proc. Camb. Phil. Soc. x (1899), p. 241; xi (1901), p. 296. On the results obtained when the gas is hydrogen, cf. H. A. Wilson, Phil. Trans. ccii (1903), p. 243; ccviii (1908), p. 247; and O. W. Richardson, Phil. Trans. ccvii (1906), p. 1.

† Proc. Camb. Phil. Soc. xi (1902), p. 286; Phil. Trans. cci (1903), p. 497. Cf. also H. A. Wilson, Phil. Trans. ccii (1903), p. 243.

‡ The same law applies to the emission from other bodies, e.g. heated alkaline earths, and to the emission of positive ions—at any rate when a steady state of emission has been reached in a gas which is at a definite pressure.

§ Phil. Trans. cci (1903), p. 497.

‖ Cf. pp. 457 *et sqq.*

x-components of velocity are comprised between u and $u + du$ is

$$\pi^{-\frac{1}{2}}a^{-1}Ne^{-\frac{u^2}{\alpha^2}}du\,dx, \quad \text{where} \quad \tfrac{3}{4}m\alpha^2 = qT,$$

m denoting the mass of an electron, T the absolute temperature, and q the universal constant previously introduced.

Now, an electron whose x-component of velocity is u will arrive at the interface within an interval dt of time, provided that at the beginning of this interval it is within a distance $u\,dt$ of the interface. So the number of electrons whose x-components of velocity are comprised between u and $u + du$ which arrive at unit area of the interface in the interval dt is

$$\pi^{-\frac{1}{2}}a^{-1}Ne^{-\frac{u^2}{\alpha^2}}u\,du\,dt.$$

If the work which an electron must perform in order to escape through the surface layer be denoted by ϕ, the number of electrons emitted by unit area of metal in unit time is therefore

$$\int_{\frac{1}{2}mu^2=\phi}^{\infty}\pi^{-\frac{1}{2}}a^{-1}Ne^{-\frac{u^2}{\alpha^2}}u\,du, \quad \text{or} \quad \tfrac{1}{2}\pi^{-\frac{1}{2}}Nae^{-\frac{2\phi}{m\alpha^2}}.$$

The current issuing from unit area of the hot metal is thus

$$\tfrac{1}{2}\pi^{-\frac{1}{2}}N\epsilon ae^{-\frac{2\phi}{m\alpha^2}}, \quad \text{or} \quad N\epsilon\,.\,(qT/3\pi m)^{\frac{1}{2}}e^{-\frac{3\phi}{2qT}},$$

where ϵ denotes the charge on an electron. This expression, being of the form

$$AT^{\frac{1}{2}}e^{-\frac{b}{T}},$$

agrees with the experimental measures; and the comparison furnishes the value of the superficial discontinuity of potential which is implied in the existence of ϕ.*

A few years after the date of this investigation, a plan was

*** This discontinuity of potential was found to be 2·45 volts for sodium, 4·1 volts for platinum, and 6·1 volts for carbon.**

devised and successfully carried out* for determining experimentally the kinetic energy possessed by the ions after emission. The mean kinetic energy of both negative and positive ions was found to be the same for various metals (platinum, gold, silver, etc.), and to be directly proportional to the absolute temperature; and the distribution of velocities among the ions proved to be that expressed by Maxwell's law. The ions may therefore be regarded as kinetically equivalent to the molecules of a gas whose temperature is the same as that of the metal.

By the investigations which have been recorded, the hypothesis of atomic electric charges has been, to all appearances, decisively established. But all the parts of the theory of electrons do not enjoy an equal degree of security; and in particular, it is possible that the future may bring important changes in the conception of the aether. The hope was formerly entertained of discovering an aether by reference to which motion might be estimated absolutely; but such a hope has been destroyed by the researches which have sprung from FitzGerald's hypothesis of contraction; and in some recent writings it is possible to recognize a tendency to replace the classical aether by other conceptions, which, however, have been as yet but indistinctly outlined.

In any event, the close of the nineteenth century brought to an end a well-marked era in the history of natural philosophy; and this is true not only with respect to the discoveries themselves, but also in regard to the conditions of scientific organization and endeavour, which in the last decades of that period became profoundly changed. The investigators who advanced the theories of aether and electricity, from the time of Descartes to that of Lord Kelvin, were, with very few exceptions, congregated within a narrow territory: from Dublin to the western provinces of Russia, and from Stockholm to the north of Italy, may be circumscribed by a circle of no more than six hundred miles radius. But throughout the whole of Kelvin's long life, the domain of culture was rapidly extending: the learning of the Germanic and Latin peoples was carried to the furthest regions of the earth: new universities were founded, and inquiries into the secrets of nature were instituted in every quarter of the globe. Let this record close with the anticipation that fellowship in the pursuit of knowledge will increase in the nations the spirit of generous emulation and mutual respect.

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* O. W. Richardson and F. C. Brown, Phil. Mag. xvi (1908), pp. 353, 890; F. C. Brown, Phil. Mag. xvii (1909), p. 355; xviii (1909), p. 649

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